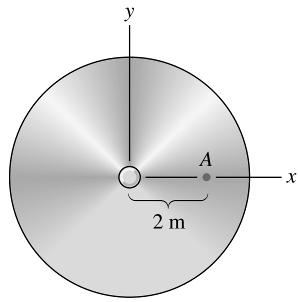
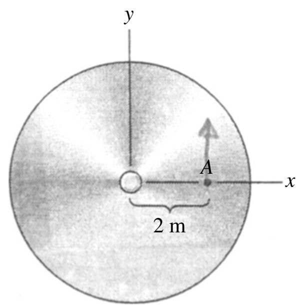
Problem 17.1 The disk rotates relative to the coordinate system about a fixed shaft that is coincident with the z axis. At the instant shown, the disk has a counterclockwise angular velocity of 3 rad/s and a counterclockwise angular acceleration of 4 rad/s^2. What are the x and y components of the velocity and acceleration of point A?



Strategy: Use Eqs. (17.2) and (17.3) and Fig. 17.8 to determine the velocity and acceleration of point A.

Solution:



$$v = r\omega$$

$$= (2)(3) = 6 \text{ m/s}$$

$$\underline{v_x = 0,\, v_y = 6 \text{ m/s}}$$

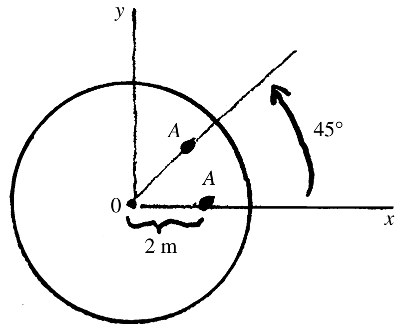
$$\underline{a_t = a_y = r\alpha = (2)(4) \text{ m/s}^2 = 8 \text{ m/s}^2}$$

$$a_n = -a_\alpha = r\omega^2 = (2)(3)^2 = 18 \text{ m/s}^2$$

$$\underline{a_x = -18 \text{ m/s}^2}$$

Problem 17.2 If the angular acceleration of the disk in Problem 17.1 is constant, what are the x and y components of the velocity and acceleration of point A when the disk has rotated 45° relative to the position shown?

Solution:



$\alpha = 4 \text{ rad/s}^2 = \text{constant}$

$\omega_0 = 3 \text{ rad/s}, \theta_0 = 0, r = 2 \text{ m}$

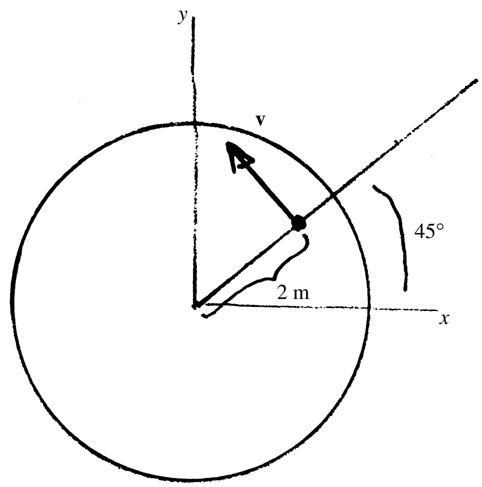
$$\alpha = \frac{d\omega}{dt}$$

$\omega = \omega_0 + \alpha t = 3 + 4t \text{ rad/s}$

$$\omega = \frac{d\theta}{dt}$$

$\theta = \theta_0 + \omega_0 t + \alpha t^2/2 \text{ rad}$

$\theta = 0 + 3t + 4t^2/2 \text{ rad}$



Set $\theta = \pi/4$ and solve for t, $t = 0.227$ s. Find ω at this time. $\omega = 3.91$ rad/s

$v = r\omega, \quad a_t = r\alpha, \quad a_n = r^2/r$

Solving,

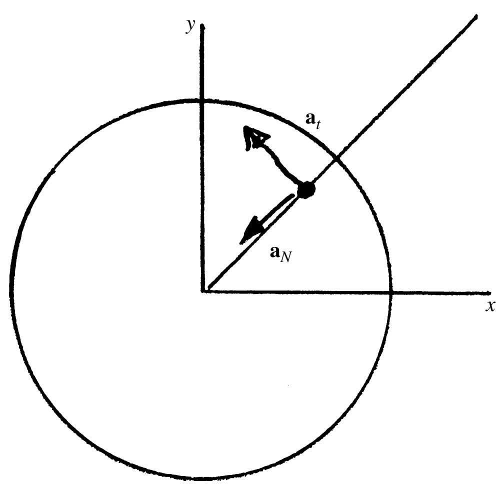
$v = 7.82 \text{ m/s}, \quad a_t = 8 \text{ m/s}^2, \quad a_n = 30.57 \text{ m/s}^2$

We now need to write the xy components of velocity and accelerations.:

$\mathbf{v} = -v \sin 45°\mathbf{i} + v \cos 45°\mathbf{j}$

$\underline{v_x = -5.53 \text{ m/s}}$

$\underline{v_y = 5.53 \text{ m/s}}$



$\mathbf{a}_t = -a_t \sin 45°\mathbf{i} + a_t \cos 45°\mathbf{j}$

$\mathbf{a}_N = -a_N \cos 45°\mathbf{i} - a_N \sin 45°\mathbf{j}$

$a_x = -a_t \sin 45° - a_N \cos 45°$

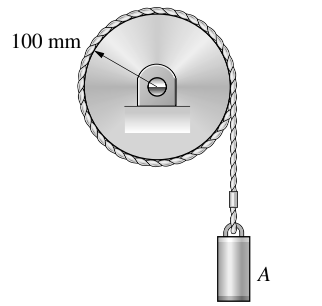
$a_y = +a_t \cos 45° - a_N \sin 45°$

$a_x = -27.3 \text{ m/s}^2$

$a_y = -16.0 \text{ m/s}^2$

Problem 17.3 The weight A starts from rest at $t = 0$ and falls with a constant acceleration of 2 m/s^2.

(a) What is the magnitude of the disk's angular velocity at $t = 1$ s?
(b) What are the magnitudes of the velocity and acceleration of a point at the outer edge of the disk at $t = 1$ s?



Solution:

$a_y = -2$ m/s^2 +

$v_y = -2\ t$ m/s

At $t = 1$ s, $v_y = -2$ m/s

$v_y = r\omega$

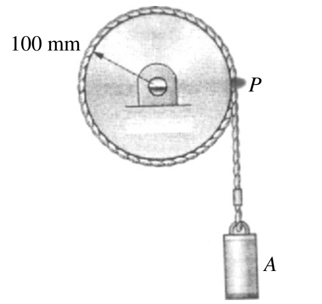
-2 m/s $= (0.1)\omega$

(a) $\underline{\omega = -20 \text{ rad/s (clockwise)}}$

(b) $\underline{|\mathbf{v}| = r\omega = (0.1)(20) = 2 \text{ m/s}}$

Use Point P — $|\mathbf{a}_t| = r\alpha = 2$ m/s^2

$|\mathbf{a}_N| = \omega^2 r = (20)^2(0.1) = 40$ m/s^2



$|\mathbf{a}| = \sqrt{a_t^2 + a_N^2} = \sqrt{1600 + 4}$

$\underline{|\mathbf{a}| = 40.0 \text{ m/s}^2}$

Problem 17.4 The weight A shown in Problem 17.3 is released from rest and falls with constant acceleration a_A. An accelerometer attached to the disk at 50 mm from the center measures a total acceleration of 12 m/s^2 magnitude at $t = 0.5$ s. What is a_A?

Solution:

$R = 0.1$ m

$r = 0.05$ m

a_y constant (negative)

$v_y = a_y t$ **(1)**

$v_y = R\omega$ **(2)**

$a_y = R\alpha$ **(3)**

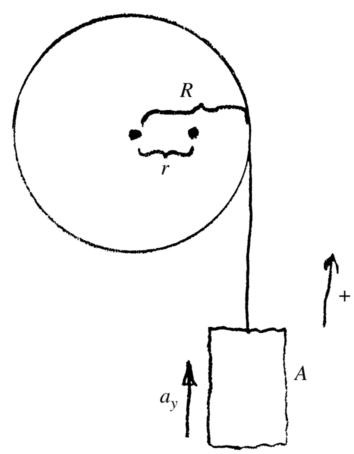
For accelerometer

$a'_y = r\alpha$ **(4)**

$a'_x = \omega^2 r$ **(5)**

$|\mathbf{a}'| = \sqrt{(r\alpha)^2 + (\omega^2 r)^2}$ **(6),**

$|\mathbf{a}| = 12$ m/s^2



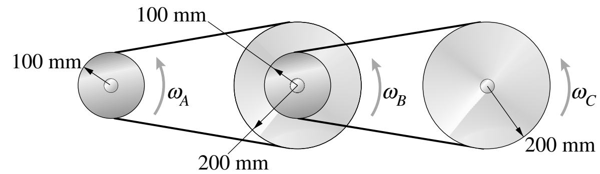
We have 6 eqns in the 6 unknowns v_y, a_y, ω, α, a'_x, a'_y
Solving, we get

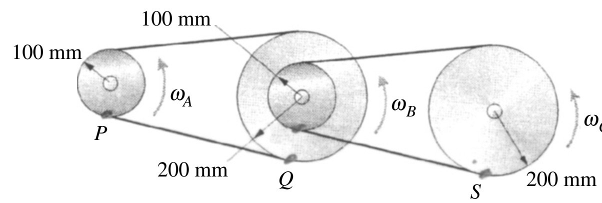
$\underline{a_y = -3.09 \text{ m/s}^2}$, $\omega = -15.43$ rad/s

$a'_x = 11.9$ m/s^2 $a'_y = -1.54$ m/s^2 $\underline{\alpha = -30.86 \text{ rad/s}^2}$

Problem 17.5 The angular velocity ω_A is zero at $t = 0$ and increases at a constant rate until $\omega_A = 100$ rad/s at $t = 5$ s.

(a) What are the angular velocities ω_B and ω_C at $t = 5$ s?
(b) Through what angle does the right disk turn from $t = 0$ to $t = 5$ s?





Solution:

$\alpha_A =$ constant

$\omega_A = \alpha_A t,$

At $t = 5$ s, $\omega_A = 100$ rad/s

$\alpha_A = 20$ rad/s^2

$\omega_A = 20\ t$ rad/s

The key is that the velocity magnitudes at all points on any one belt are the same.

(a) $v_P = r_A\omega_A \qquad r_A = 0.1$ m

$v_Q = v_P = R_B\omega_B \qquad r_B = 0.1$ m

$v_R = r_B\omega_B \qquad R_B = 0.2$ m

$v_S = v_R = r_C\omega_C \qquad r_C = 0.2$ m

$\omega_A = 100$ rad/s at $t = 5$ s

Solving, we get $\underline{\omega_B = 50 \text{ rad/s}, \omega_C = 25 \text{ rad/s}}$

(b) Disc C started at rest and went from $\omega_c = 0$ to $\omega_c = 25$ rad/s in 5 sec

$\alpha_c =$ constant

$\omega_c = \alpha_c\ t$

$\alpha_c = 5$ rad/s^2

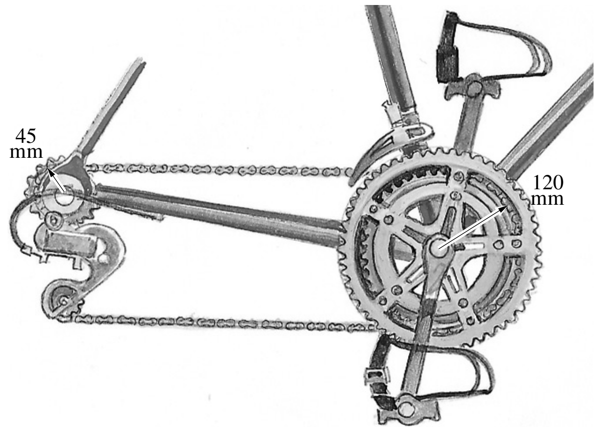
$\omega_c = 5\ t$ rad/c

$\theta_c = 2.5\ t^2$ rad

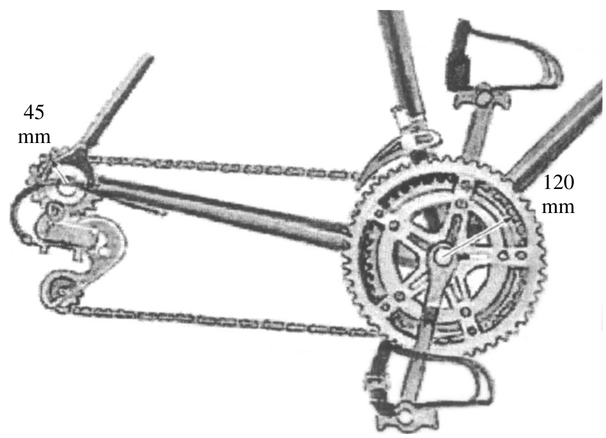
At $t = 5$ s,

$\underline{\theta_c = (2.5)(5)^2 = 62.5 \text{ rad}}$

Problem 17.6 (a) If the bicycle's 120-mm sprocket wheel rotates through one revolution, through how many revolutions does the 45-mm gear turn? (b) If the angular velocity of the sprocket wheel is 1 rad/s, what is the angular velocity of the gear?



Solution: The key is that the tangential accelerations and tangential velocities along the chain are of constant magnitude



(a) $\theta_B = 2.67$ rev

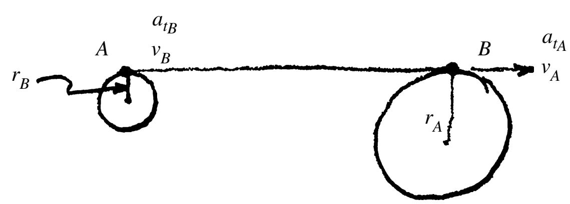
(b) $v_B = r\omega_B \qquad v_A = r_A\omega_A$

$v_A = v_B$

$v_B = (0.045)\omega_B \qquad v_A = (0.120)(1)$

$$\omega_B = \left(\frac{120}{45}\right) \text{ rad/s} = 2.67 \text{ rad/s}$$

$$r_B \frac{d\theta_B}{dt} = r_A \frac{d\theta_A}{dt}$$



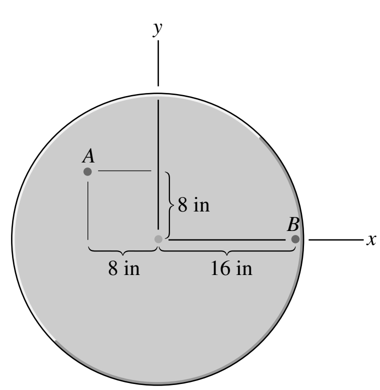
Integrating, we get

$r_B\theta_B = r_A\theta_A \qquad r_A = 0.120$ m

$r_B = 0.045$ m

$$\theta_B = \left(\frac{120}{45}\right)(1) \text{ rev} \qquad \theta_A = 1 \text{ rev.}$$

Problem 17.7 The rear wheel of the bicycle in Problem 17.6 has a 330-mm radius and is rigidly attached to the 45-mm gear. It the rider turns the pedals, which are rigidly attached to the 120-mm sprocket wheel, at one revolution per second, what is the bicycle's velocity?



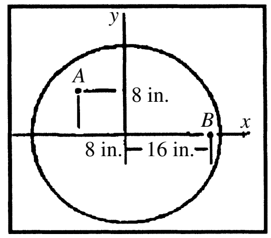
Solution: The angular velocity of the 120 mm sprocket wheel is $\omega = 1$ rev/s $= 2\pi$ rad/s. Use the solution to Problem 17.6. The angular velocity of the 45 mm gear is

$$\omega_{45} = 2\pi\left(\frac{120}{45}\right) = 16.76 \text{ rad/s.}$$

This is also the angular velocity of the rear wheel, from which the velocity of the bicycle is

$$v = \omega_{45}(330) = 5.53 \text{ m/s.}$$

Problem 17.8 Relative to the given coordinate system, the disk rotates about the origin with a constant counterclockwise angular velocity of 10 rad/s. What are the x and y components of the velocity and acceleration of points A and B at the instant shown?

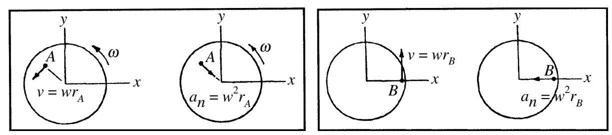


Solution: The radial distances to A and B are

$$r_A = \sqrt{8^2 + 8^2} = 11.3 \text{ in} = 0.943 \text{ ft}$$

$$r_B = 16 \text{ in} = 1.33 \text{ ft}.$$

The velocity and acceleration of A are shown: In terms of x and y components,



$$\mathbf{v}_A = -\omega r_A \cos 45°\mathbf{i} - \omega r_A \cos 45°\mathbf{j}$$

$$= -(10)(0.943)\cos 45°(\mathbf{i} + \mathbf{j}) = -6.67(\mathbf{i} + \mathbf{j}) \text{ ft/s}.$$

$$\mathbf{a}_A = \omega^2 r_A \cos 45°\mathbf{i} - \omega^2 r_A \cos 45°\mathbf{j}$$

$$= (10)^2(0.943)\cos 45°(\mathbf{i} - \mathbf{j}) = 66.7(\mathbf{i} - \mathbf{j}) \text{ ft/s}^2.$$

The velocity and acceleration of B are shown: In terms of x and y components,

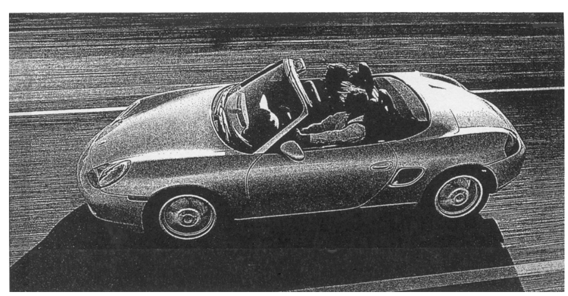
$$\mathbf{v}_B = \omega r_B \mathbf{j} = (10)(1.33)j = 13.3\mathbf{j} \text{ (ft/s)};$$

$$a_B = \omega^2 r_B \mathbf{i} = -(10)^2(1.33)\mathbf{i} = -133\mathbf{i} \text{ (ft/s}^2).$$

Problem 17.9 The car is traveling at a constant speed of 120 km/hr. Its tires are 610 mm in diameter.

(a) What is the angular velocity of the car's wheels?
(b) Relative to a reference frame with its origin at the center of a wheel, what is the magnitude of the acceleration of a point of the wheel at a radial distance of 200 mm from the center of the wheel?

Solution:



120 km/hr = 33.33 m/s

$r = 0.305$ m

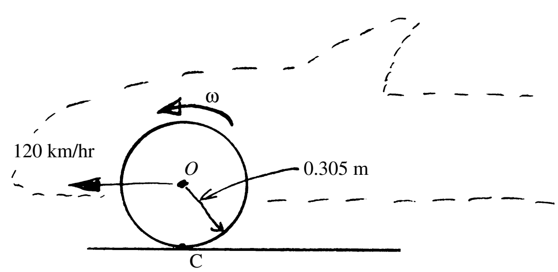
$v = r\omega$

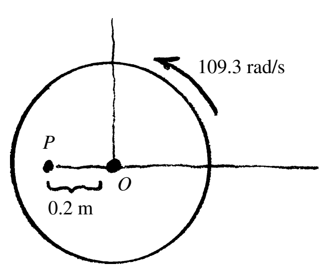
$33.33 = 0.305\omega$

$\underline{\omega = 109 \text{ rad/s}}$

$a_P = \omega^2 r_P = (109.3)^2(0.2) \text{ m/s}^2$

$\underline{a_P = 2390 \text{ m/s}^2}$



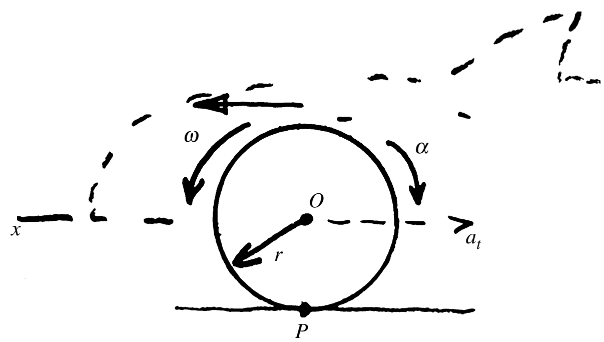


Problem 17.10 The driver of the car in Problem 17.9 applies the brakes, subjecting the car to a constant deceleration of 6 m/s^2.

(a) What is the magnitude of the angular acceleration of the wheels?

(b) At the instant the brakes are applied, what are the magnitudes of the tangential and normal components of acceleration of a point of a wheel at a radial distance of 200 mm from the center of the wheel relative to a reference frame with its origin at the center at the wheel?

Solution:



$v = 33.33$ m/s

$a_t = -6$ m/s^2 to right

$r = 0.305$ m

$v = 33.33$ m/s

$\omega = 109.3$ rad/s

$v = rw$

$a_t = r\alpha$

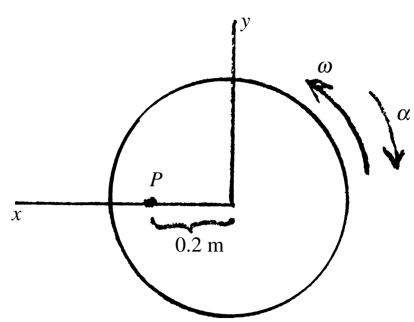
$-6 = (0.305)\alpha \qquad \alpha = -19.7$ rad/s^2

$|\mathbf{a}_{P_t}| = r_{OP} \quad \alpha = 0.2\alpha$

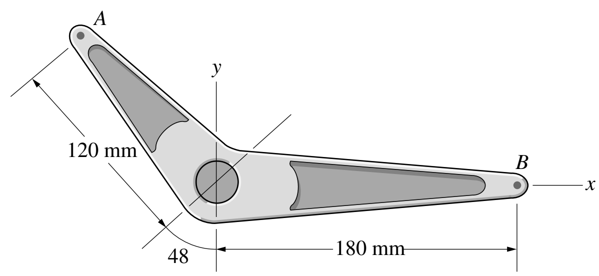
$|\mathbf{a}_{P_t}| = 3.93$ m/s^2

$|\mathbf{a}_{P_N}| = \omega^2 r_{OP}$

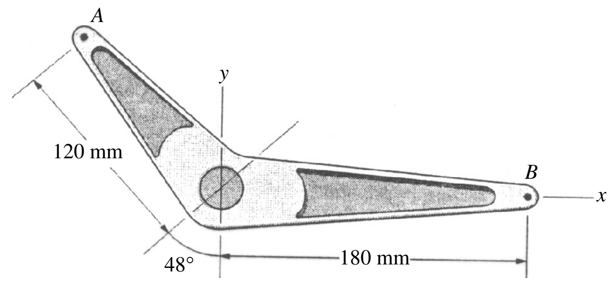
$|\mathbf{a}_{P_N}| = 2390$ m/s^2



Problem 17.11 The bracket rotates relative to the coordinate system about a fixed shaft that is coincident with the z axis. If the device has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 200 rad/s^2, what are the magnitudes of the accelerations of points A and B?



Solution:



Point A:

$\omega = 20$ rad/s, $\qquad \alpha = -200$ rad/s^2

$r_A = 0.120$ m

$a_t = r_A\alpha \qquad a_N = r_A\omega^2$

$a_t = -24$ m/s $\qquad a_N = 48$ m/s

$|\mathbf{a}_A| = \sqrt{a_t^2 + a_N^2} = 53.7$ m/s^2

Point B:

$r_B = 0.180$ m

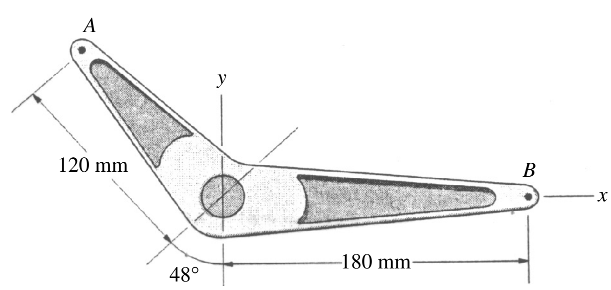
$a_t = r_B\alpha \qquad a_N = r_B\omega^2$

$a_t = -36$ m/s $\qquad a_N = 72$ m/s^2

$|\mathbf{a}_B| = \sqrt{a_t^2 + a_N^2} = 80.5$ m/s^2

Problem 17.12 Consider the rotating bracket shown in Problem 17.11. At the instant shown, $|\mathbf{v}_A| = 2$ m/s and $|\mathbf{a}_A| = 40$ m/s^2. What are $|\mathbf{v}_B|$ and $|\mathbf{a}_B|$?

Solution:



$v_A = r_A\omega$

$z = (0.120)\omega$

$\omega = 16.67$ rad/s (direction unknown)

$|a_A| = 40 = \sqrt{(r_A\alpha)^2 + (\omega^2 r_A)^2}$ (α unknown)

Solving, $\alpha = 184.26$ rad/s^2

Point B:

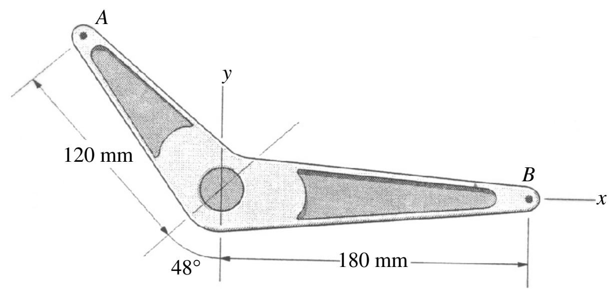
$|\mathbf{v}_B| = r_B\omega = (0.180)(16.67)$ m/s

$|\mathbf{v}_B| = 3$ m/s

$|\mathbf{a}_B| = \sqrt{(r_B\alpha)^2 + (\omega^2 r_B)^2}$

$|\mathbf{a}_B| = 60$ m/s^2

Problem 17.13 Suppose that the rotating bracket shown in Problem 17.11 starts from rest in the position shown and has a constant counterclockwise angular acceleration. When it has rotated 48°, the x component of the velocity of point B is $v_{Bx} = -0.4$ m/s. At that instant, what are the x and y components of the velocity and acceleration of point A?



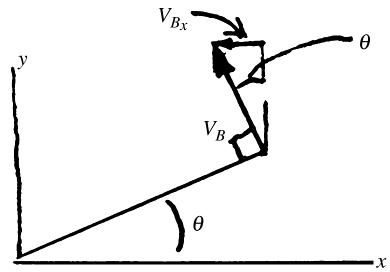
Solution:

$\alpha =$ constant, $\omega_0 = 0$, $\theta_0 = 0$

$\omega = \alpha t$

$\theta = \alpha t^2/2$

when $\theta = 48°$, $V_{B_x} = -0.4$ m/s. From the figure,



$v_{B_x} = -v_B \sin\theta$

$v_B = r_B \omega$

Thus $(-0.4) = -r_B \omega \sin(48°)$ with $r_B = 0.180$ m. Solving, $\omega =$ 2.99 rad/s. (Note $48° = 0.8378$ rad).

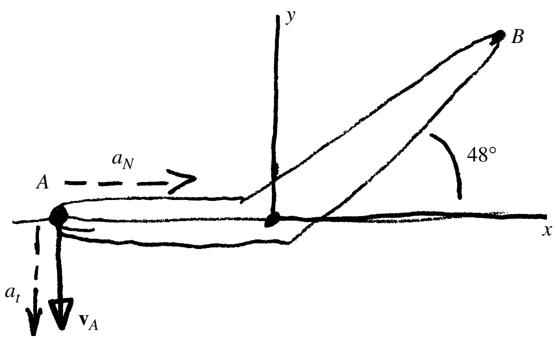
We have $\theta = 0.8378$ rad (48°)

When $\omega = 2.99$ rad/s

$\omega = \alpha t$

$\theta = \alpha t^2/2 = \omega t/2$

Solving for t and α, we get $t = 0.560$ s, $\alpha = 5.34$ rad/s^2



Point A:

$|\mathbf{v}_A| = r_A \omega = (0.12)(2.99) = 0.359$ m/s

$v_{A_x} = 0$

$\underline{v_{A_y} = -0.359 \text{ m/s}}$

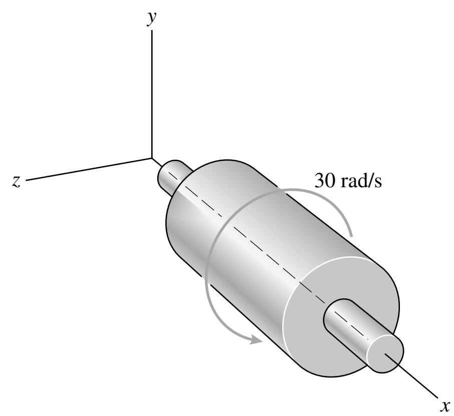
$a_{A_T} = r_A \alpha = (0.12)(5.34) = 0.641$ m/s^2

$a_y = -a_t = -0.641$ m/s^2

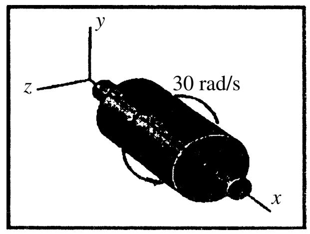
$\mathbf{a}_N = r_A \omega^2 = (0.12)(2.99) = 1.073$ m/s^2

$a_x = a_N = 1.073$ m/s^2

Problem 17.14 The turbine rotates relative to the coordinate system at 30 rad/s about a fixed axis coincident with the x axis. What is angular velocity vector?

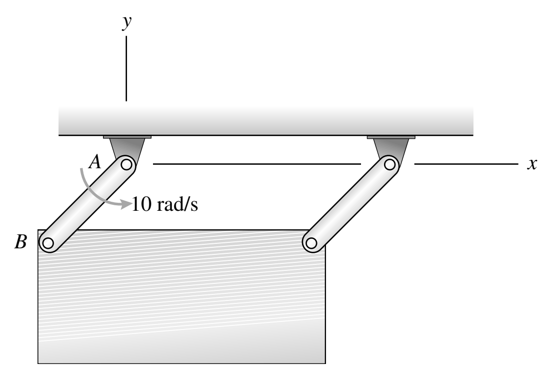


Solution: The angular velocity vector is parallel to the x axis, with magnitude 30 rad/s. By the right hand rule, the positive direction coincides with the positive direction of the x axis.

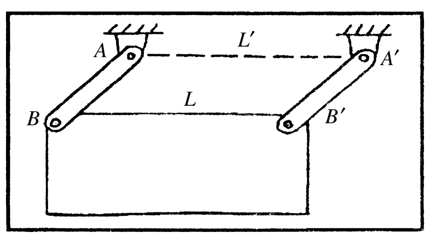


$\boxed{\boldsymbol{\omega} = 30\mathbf{i}}$ (rad/s).

Problem 17.15 The rectangular plate swings in the x-y plane from arms of equal length. Determine the angular velocity of the (a) the rectangular plate and (b) the bar AB.



Solution: Denote the upper corners of the plate by B and B', and denote the distance between these points (the length of the plate) by L. Denote the suspension points by A and A', the distance separating them by L'. By inspection, since the arms are of equal length, and since $L = L'$, the figure $AA'B'B$ is a parallelogram. By definition, the opposite sides of a parallelogram remain parallel, and since the fixed side AA' does not rotate, then BB' cannot rotate, so that the plate does not rotate and



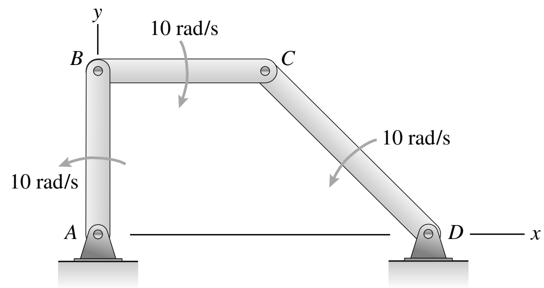
$\boxed{\boldsymbol{\omega}_{BB'} = 0}$.

Similarly, by inspection the angular velocity of the bar AB is

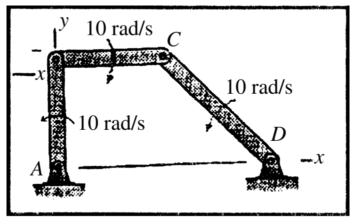
$\boxed{\boldsymbol{\omega}_{AB} = 10\mathbf{k}}$ (rad/s),

where by the right hand rule the direction is along the positive z axis (out of the paper).

Problem 17.16 What are the angular velocity vectors of each bar of the linkage?



Solution: The strategy is to use the definition of angular velocity, including the application of the right hand rule. For bar AB the magnitude is 10 rad/s, and the right hand rule indicates a direction in the positive z direction. (out of the paper).



$\boxed{\boldsymbol{\omega}_{AB} = 10\mathbf{k}\ \text{(rad/s)}}$.

For bar BC,

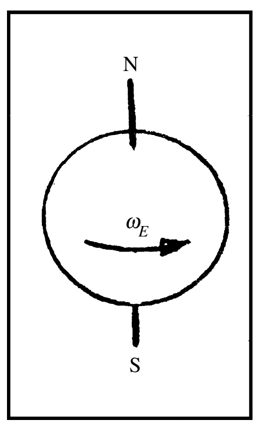
$\boxed{\boldsymbol{\omega}_{BC} = -10\mathbf{k}\ \text{(rad/s)}}$.

For bar CD,

$\boxed{\boldsymbol{\omega}_{CD} = 10\mathbf{k}\ \text{(rad/s)}}$.

Problem 17.17 If you model the earth as a rigid body, what is the magnitude of its angular velocity vector $\boldsymbol{\omega}_E$? Does $\boldsymbol{\omega}_E$ point north or south?

Solution: With respect to an inertial reference (the "fixed" stars) the earth rotates 2π radians per day relative to the sun plus one revolution per year relative to the fixed direction. Take the length of the year to be 365.242 days. The revolutions per day is



$$\text{rpd} = 1 + \frac{1}{\text{year}} = 1.00274 \text{ rev/day}$$

$$= 6.300 \text{ rad/day}$$

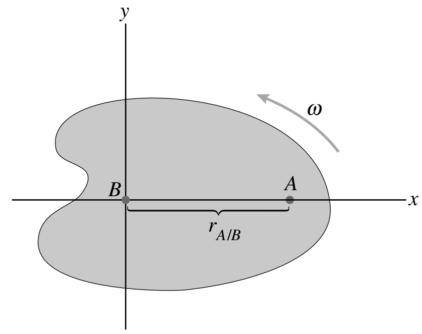
The magnitude of the angular velocity is

$\boxed{\omega_E = 6.300 \text{ rad/day} = 7.292 \times 10^{-5} \text{ rad/s}}$.

The earth turns to the east, or counterclockwise when viewing it from the north pole. From the right hand rule, the *angular velocity vector points north*.

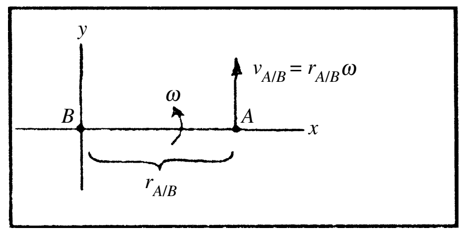
[*Note*: If the earth's orbital motion around the sun is ignored, then revolution per day = 1, and $\omega_E = 7.27 \times 10^{-5}$ rad/s.]

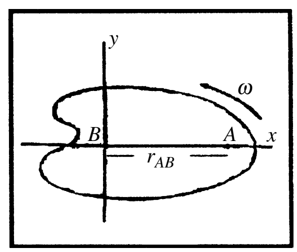
Problem 17.18 The rigid body rotates with a counter-clockwise angular velocity ω about a fixed axis through B that is coincident with the z axis. Determine the x and y components of the velocity of A relative to B by representing that velocity as shown in Fig. 17.11b.



Solution: From the Figure, we see that

$\mathbf{v}_{A/B} = r_{A/B}\omega\mathbf{j}$.

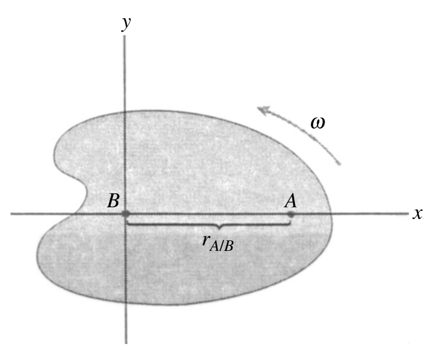




Problem 17.19 Consider the rotating rigid body in Problem 17.18. The angular velocity $\omega = 4$ rad/s and the distance $r_{A/B} = 0.6$ m.

(a) What is the rigid body's angular velocity vector?
(b) Use Eq. (17.5) to determine the velocity of A relative to B.

Solution:



$\mathbf{v}_B = 0$

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \qquad \therefore \mathbf{v}_A = \mathbf{v}_{A/B}$

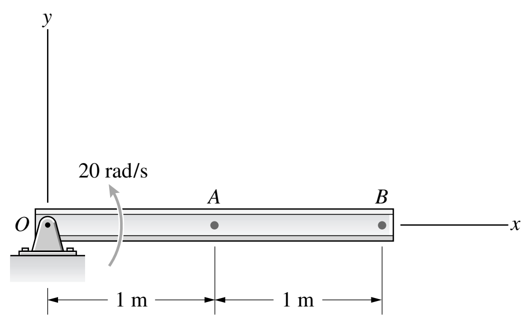
$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}_{A/B}$

(a) $\boldsymbol{\omega} = \omega\mathbf{k} = 4\mathbf{k}$

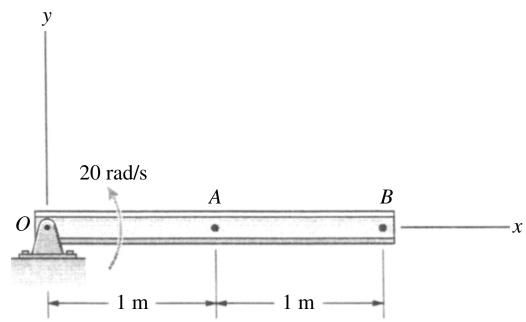
(b) $\mathbf{v}_{A/B} = 4\mathbf{k} \times 0.6\mathbf{i}$

$\mathbf{v}_{A/B} = 2.4\mathbf{j}$ (m/s).

Problem 17.20 The bar is rotating about the fixed point O with a counterclockwise angular velocity of 20 rad/s. By using Eq. (17.5), determine (a) the velocity of A relative to B and (b) the velocity of B relative to A.



Solution:



$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}_{A/B} = 20\mathbf{k} \times (-1\mathbf{i})$

$\mathbf{v}_{A/B} = -20\mathbf{j}$ (m/s)

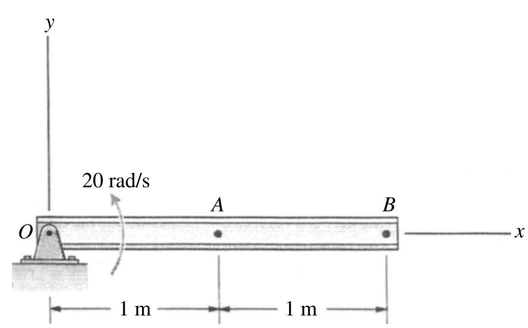
$\mathbf{v}_{B/A} = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 20\mathbf{k} \times (1\mathbf{i})$

$\underline{\mathbf{v}_{B/A} = 20\mathbf{j} \text{ (m/s)}}$.

Problem 17.21 Consider the rotating bar in Problem 17.20.

(a) By applying Eq. (17.6) to point A and the fixed point O, determine the velocity of A. (that is, the velocity of A relative to the fixed, nonrotating reference frame.)
(b) By using the result of part (a) and applying Eq. (17.6) to points A and B, determine the velocity of B.

Solution:



$\mathbf{v}_0 = 0 \qquad \boldsymbol{\omega} = 20\mathbf{k}$ rad/s

$\mathbf{v}_A = \mathbf{v}_0 + \mathbf{v}_{A/O} \quad = \boldsymbol{\omega} \times \mathbf{r}_{A/O} = 20\mathbf{k} \times (1\mathbf{i})$

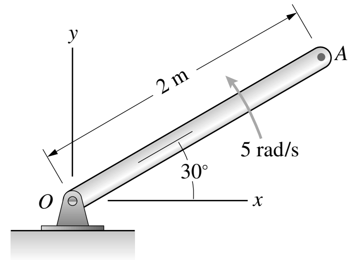
$\underline{\mathbf{v}_A = 20\mathbf{j} \text{ (m/s)}}$.

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

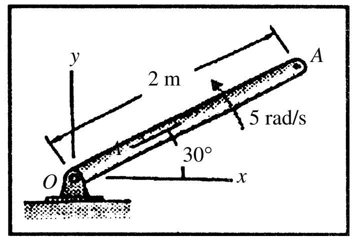
$\quad = 20\mathbf{j} + 20\mathbf{k} \times (1\mathbf{i})$

$\underline{\mathbf{v}_B = 40\mathbf{j} \text{ (m/s)}}$.

Problem 17.22 Determine the x and y components of the velocity of point A.



Solution: The velocity of point A is given by:



$$\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O}.$$

Hence, $$\mathbf{v}_A = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 2\cos 30^\circ & 2\sin 30^\circ & 0 \end{vmatrix}$$

$$= -10\sin 30^\circ \mathbf{i} + 10\cos 30^\circ \mathbf{j},$$

or $\mathbf{v}_A = -5\mathbf{i} + 8.66\mathbf{j}$ (m/s).

Problem 17.23 If the angular velocity of the bar in Problem 17.22 is constant, what are the x and y components of the velocity of Point A 0.1 s after the instant shown?

Solution: The angular velocity is given by

$$\omega = \frac{d\theta}{dt} = 5 \text{ rad/s},$$

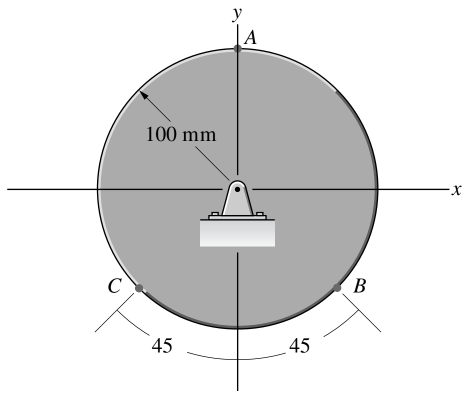
$$\int_0^\theta d\theta = \int_0^t 5\, dt,$$

and $\theta = 5t$ rad.
At $t = 0.1$ s, $\theta = 0.5$ rad $= 28.6^\circ$.

$$\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 2\cos 58.6^\circ & 2\sin 58.6^\circ & 0 \end{vmatrix}.$$

Hence, $\mathbf{v}_A = -10\sin 58.6^\circ \mathbf{i} + 10\cos 58.6^\circ \mathbf{j} = -8.54\mathbf{i} + 5.20\mathbf{j}$ (m/s).

Problem 17.24 The disk is rotating about the z axis at 50 rad/s in the clockwise direction. Determine the x and y components of the velocities of points A, B, and C.

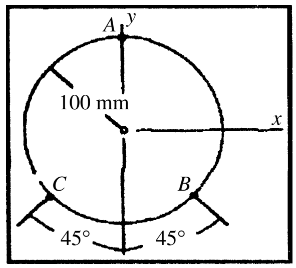


Solution: The velocity of A is given by $\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$, or $\mathbf{v}_A = 0 + (-50\mathbf{k}) \times (0.1\mathbf{j}) = 5\mathbf{i}$ (m/s).

For B, we have

$$\mathbf{v}_B = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{B/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -50 \\ 0.1\cos 45^\circ & -0.1\sin 45^\circ & 0 \end{vmatrix}$$

$$= -3.54\mathbf{i} - 3.54\mathbf{j} \text{ (m/s)},$$



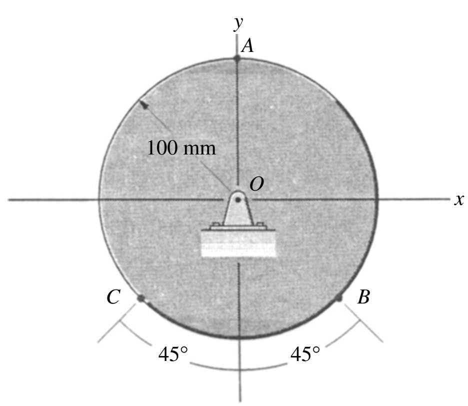
For C, we have

$$\mathbf{v}_c = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{C/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -50 \\ -0.1\cos 45^\circ & -0.1\sin 45^\circ & 0 \end{vmatrix}$$

$$= -3.54\mathbf{i} + 3.54\mathbf{j} \text{ (m/s)}.$$

Problem 17.25 Consider the rotating disk shown in Problem 17.24. If the magnitude of the velocity of point A relative to point B is 4 m/s, what is the magnitude of the disk's angular velocity?

Solution:



$$\mathbf{v}_0 = 0 \quad \boldsymbol{\omega} = \omega\mathbf{k} \quad r = 0.1 \text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_0 + \omega\mathbf{k} \times \mathbf{r}_{OB}$$

$$= \omega\mathbf{k} \times (r\cos 45^\circ\mathbf{i} - r\sin 45^\circ\mathbf{j})$$

$$= (r\omega\cos 45^\circ)\mathbf{j} + (r\omega\sin 45^\circ)\mathbf{i}.$$

$$\mathbf{v}_A = \mathbf{v}_0 + \omega\mathbf{k} \times \mathbf{r}_{OA} = \omega\mathbf{k} \times r\mathbf{j}$$

$$= -r\omega\mathbf{i}.$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B,$$

$$\mathbf{v}_{A/B} = (-r\omega - r\omega\sin 45^\circ)\mathbf{i} - r\omega\cos 45^\circ\mathbf{j}$$

$$= r\omega(-1 - \sin 45^\circ)\mathbf{i} - r\omega\cos 45^\circ\mathbf{j}.$$

We know

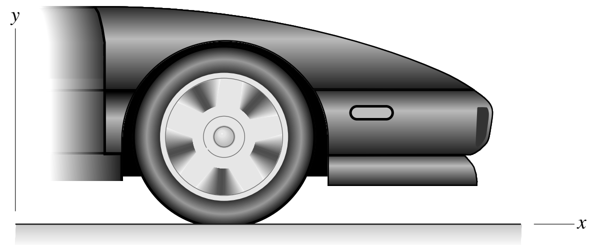
$$|\mathbf{v}_{A/B}| = 4 \text{ m/s}, \quad r = 0.1 \text{ m}$$

$$|\mathbf{v}_{A/B}| = \sqrt{[r\omega(-1 - \sin 45^\circ)]^2 + [-r\omega\cos 45^\circ]^2}$$

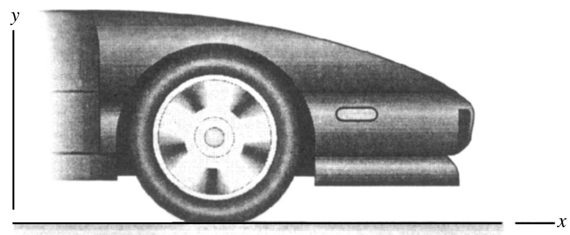
Solving for ω, $\omega = 21.6$ rad/s (direction undetermined).

Problem 17.26 The car is moving to the right at 100 km/hr. The tire is 600 mm in diameter.

(a) What is the tire's angular velocity vector?
(b) Determine the velocity of the center of the wheel.
(c) Determine the velocity of the highest point of the tire.



Solution: For a wheel rolling without slipping, $\mathbf{v}_c \equiv 0$.



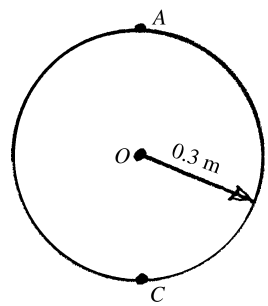
$v_0 = v_{\text{CAR}} = 100$ km/hr

(a) $\underline{\boldsymbol{\omega} = -92.6\mathbf{k} \text{ (rad/s)}}$.

$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \omega\mathbf{k} \times 0.6\mathbf{j}$

$\mathbf{v}_A = -92.6\mathbf{k} \times (0.6\mathbf{j})$

$\underline{\mathbf{v}_A = 55.6\mathbf{i} \text{ (m/s)}}$.



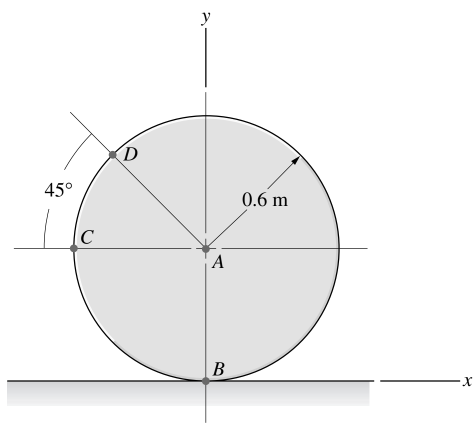
(b) $\underline{\mathbf{v}_0 = 27.8\mathbf{i}}$ (m/s).

$\mathbf{v}_0 = \mathbf{v}_c + \boldsymbol{\omega} \times \mathbf{r}_{O/C} = \omega\mathbf{k} \times 0.3\mathbf{j}$

$27.78\mathbf{i} = -0.3\omega\mathbf{i}$

$\omega = -92.6$ rad/s

Problem 17.27 Point A of the rolling disk is moving to the right. The magnitude of the velocity of point C relative to point B is 8 m/s. What is the velocity of point D?



Solution:

$\mathbf{v}_B = 0$ for rolling (no slip) disk.

$\mathbf{v}_C = \mathbf{v}_B + \omega\mathbf{k} \times \mathbf{r}_{C/B} = \mathbf{v}_{C/B}$

$\mathbf{v}_C = \omega\mathbf{k} \times (-0.6\mathbf{i} + 0.6\mathbf{j})$

$\mathbf{v}_C = -0.6\omega\mathbf{j} - 0.6\omega\mathbf{i}$

Rolling to right — $\therefore \omega < 0$

$|\mathbf{v}_{C/B}| = |\mathbf{v}_C| = 8$ m/s

$|\mathbf{v}_C| = \sqrt{(0.6\omega)^2 + (0.6\omega)^2} = 8$

Solving, we get

$\omega = \pm 9.43$ rad/s

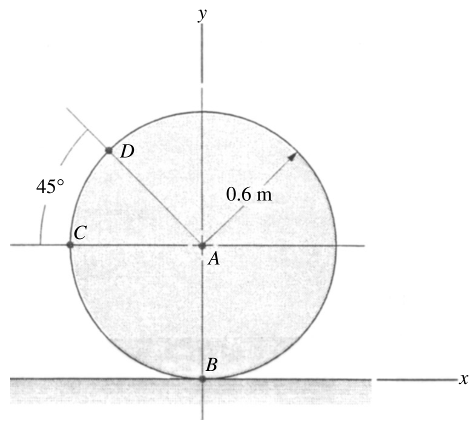
$\boldsymbol{\omega} = -9.43\mathbf{k}$ rad/s

$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{D/B} = 0 + \omega\mathbf{k} \times \mathbf{r}_{D/B}$

From the figure

$\mathbf{r}_{D/B} = -(0.6\cos 45^\circ)\mathbf{i} + 0.6 - 0.6\sin 45^\circ)\mathbf{j}$

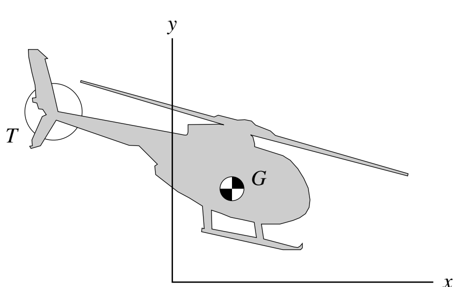
$\mathbf{r}_{D/B} = -0.424\mathbf{i} + 1.024\mathbf{j}$ (m).



$\mathbf{v}_D = -9.43\mathbf{k} \times (-0.424\mathbf{i} + 1.024\mathbf{j})$

$\underline{\mathbf{v}_D = 9.66\mathbf{i} + 4.00\mathbf{j} \text{ (m/s)}}$.

Problem 17.28 The helicopter is in planar motion in the x-y plane. At the instant shown, the position of its center of mass G, is $x = 2$ m, $y = 2.5$ m, and its velocity is $\mathbf{v}_G = 12\mathbf{i} + 4\mathbf{j}$ (m/s). The position of point T, where the tail rotor is mounted, is $x = -3.5$ m, $y = 4.5$ m. The helicopter's angular velocity is 0.2 (rad/s) clockwise. What is the velocity of point T?



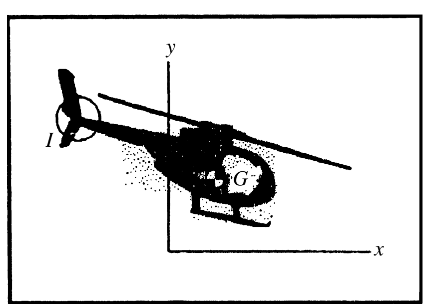
Solution: The position of T relative to G is

$\mathbf{r}_{T/G} = (-3.5 - 2)\mathbf{i} + (4.5 - 2.5)\mathbf{j} = -5.5\mathbf{i} + 2\mathbf{j}$ (m).

The velocity of T is

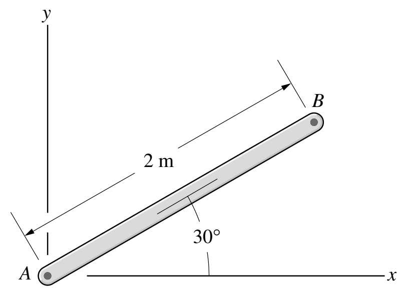
$$\mathbf{v}_T = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{T/G} = 12\mathbf{i} + 4\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.2 \\ -5.5 & 2 & 0 \end{vmatrix}$$

$= 12.4\mathbf{i} + 5.1\mathbf{j}$ (m/s)



Problem 17.29 The bar is in planar motion. The velocity of point A is $\mathbf{v}_A = 4\mathbf{i} - 2\mathbf{j}$ (m/s). The x component of the velocity of point B is -2 m/s.

(a) What is the bar's angular velocity vector?
(b) What is the velocity of point B?



Solution:

(a) $\mathbf{v}_A = 4\mathbf{i} - 2\mathbf{j}$ m/s

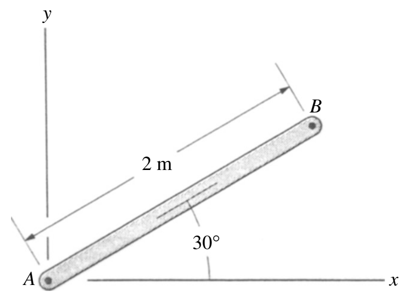
$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$\mathbf{v}_B = 4\mathbf{i} - 2\mathbf{j} + \omega\mathbf{k} \times (2\cos 30\mathbf{i} + \mathbf{j})$

$$\begin{cases} v_{B_x} = 4 - \omega \\ v_{B_y} = -2 + 2\omega\cos 30^\circ \\ . \end{cases}$$

$v_{B_x} = -2$ m/s given

$\therefore \omega = 6$ rad/s $\Rightarrow \underline{\boldsymbol{\omega} = 6\mathbf{k} \text{ (rad/s)}}$.



(b) Now, solving for the components of v_B, we get

$v_{B_x} = -2$ m/s

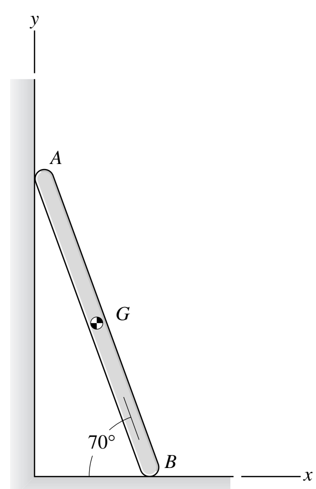
$v_{B_y} = 8.39$ m/s

$\underline{\mathbf{v}_B = -2\mathbf{i} + 8.39\mathbf{j} \text{ (m/s)}}$.

Problem 17.30 Points A and B of the 1-m bar slide on the plane surfaces. The velocity of point B is $2\mathbf{i}$ (m/s).

(a) What is the velocity of point A?
(b) What is the bar's angular velocity vector?

Strategy: Apply Eq. (17.6) to points A and B, and use the fact that A is moving vertically and B is moving horizontally.



Solution:

$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$:

$$v_A\mathbf{j} = 2\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ -(1)\cos 70^\circ & (1)\sin 70^\circ & 0 \end{vmatrix}.$$

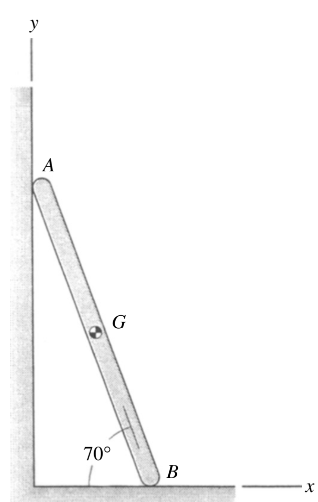
The $\mathbf{i}$ and $\mathbf{j}$ components of this equation are

$0 = 2 - \omega\sin 70^\circ$,

$v_A = -\omega\cos 70^\circ$.

Solving, we obtain $\omega = 2.13$ rad/s and $v_A = -0.728$ m/s, so

(a) $\mathbf{v}_A = -0.728\mathbf{j}$ (m/s),
(b) $\boldsymbol{\omega} = 2.13\mathbf{k}$ (rad/s).



Problem 17.31 In Problem 17.30, what is the velocity of the midpoint G of the bar?

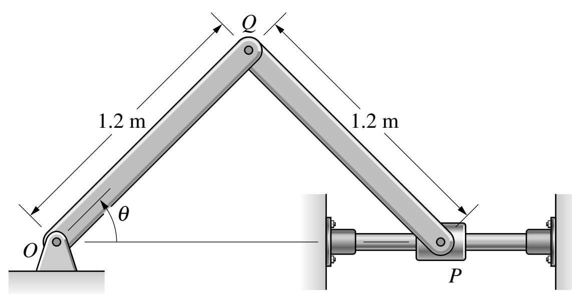
Solution: See the solution of Problem 17.30.

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

$$= 2\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.13 \\ -0.5\cos 70^\circ & 0.5\sin 70^\circ & 0 \end{vmatrix}$$

$$= \mathbf{i} - 0.364\mathbf{j} \text{ (m/s)}.$$

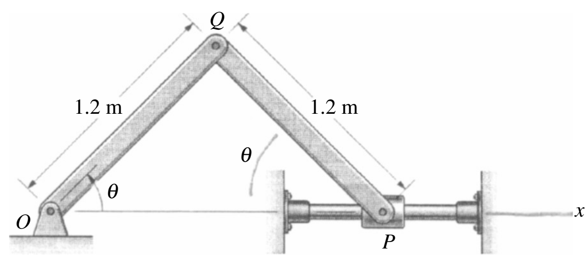
Problem 17.32 If $\theta = 45^\circ$ and the sleeve P is moving to the right at 2 m/s, what are the angular velocities of the bars OQ and PQ?



$L = 1.2$ m

$\theta = 45^\circ$

Solution:
From the figure, $\mathbf{v}_0 = 0$, $\mathbf{v}_P = v_P\mathbf{i} = 2\ \mathbf{i}$ (m/s)



$$\mathbf{v}_Q = \mathbf{v}_0 + \omega_{OQ}\mathbf{k} \times (L\cos\theta\mathbf{i} + L\sin\theta\mathbf{j})$$

$$\begin{cases} \mathbf{i}: & v_{Q_x} = -\omega_{OQ} L \sin\theta \qquad \textbf{(1)} \\ \mathbf{j}: & v_{Q_y} = \omega_{OQ} L \cos\theta \qquad \textbf{(2)} \end{cases}$$

$$\mathbf{v}_P = \mathbf{v}_Q + \omega_{QP}\mathbf{k} \times (L\cos\theta\mathbf{i} - L\sin\theta\mathbf{j})$$

$\mathbf{i}$: $2 = v_{Q_x} + \omega_{QP} L \sin\theta$ **(3)**

$\mathbf{j}$: $0 = v_{Q_y} + \omega_{QP} L \cos\theta$ **(4)**

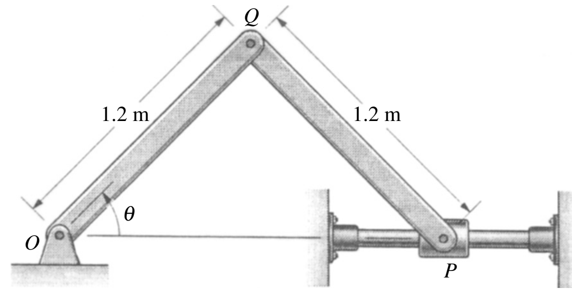
Eqns (1)–(4) are 4 eqns in the 4 unknowns ω_{OQ}, ω_{QP}, v_{Q_x}, v_{Q_y}
Solving, we get

$v_{Q_x} = 1$ m/s, $v_{Q_y} = -1$ m/s

$\underline{\boldsymbol{\omega}_{OQ} = -1.18\mathbf{k}}$ (rad/s),

$\boldsymbol{\omega}_{QP} = 1.18\mathbf{k}$ (rad/s).

Problem 17.33 Consider the system shown in Problem 17.32. If the sleeve P is moving to the right at 2 m/s and bar PQ is rotating counterclockwise at 1 rad/s, what is the angle θ?



Solution:

$\mathbf{v}_0 = 0, \quad \mathbf{v}_P = 2\ \mathbf{i}$ m/s

$$\mathbf{v}_Q = \mathbf{v}_0 + \omega_{OQ}\mathbf{k} \times (L\cos\theta\mathbf{i} + L\sin\theta\mathbf{j})$$

$$\mathbf{v}_P = \mathbf{v}_Q + \omega_{QP}\mathbf{k} \times (L\cos\theta\mathbf{i} - L\sin\theta\mathbf{j})$$

$$\begin{cases} v_{Q_x} = -\omega_{OQ} L\sin\theta \\ v_{Q_y} = \omega_{OQ} L\cos\theta \end{cases}$$

$$\begin{cases} 2 = v_{Q_x} + \omega_{QP} L\sin\theta \\ 0 = v_{Q_y} + \omega_{QP} L\cos\theta \end{cases}$$

$\boldsymbol{\omega}_{PQ} = 1\ \mathbf{k}$ rad/s $\quad \omega_{QP} = 1$ rad/s

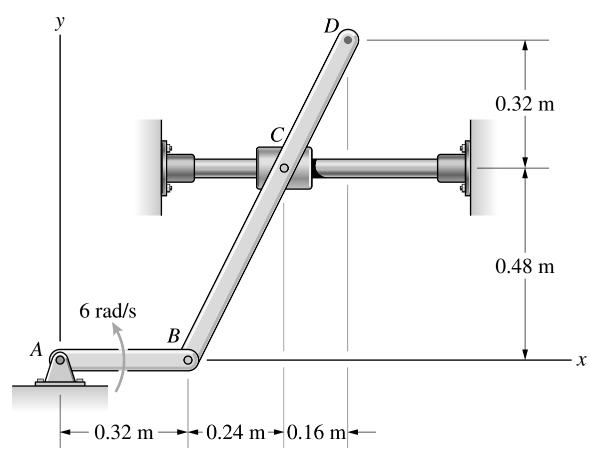
Solving, $v_{Q_x} = 1$ m/s,

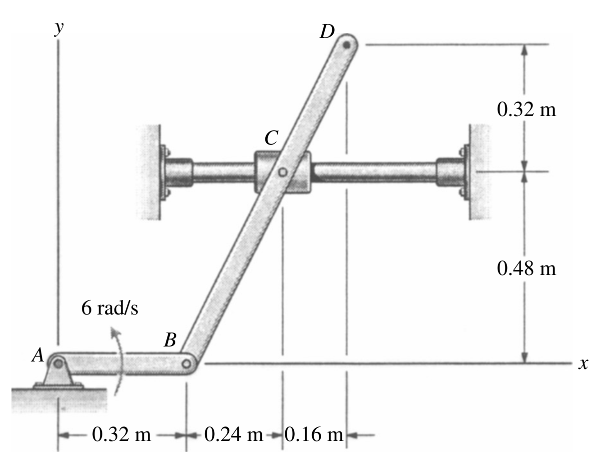
$v_{Q_y} = -0.663$ m/s,

$\omega_{OQ} = -1$ rad/s,

$\underline{\theta = 56.4^\circ}$.

Problem 17.34 Bar AB rotates in the counterclockwise direction at 6 rad/s. Determine the angular velocity of bar BD and the velocity of point D.





Solution:

$\mathbf{v}_A = 0 \quad \boldsymbol{\omega}_{AB} = 6\ \mathbf{k}$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} = 6\mathbf{k} \times 0.32\mathbf{i} = 1.92\mathbf{j} \text{ m/s.}$$

$$\mathbf{v}_C = v_C\mathbf{i} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{C/B} :$$

$$v_c\mathbf{i} = 1.92\mathbf{j} + \omega_{BD}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j}).$$

$$\begin{cases} \mathbf{i}: \quad v_C = -0.48\omega_{BD} \\ \mathbf{j}: \quad 0 \ \ = 1.92 + 0.24\omega_{BD} \end{cases}$$

Solving, $\omega_{BD} = -8$,

$$\underline{\boldsymbol{\omega}_{BD} = -8\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right)},$$

$$\underline{\mathbf{v}_C = 3.84\mathbf{i} \text{ (m/s)}}.$$

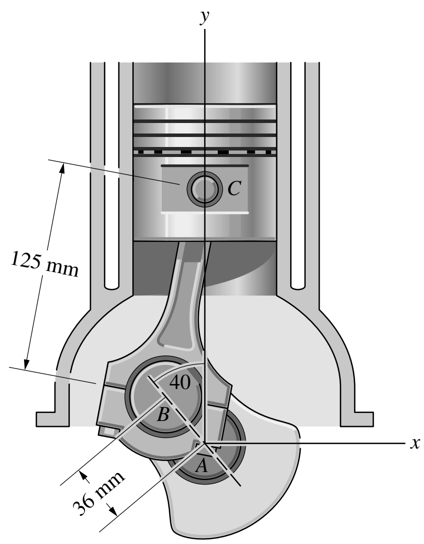
Now for the velocity of point D

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

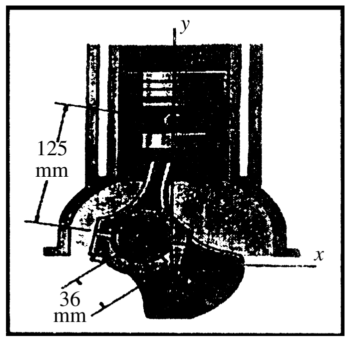
$$= 1.92\mathbf{j} + (-8\mathbf{k}) \times (0.4\mathbf{i} + 0.8\mathbf{j})$$

$$\underline{\mathbf{v}_D = 6.40\mathbf{i} - 1.28\mathbf{j} \text{ (m/s)}}.$$

Problem 17.35 If the crankshaft AB rotates at 6000 rpm in the counterclockwise direction, what is the velocity of the piston at the instant shown?



Solution: The angle between the crank and the vertical is 40°. The piston is constrained to move in parallel to the y-axis. The angular velocity of the crank is:



$$\omega_{AB} = \frac{6000(2\pi)}{60} = 200\pi \text{ rad/s.}$$

The radius vector of the crankshaft is $\boldsymbol{r}_{B/A} = 36(-\mathbf{i}\sin 40^\circ + \mathbf{j}\cos 40^\circ) = -23.1\mathbf{i} + 27.6\mathbf{j}$ (mm). The velocity of the end of the crank is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{B/A} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 200\pi \\ -23.1 & 27.6 & 0 \end{bmatrix},$$

from which $\mathbf{v}_B = -17327.5\mathbf{i} - 14539.5\mathbf{j}$ (mm/s). The angle of the connecting rod with the horizontal is

$$\theta = 90^\circ - \sin^{-1}\left(\frac{36\sin 40^\circ}{125}\right) = 79.33^\circ.$$

The vector distance from B to C is $\boldsymbol{r}_{C/B} = 125(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = 23.1\mathbf{i} + 122.8\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{BC} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 23.1 & 122.8 & 0 \end{bmatrix}$$

$$= \mathbf{v}_B - 122.8\omega_{BC}\mathbf{i} + 23.1\omega_{BC}\mathbf{j}, \text{ from which}$$

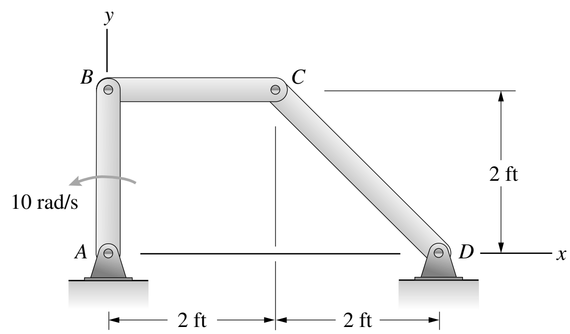
$\mathbf{v}_C = (-17327.5 - \omega_{BC}122.8)\mathbf{i} + (-14539.3 + \omega_{BC}23.1)\mathbf{j}$. From the constraint on the piston's motion, $0 = (-17327.5 - \omega_{BC}122.8)\mathbf{i}$, from which

$$\omega_{BC} = -\frac{17327.5}{122.8} = -141.1 \text{ (rad/s).}$$

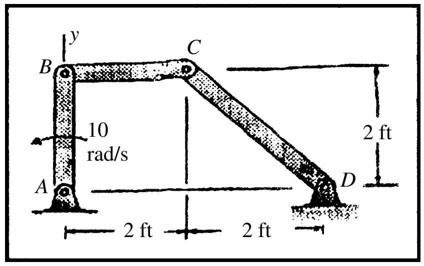
Substitute:

$$\boxed{\mathbf{v}_C = (-14539.5 - 141.1(23.1))\mathbf{j} = -177.98\mathbf{j} \text{ (mm/s).}}$$

Problem 17.36 Bar AB rotates at 10 rad/s in the counterclockwise direction. Determine the angular velocity of bar CD.



Strategy: Since the angular velocity of the bar AB is known, the velocity of B can be determined. Apply Eq. (17.6) to points B and C to obtain an equation for $\mathbf{v}_C$ in terms of the angular velocity of bar BC, then apply Eq. (17.6) to points C and D to obtain an equation for $\mathbf{v}_C$ in terms of the angular velocity of bar CD. By equating the two expressions, you will obtain a vector equation in two unknowns: the angular velocities of bars BC and CD.



Solution: The velocity of point B is

$$\mathbf{v}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0 & 2 & 0 \end{bmatrix} = -20\mathbf{i} \text{ (ft/s)}$$

The vector BC is $\mathbf{r}_{C/B} = 2\mathbf{i}$ (ft). The vector DC is $\mathbf{r}_{C/D} = -2\mathbf{i} + 2\mathbf{j}$ (ft). The velocity of point C is $\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$.

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2 & 0 & 0 \end{bmatrix}$$

$$= -20\mathbf{i} + 2\omega_{BC}\mathbf{j} \text{ (ft/s)}.$$

Similarly, since D is a fixed point:

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -2 & 2 & 0 \end{bmatrix}$$

$$= -2\omega_{CD}(\mathbf{i} + \mathbf{j}).$$

Equating the two expressions for the velocity of point C and separating components:

$$(-20 + 2\omega_{CD})\mathbf{i} = 0,$$

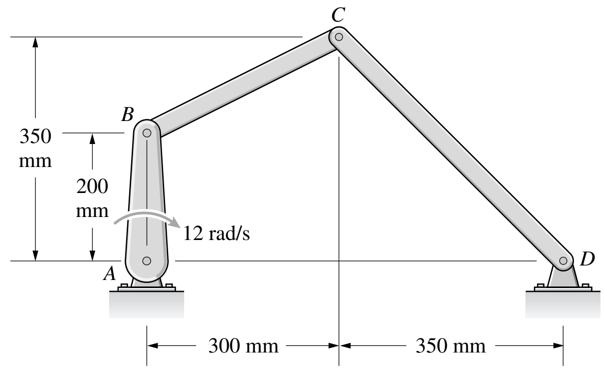
and $(2\omega_{BC} + 2\omega_{CD})\mathbf{j} = 0.$

Solve: $\omega_{CD} = 10$ rad/s,

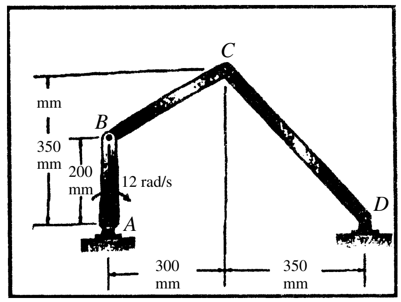
$$\boxed{\boldsymbol{\omega}_{CD} = 10\mathbf{k} \text{ (rad/s)}}$$

and $\omega_{BC} = -\omega_{CD}$ rad/s.

Problem 17.37 Bar AB rotates at 12 rad/s in the clockwise direction. Determine the angular velocities of bars BC and CD.



Solution: The strategy is analogous to that used in Problem 17.28. The radius vector AB is $\mathbf{r}_{B/A} = 200\mathbf{j}$ (mm). The angular velocity of AB is $\boldsymbol{\omega} = -12\mathbf{k}$ (rad/s). The velocity of point B is



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 300\mathbf{i} + (350 - 200)\mathbf{j} = 300\mathbf{i} + 150\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 300 & 150 & 0 \end{bmatrix}$$

$$= (2400 - 150\omega_{BC})\mathbf{i} + \omega_{BC}300\mathbf{j} \text{ (mm/s)}.$$

The radius vector DC is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}(\mathbf{i} + \mathbf{j}).$$

Equate the two expressions for $\mathbf{v}_C$, and separate components:

$$(2400 - 150\omega_{BC} + 350\omega_{CD})\mathbf{i} = 0,$$

and $(300\omega_{BC} + 350\omega_{CD})\mathbf{j} = 0.$

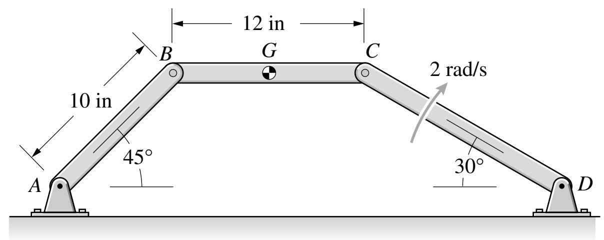
Solve: $\omega_{BC} = 5.33$ rad/s,

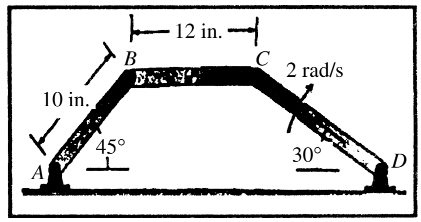
$\boxed{\boldsymbol{\omega}_{BC} = 5.33\mathbf{k} \text{ (rad/s)}}$.

$\omega_{CD} = -4.57$ rad/s,

$\boxed{\boldsymbol{\omega}_{CD} = -4.57\mathbf{k} \text{ (rad/s)}}$.

Problem 17.38 Bar CD rotates at 2 rad/s in the clockwise direction. Determine the angular velocities of bars AB and BC.





Solution: The strategy is analogous to that used in Problem 17.36, except that the computation is started with bar CD. Denote the length of CD by

$$L_{CD} = \frac{10\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}(10) = 14.14 \text{ in.}$$

The radius vector DC is

$$\mathbf{r}_{C/D} = L_{CD}(-\mathbf{i}\cos 30^\circ) + \mathbf{j}\sin 30^\circ)$$

$$= (-12.25\mathbf{i} + 7.07\mathbf{j}) \text{ (in.)}.$$

The angular velocity of CD is $\boldsymbol{\omega}_{CD} = -2\mathbf{k}$ (rad/s). The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ -12.25 & 7.07 & 0 \end{bmatrix}$$

$$= (14.14\mathbf{i} + 24.49\mathbf{j}) \text{ (in/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 12\mathbf{i}$ (in.). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \boldsymbol{\omega}_{BC} \\ 12 & 0 & 0 \end{bmatrix}$$

$$= \mathbf{v}_B + 12\omega_{BC}\mathbf{j} \text{ (in/s)}.$$

The radius vector AB is $\mathbf{r}_{B/A} = \dfrac{10}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$ (in.). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ \dfrac{10}{\sqrt{2}} & \dfrac{10}{\sqrt{2}} & 0 \end{bmatrix} = \left(\frac{10}{\sqrt{2}}\right)\omega_{AB}(-\mathbf{i} + \mathbf{j}) \text{ (in/s)}.$$

Substitute:

$$\mathbf{v}_C = \frac{10}{\sqrt{2}}\omega_{AB}(-\mathbf{i} + \mathbf{j}) + 12\omega_{BC}\mathbf{j}.$$

Equate the two expressions for $\mathbf{v}_C$ and separate components:

$$0 = \left(14.14 + \frac{10}{\sqrt{2}}\omega_{AB}\right)\mathbf{i},$$

$$0 = \left(24.49 - \frac{10}{\sqrt{2}}\omega_{AB} - 12\omega_{BC}\right)\mathbf{j}.$$

Solve: $\omega_{AB} = -2$ rad/s,

$$\boxed{\boldsymbol{\omega}_{AB} = -2\mathbf{k} \text{ (rad/s)}}.$$

$\omega_{BC} = 3.22$ rad/s,

$$\boxed{\boldsymbol{\omega}_{BC} = 3.22\mathbf{k} \text{ (rad/s)}}.$$

Problem 17.39 In Problem 17.38, what is the magnitude of the velocity of the midpoint G of bar BC?

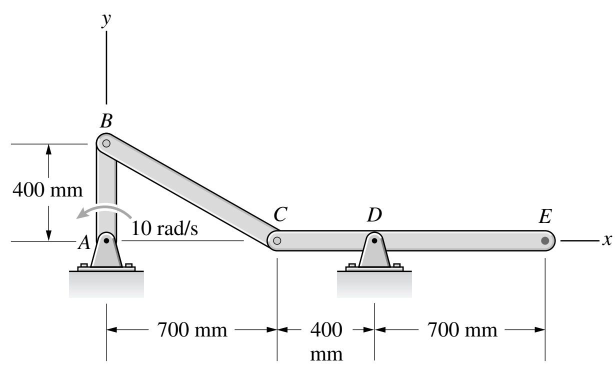
Solution: Use the solution to Problem 17.38. The radius vector from C to G is $\mathbf{r}_{G/C} = -6\mathbf{i}$. The velocity of point G is $\mathbf{v}_G = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{G/C}$. From the solution to Problem 17.38, $\mathbf{v}_C = (14.14\mathbf{i} + 24.49\mathbf{j})$ (in./s), and $\boldsymbol{\omega}_{BC} = 3.22\mathbf{k}$ (rad/s), from which

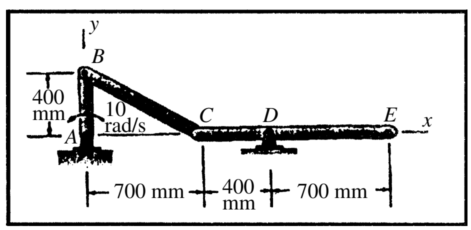
$$\mathbf{v}_G = 14.14\mathbf{i} + 24.49\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 3.22 \\ -6 & 0 & 0 \end{bmatrix}$$

$$= 14.14\mathbf{i} + 5.18\mathbf{j} \text{ (in/s)}.$$

The magnitude is $\boxed{|\mathbf{v}_G| = \sqrt{14.14^2 + 5.18^2} = 15.0 \text{ in/s}}$.

Problem 17.40 Bar AB rotates at 10 rad/s in the counterclockwise direction. Determine the velocity of point E.





Solution: The strategy is analogous to that used in Problem 17.28. The radius vector AB is $\mathbf{r}_{B/A} = 400\mathbf{j}$ (mm). The angular velocity of bar AB is $\boldsymbol{\omega}_{AB} = 10\mathbf{k}$ (rad/s). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 400 & 0 \end{bmatrix} = -4000\mathbf{i} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 700\mathbf{i} - 400\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = -4000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 700 & -400 & 0 \end{bmatrix}$$

$$= (-4000 + 400\omega_{BC})\mathbf{i} + 700\omega_{BC}\mathbf{j}.$$

The radius vector CD is $\mathbf{r}_{C/D} = -400\mathbf{i}$ (mm). The point D is fixed (cannot translate). The velocity at point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = (\omega_{CD}(\mathbf{k}) \times (-400\mathbf{i})) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

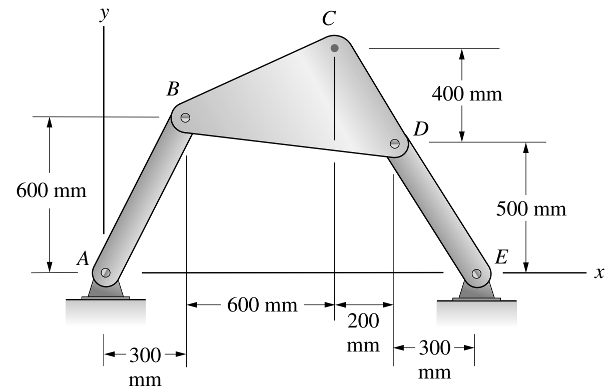
$$= -400\omega_{CD}\mathbf{j}.$$

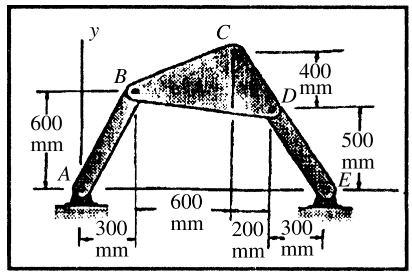
Equate the two expressions for the velocity at point C, and separate components: $0 = (-4000 + 400\omega_{BC})\mathbf{i}$, $0 = (700\omega_{BC} + 400\omega_{CD})\mathbf{j}$. Solve: $\omega_{BC} = 10$ rad/s, $\omega_{CD} = -17.5$ rad/s. The radius vector DE is $\mathbf{r}_{D/E} = 700\mathbf{i}$ (mm). The velocity of point E is

$$\mathbf{v}_E = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -17.5 \\ 700 & 0 & 0 \end{bmatrix}$$

$\boxed{\mathbf{v}_E = -12250\mathbf{j} \text{ (mm/s)}}$.

Problem 17.41 Bar AB rotates at 4 rad/s in the counterclockwise direction. Determine the velocity of point C.





Solution: The strategy is analogous to that used in Problem 17.36. The angular velocity of bar AB is $\boldsymbol{\omega} = 4\mathbf{k}$ (rad/s). The radius vector AB is $\mathbf{r}_{B/A} = 300\mathbf{i} + 600\mathbf{j}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 300 & 600 & 0 \end{bmatrix},$$

from which $\mathbf{v}_B = -2400\mathbf{i} + 1200\mathbf{j}$ (mm/s). The vector radius from B to C is $\mathbf{r}_{C/B} = 600\mathbf{i} + (900 - 600)\mathbf{j} = 600\mathbf{i} + 300\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 600 & 300 & 0 \end{bmatrix}$$

$$= (-2400 - 300\omega_{BC})\mathbf{i} + (1200 + 600\omega_{BC})\mathbf{j} \text{ (mm/s)}.$$

The radius vector from C to D is $\mathbf{r}_{D/C} = 200\mathbf{i} - 400\mathbf{j}$ (mm). The velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 200 & -400 & 0 \end{bmatrix}$$

$$= \mathbf{v}_C + 400\omega_{BC}\mathbf{i} + 200\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The radius vector from E to D is $\mathbf{r}_{D/E} = -300\mathbf{i} + 500\mathbf{j}$ (mm). The velocity of point D is

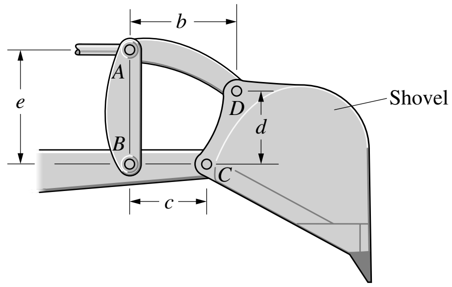
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -300 & 500 & 0 \end{bmatrix}$$

$$= -500\omega_{DE}\mathbf{i} - 300\omega_{DE}\mathbf{j} \text{ (mm/s)}.$$

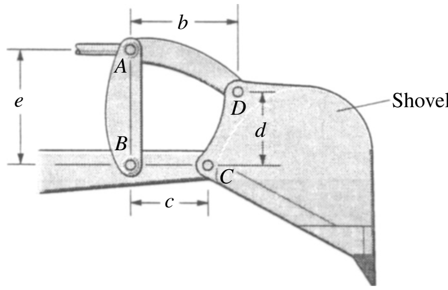
Equate the expressions for the velocity of point D; solve for $\mathbf{v}_C$, to obtain one of two expressions for the velocity of point C. Equate the two expressions for $\mathbf{v}_C$, and separate components: $0 = (-500\omega_{DE} - 100\omega_{BC} + 2400)\mathbf{i}$, $0 = (1200 + 300\omega_{DE} + 800\omega_{BC})\mathbf{j}$. Solve $\omega_{DE} = 5.51$ rad/s, $\omega_{BC} = -3.57$ rad/s. Substitute into the expression for the velocity of point C to obtain

$$\boxed{\mathbf{v}_C = -1330\mathbf{i} - 941\mathbf{j} \text{ (mm/s)}.}$$

Problem 17.42 At the instant shown, the dimensions are $b = 300$ mm, $c = 200$ mm, $d = 150$ mm, and $e = 260$ mm. If the link AB has a clockwise angular velocity of 0.2 rad/s. what is the angular velocity of the excavator's shovel?



Solution: Assume link BC is stationary. $\mathbf{v}_B = \mathbf{v}_C = 0$, we know $\boldsymbol{\omega}_{AB} = -0.2\mathbf{k}$ rad/s.



$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

$\mathbf{v}_A = 0 + (-0.2\mathbf{k}) \times (0.260\mathbf{j})$

$\underline{\mathbf{v}_A = 0.052\mathbf{i} \text{ m/s}}$

$\mathbf{v}_D = \mathbf{v}_A + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{D/A}$

$\mathbf{v}_D = 0.052\mathbf{i} + \omega_{AD}\mathbf{k} \times (0.3\mathbf{i} - 0.11\mathbf{j})$

$$\begin{cases} v_{Dx} = 0.052 + 0.11\omega_{AD} & \textbf{(1)} \\ v_{Dy} = 0.3\omega_{AD} & \textbf{(2)} \end{cases}$$

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$

$\mathbf{v}_D = 0 + \omega_{CD}\mathbf{k} \times (0.10\mathbf{i} + 0.15\mathbf{j})$

$$\begin{cases} v_{Dx} = -0.15\omega_{CD} & \textbf{(3)} \\ v_{Dy} = 0.10\omega_{CD} & \textbf{(4)} \end{cases}$$

Eliminate v_{Dx} and v_{Dy} between eqns. (1), (2), (3), and (4). We get

$0.052 + 0.11\omega_{AD} = -0.15\omega_{CD}$

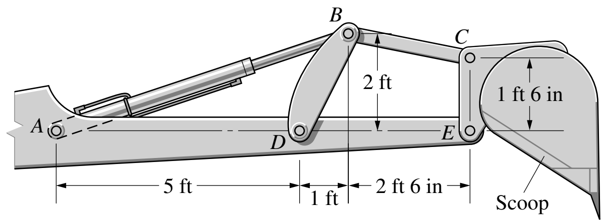
$0.3\omega_{AD} = 0.10\omega_{CD}$

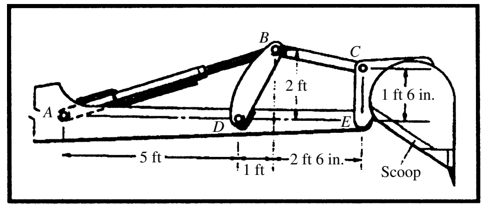
Solving,

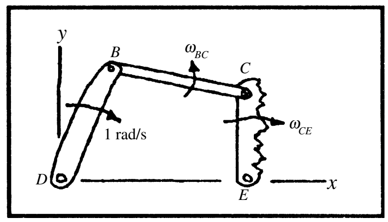
$\boldsymbol{\omega}_{AB} = -0.092\mathbf{k}$ rad/s

$\underline{\boldsymbol{\omega}_{CD} = -0.279\mathbf{k} \text{ mm/s}}$ (clockwise)

Problem 17.43 The horizontal member ADE supporting the scoop is stationary. If the link BD is rotating in the clockwise direction at 1 rad/s, what is the angular velocity of the scoop?







Solution: The velocity of B is $\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$. Expanding, we get

$$\mathbf{v}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 1 & 2 & 0 \end{vmatrix} = 2\mathbf{i} - 1\mathbf{j} \text{ (ft/s)}.$$

The velocity of C is

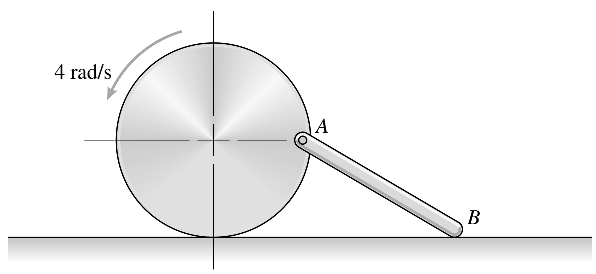
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 2\mathbf{i} - 1\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & +\omega_{BC} \\ 2.5 & -0.5 & 0 \end{vmatrix} \quad \textbf{(1)}.$$

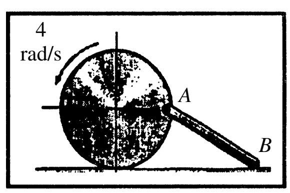
We can also express the velocity of C as $\mathbf{v}_C = \mathbf{v}_E + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{C/E}$ or

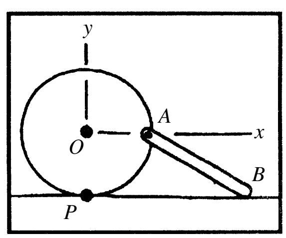
$$\mathbf{v}_C = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & +\omega_{CE} \\ 0 & 1.5 & 0 \end{vmatrix} \quad \textbf{(2)}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (1) and (2) and solving, we obtain $\omega_{BC} = 0.4$ rad/s and $\omega_{CE} = -1.47$ rad/s.

Problem 17.44 The diameter of the disk is 1 m, and the length of the bar AB is 1 m. The disk is rolling, and point B slides on the plane surface. Determine the angular velocity of the bar AB and the velocity of point B.







Solution: Choose a coordinate system with the origin at O, the center of the disk, with x axis parallel to the horizontal surface. The point P of contact with the surface is stationary, from which

$$\mathbf{v}_P = 0 = \mathbf{v}_0 + \boldsymbol{\omega}_0 \times -\mathbf{R} = \mathbf{v}_0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_0 \\ 0 & -0.5 & 0 \end{bmatrix} = \mathbf{v}_0 + 2\mathbf{i},$$

from which $\mathbf{v}_0 = -2\mathbf{i}$ (m/s). The velocity at A is $\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega}_0 \times \mathbf{r}_{A/O}$.

$$\mathbf{v}_A = -2\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_0 \\ 0.5 & 0 & 0 \end{bmatrix} = -2\mathbf{i} + 2\mathbf{j} \text{ (m/s)}.$$

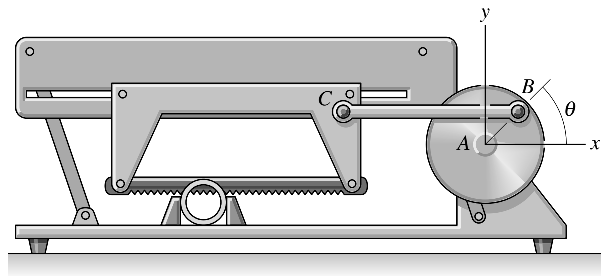
The vector from B to A is $\mathbf{r}_{A/B} = -\mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ (m), where $\theta = \sin^{-1} 0.5 = 30^\circ$. The motion at point B is parallel to the x axis. The velocity at A is

$$\mathbf{v}_A = v_B\mathbf{i} + \boldsymbol{\omega} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -0.866 & 0.5 & 0 \end{bmatrix}$$

$$= (v_B - 0.5\omega_{AB})\mathbf{i} - 0.866\omega_{AB}\mathbf{j} \text{ (m/s)}.$$

Equate and solve: $(-2 - 0.866\omega_{AB})\mathbf{j} = 0$, $(v_B - 0.5\omega_{AB} + 2)\mathbf{i} = 0$, from which $\boldsymbol{\omega}_{AB} = -2.31\mathbf{k}$ (rad/s), $\mathbf{v}_B = -3.15\mathbf{i}$ (m/s).

Problem 17.45 A motor rotates the circular disk mounted at A, moving the saw back and forth. (The saw is supported by a horizontal slot so that point C moves horizontally). The radius at AB is 4 in, and the link BC is 14 in long. In the position shown, $\theta = 45^\circ$ and the link BC is horizontal. If the angular velocity of the disk is one revolution per second counterclockwise, what is the velocity of the saw?



Solution: The radius vector from A to B is

$$\mathbf{r}_{B/A} = 4(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ) = 2\sqrt{2}(\mathbf{i}+\mathbf{j}) \text{ (in.)}.$$

The angular velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A},$$

$$\mathbf{v}_B = 0 + 2\pi(2\sqrt{2})\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 4\pi\sqrt{2}(-\mathbf{i}+\mathbf{j}) \text{ (in/s)}.$$

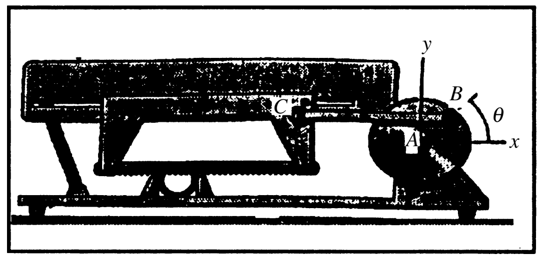
The radius vector from B to C is $\mathbf{r}_{C/B} = (4\cos 45^\circ - 14)\mathbf{i}$. The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2\sqrt{2}-14 & 0 & 0 \end{bmatrix}$$

$$= -\sqrt{24}\pi\mathbf{i} + ((2\sqrt{2}-14)\omega_{BC}+1)\mathbf{j}.$$

The saw is constrained to move parallel to the x axis, hence $(\sqrt{22}-14)\omega_{BC} + 1 = 0$, and the saw velocity is

$$\boxed{\mathbf{v}_S = -4\sqrt{2}\pi\mathbf{i} = -17.8\mathbf{i} \text{ (in./s)}}.$$



Problem 17.46 In Problem 17.45, if the singular velocity of the disk is one revolution per second counter clockwise and $\theta = 270^\circ$, what is the velocity of the saw?

Solution: The radius vector from A to B is $\mathbf{r}_{B/A} = -4\mathbf{j}$ (in.). The velocity of B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 2\pi(-4)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 8\pi\mathbf{i} \text{ (in/s)}.$$

The coordinates of point C are

$$(-14\cos\beta, +4\sin 45^\circ) = (-12.22, 2\sqrt{2}) \text{ in.},$$

where $\beta = \sin^{-1}\left(\dfrac{4(1+\sin 45^\circ)}{14}\right) = 29.19^\circ.$

The coordinates of point B are $(0, -4)$ in. The vector from C to B is

$$\mathbf{r}_{C/B} = (-12.22 - 0)\mathbf{i} + (2\sqrt{2} - (-4))\mathbf{j} = -12.22\mathbf{i} + 6.828\mathbf{j} \text{ (in.)}$$

The velocity at point C is

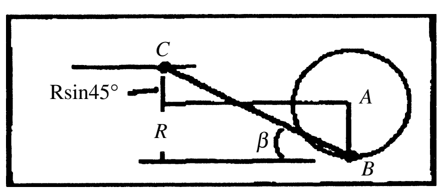
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -12.22 & 6.828 & 0 \end{bmatrix}$$

$$= (8\pi - 6.828\omega_{BC})\mathbf{i} - 12.22\omega_{BC}\mathbf{j}.$$

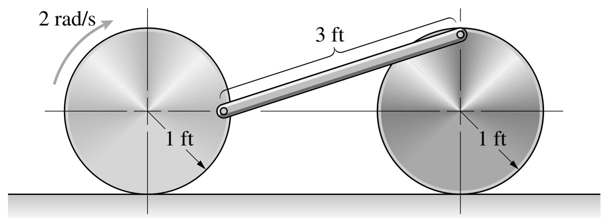
Since the saw is constrained to move parallel to the x axis, $-12.22\omega_{BC}\mathbf{j} = 0$, from which $\omega_{BC} = 0$, and the velocity of the saw is

$$\boxed{\mathbf{v}_C = 8\pi\mathbf{i} = 25.1\mathbf{i} \text{ (in./s)}}$$

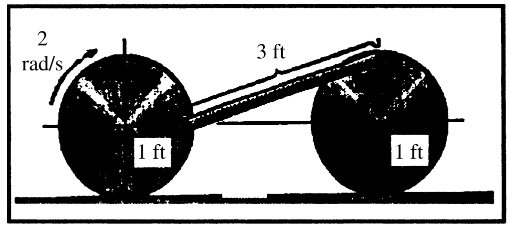
[*Note*: Since the vertical velocity at B reverses direction at $\theta = 270^\circ$, the angular velocity $\omega_{BC} = 0$ can be determined on physical grounds by inspection, simplifying the solution.]



Problem 17.47 The disks roll on a plane surface. The angular velocity of the left disk is 2 rad/s in the clockwise direction. What is the angular velocity of the right disk?

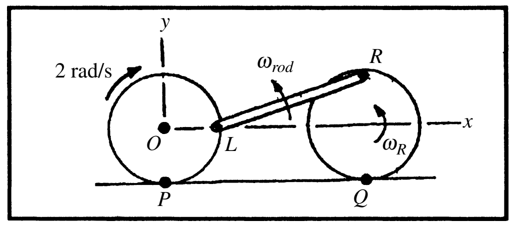


Solution: The velocity at the point of contact P of the left disk is zero. The vector from this point of contact to the center of the left disk is $\mathbf{r}_{O/P} = 1\mathbf{j}$ (ft). The velocity of the center of the left disk is



$$\mathbf{v}_O = \boldsymbol{\omega} \times \mathbf{r}_{O/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix} = 2\mathbf{i} \text{ (ft/s).}$$

The vector from the center of the left disk to the point of attachment of the rod is $\mathbf{r}_{L/O} = 1\mathbf{i}$ (ft). The velocity of the point of attachment of the rod to the left disk is



$$\mathbf{v}_L = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_{L/O} = 2\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= 2\mathbf{i} - 2\mathbf{j} \text{ (ft/s),}$$

The vector from the point of attachment of the left disk to the point of attachment of the right disk is

$$\mathbf{r}_{R/L} = 3(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) \text{ (ft),}$$

where $\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.47°.$

The velocity of the point on attachment on the right disk is

$$\mathbf{v}_R = \mathbf{v}_L + \boldsymbol{\omega}_{\text{rod}} \times \mathbf{r}_{R/L} = \mathbf{v}_L + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{rod}} \\ 2.83 & 1 & 0 \end{bmatrix}$$

$$= (2 - \omega_{\text{rod}})\mathbf{i} + (-2 + 2.83\omega_{\text{rod}})\mathbf{j} \text{ (ft/s).}$$

The velocity of point R is also expressed in terms of the contact point Q,

$$\mathbf{v}_R = \boldsymbol{\omega}_{RO} \times \mathbf{r}_{R/O} = \omega_{RO}(2) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= -2\omega_{RO}\mathbf{i} \text{ (ft/s).}$$

Equate the two expressions for the velocity $\mathbf{v}_R$ and separate components:

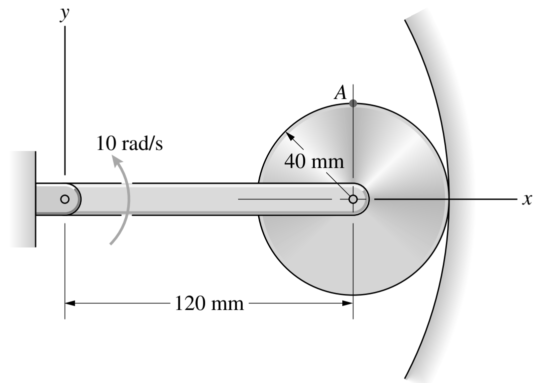
$$(2 - \omega_{\text{rod}} + 2\omega_{RO})\mathbf{i} = 0,$$

$$(-2 + 2.83\omega_{\text{rod}})\mathbf{j} = 0,$$

from which $\boxed{\boldsymbol{\omega}_{RO} = -0.65\mathbf{k} \text{ (rad/s)}}$

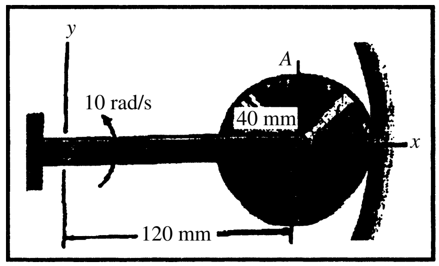
and $\omega_{\text{rod}} = 0.707$ rad/s.

Problem 17.48 The disk rolls on the curved surface. The bar rotates at 10 rad/s in the counterclockwise direction. Determine the velocity of point A.



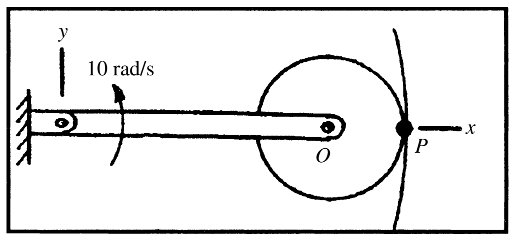
Solution: The radius vector from the left point of attachment of the bar to the center of the disk is $\mathbf{r}_{\text{bar}} = 120\mathbf{i}$ (mm). The velocity of the center of the disk is

$$\mathbf{v}_O = \boldsymbol{\omega}_{\text{bar}} \times \mathbf{r}_{\text{bar}} = 10(120)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = 1200\mathbf{j} \text{ (mm/s)}.$$



The radius vector from the point of contact with the disk and the curved surface to the center of the disk is $\mathbf{r}_{O/P} = -40\mathbf{i}$ (m). The velocity of the point of contact of the disk with the curved surface is zero, from which

$$\mathbf{v}_O = \boldsymbol{\omega}_O \times \mathbf{r}_{O/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ -40 & 0 & 0 \end{bmatrix} = -40\omega_O\mathbf{j}.$$

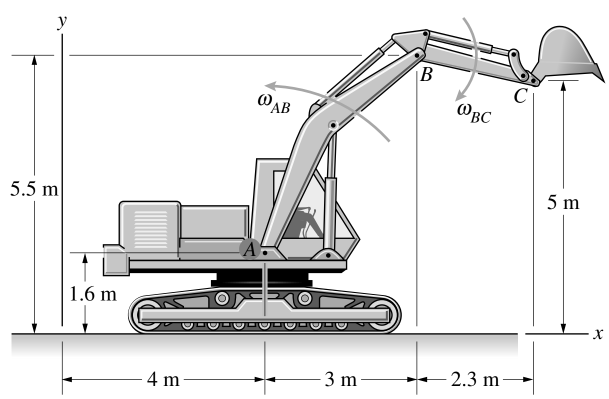


Equate the two expressions for the velocity of the center of the disk and solve: $\omega_O = -30$ rad/s. The radius vector from the center of the disk to point A is $\mathbf{r}_{A/O} = 40\mathbf{j}$ (mm). The velocity of point A is

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_O \times \mathbf{r}_{A/O} = 1200\mathbf{j} - (30)(40)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{= 1200\mathbf{i} + 1200\mathbf{j} \text{ (mm/s)}}$$

Problem 17.49 If $\omega_{AB} = 2$ rad/s and $\omega_{BC} = 4$ rad/s, what is the velocity of point C, where the excavator's bucket is attached?

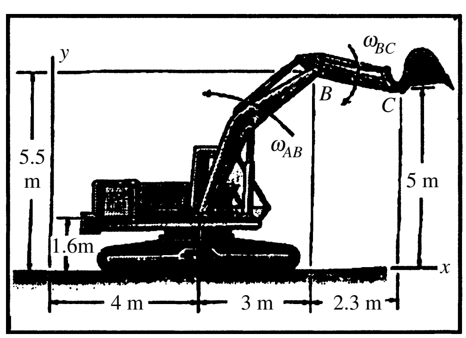


Solution: The radius vector AB is

$$\mathbf{r}_{B/A} = 3\mathbf{i} + (5.5 - 1.6)\mathbf{j} = 3\mathbf{i} + 3.9\mathbf{j} \text{ (m)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 3 & 3.9 & 0 \end{bmatrix} = -7.8\mathbf{i} + 6\mathbf{j} \text{ (m/s)}.$$



The radius vector BC is $\mathbf{r}_{C/B} = 2.3\mathbf{i} + (5 - 5.5)\mathbf{j} = 2.3\mathbf{i} - 0.5\mathbf{j}$ (m). The velocity at point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = -7.8\mathbf{i} + 6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -4 \\ 2.3 & -0.5 & 0 \end{bmatrix}$$

$$\boxed{= -9.8\mathbf{i} - 3.2\mathbf{j} \text{ (m/s)}}$$

Problem 17.50 In Problem 17.49, if $\omega_{AB} = 2$ rad/s, what clockwise angular velocity ω_{BC} will cause the vertical component of the velocity of point C to be zero? What is the resulting velocity of point C?

Solution: Use the solution to Problem 17.49. The velocity of point B is

$\mathbf{v}_B = -7.8\mathbf{i} + 6\mathbf{j}$ (m/s).

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= -7.8\mathbf{i} + 6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix},$$

$\mathbf{v}_C = (-7.8 - 0.5\omega_{BC})\mathbf{i} + (6 - 2.3\omega_{BC})\mathbf{j}$ (m/s).

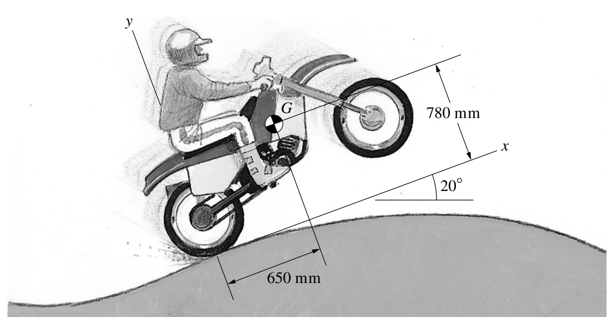
For the vertical component to be zero,

$$\boxed{\omega_{BC} = \frac{6}{2.3} = 2.61 \text{ rad/s clockwise.}}$$

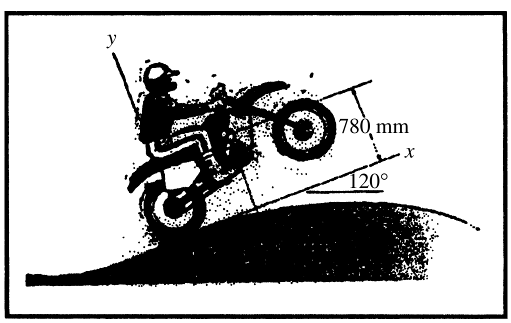
The velocity of point C is

$$\boxed{\mathbf{v}_C = -9.1\mathbf{i} \text{ (m/s)}}$$

Problem 17.51 The motorcycle's rear wheel is rolling on the ground (the velocity of its point of contact with the ground is zero) at 500 rpm, the wheel's radius is 280 mm, and the body of the motorcycle is rotating in the clockwise direction at 6 rad/s. Determine the velocity of the center of mass G.



Solution: The rear wheel's clockwise angular velocity is $\omega = (500)(2\pi)/60 = 52.4$ rad/s so the velocity of the center C of the wheel is $\mathbf{v}_C = (52.4)(0.28)\mathbf{i} = 14.7\mathbf{i}$ (m/s). The velocity of the center of mass is

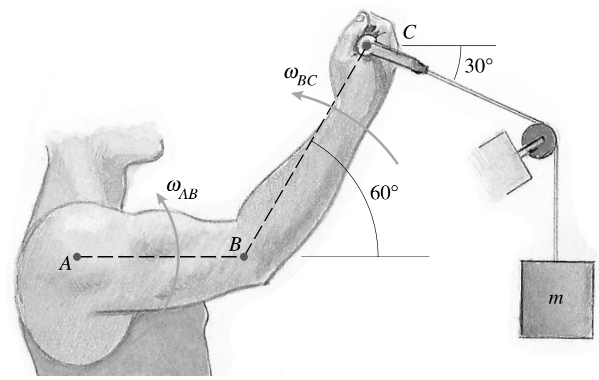


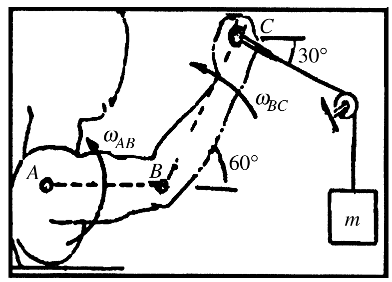
$$\mathbf{v}_G = \mathbf{v}_c + \boldsymbol{\omega} \times \mathbf{r}_{G/c}$$

$$= 14.7\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -6 \\ 0.65 & 0.5 & 0 \end{vmatrix}$$

$$= 17.7\mathbf{i} - 3.9\mathbf{j} \text{ (m/s)}.$$

Problem 17.52 An athlete exercises his arm by raising the mass m. The shoulder joint A is stationary. The distance AB is 300 mm, and the distance BC is 400 mm. At the instant shown, $\omega_{AB} = 1$ rad/s and $\omega_{BC} = 2$ rad/s. How fast is the mass m rising?





Solution: The magnitude of the velocity of the point C parallel to the cable at C is also the magnitude of the velocity of the mass m. The radius vector AB is $\mathbf{r}_{B/A} = 300\mathbf{i}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 300 & 0 & 0 \end{bmatrix} = 300\mathbf{j} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 400(\mathbf{i}\cos 60^\circ + \mathbf{j}\sin 60^\circ) = 200\mathbf{i} + 346.4\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 300\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 200 & 346.4 & 0 \end{bmatrix}$$

$$= -692.8\mathbf{i} + 700\mathbf{j} \text{ (mm/s)}.$$

The unit vector parallel to the cable at C is $\mathbf{e}_C = -\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ = -0.866\mathbf{i} + 0.5\mathbf{j}$. The component of the velocity parallel to the cable at C is

$$\boxed{\mathbf{v}_C \cdot \mathbf{e}_C = 950 \text{ mm/s}},$$

which is the velocity of the mass m.

Problem 17.53 In Problem 17.52, suppose that the distance AB is 12 in., the distance BC is 16 in., $\omega_{AB} = 0.6$ rad/s, and the mass m is rising at 24 in./s. What is the angular velocity ω_{BC}?

Solution: The radius vector AB is $\mathbf{r}_{B/A} = 12\mathbf{i}$ (in.). The velocity at point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ 12 & 0 & 0 \end{bmatrix} = 7.2\mathbf{j} \text{ (in/s)}.$$

The radius vector BC is

$$\mathbf{r}_{C/B} = 16(\mathbf{i}\cos 60 + \mathbf{j}\sin 60) = 13.9\mathbf{i} + 8\mathbf{j} \text{ (in.)}.$$

The velocity at C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 7.2\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 13.9 & 8 & 0 \end{bmatrix}$$

$$= -13.9\omega_{BC}\mathbf{i} + (7.2 + 8\omega_{BC})\mathbf{j}.$$

The unit vector parallel to the cable at C is

$$\mathbf{e}_C = -\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 60^\circ = -0.5\mathbf{i} + 0.866\mathbf{j}.$$

The component of the velocity at C parallel to the cable is

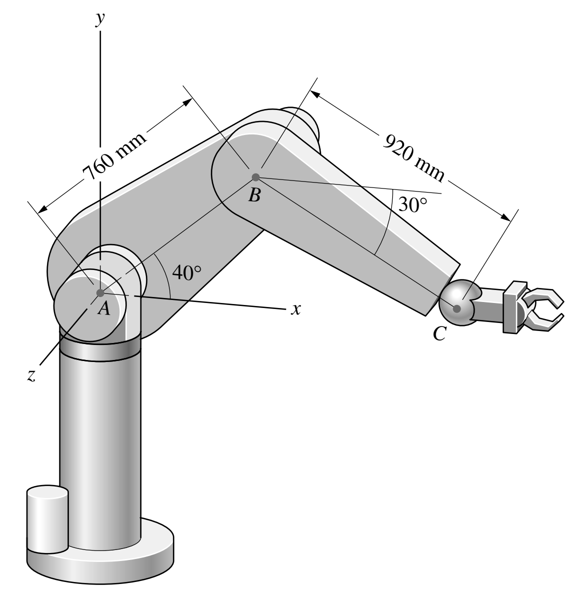
$$|\mathbf{v}_{CP}| = \mathbf{v}_C \cdot \mathbf{e}_C = +6.93\omega_{BC} + 6.93\omega_{BC} + 6.24 \text{ (in/s)}.$$

This is also the velocity of the rising mass, from which

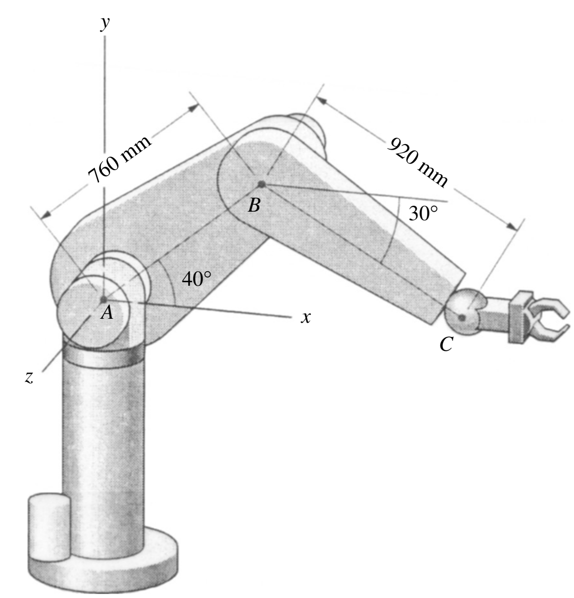
$$13.86\omega_{BC} + 6.24 = 24,$$

$$\boxed{\omega_{BC} = 1.28 \text{ rad/s}}$$

Problem 17.54 Points B and C are in the x-y plane. The angular velocity vectors of the arms AB and BC are $\omega_{AB} = -0.2\mathbf{k}$ (rad/s), and $\omega_{BC} = 0.4\mathbf{k}$ (rad/s). What is the velocity of point C.



Solution: Locations of Points:



A: $(0, 0, 0)$ m

B: $(0.76\cos 40^\circ, 0.76\sin 40^\circ, 0)$ m

C: $(x_B + 0.92\cos 30^\circ, y_B - 0.92\sin 30^\circ, 0)$ m

or B: $(0.582, 0.489, 0)$,

C: $(1.379, 0.0285, 0)$ m

$\mathbf{r}_{B/A} = 0.582\mathbf{i} + 0.489\mathbf{j}$ (m)

$\mathbf{r}_{C/B} = 0.797\mathbf{i} - 0.460\mathbf{j}$ (m)

$\mathbf{v}_A = 0, \omega_{AB} = -0.2\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right), \quad \omega_{BC} = 0.4\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right)$

$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$\mathbf{v}_B = (-0.2\ k) \times (0.582\mathbf{i} + 0.489\mathbf{j})$

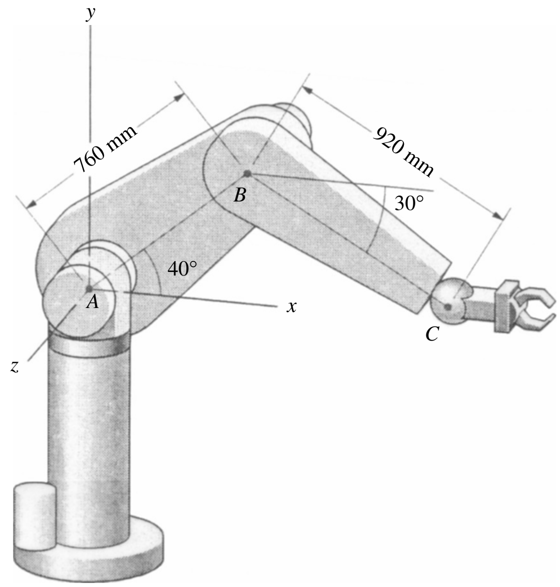
$\mathbf{v}_B = 0.0977\mathbf{i} - 0.116\mathbf{j}$ (m/s).

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = \mathbf{v}_B + 0.184\mathbf{i} + 0.319\mathbf{j}$ (m/s)

$\underline{\mathbf{v}_C = 0.282\mathbf{i} + 0.202\mathbf{j}\ \text{(m/s)}}$.

Problem 17.55 If the velocity at point C of the robotic arm shown in Problem 17.54 is $\mathbf{v}_C = 0.15\mathbf{i} + 0.42\mathbf{j}$ (m/s), what are the angular velocities of the arms AB and BC?



Solution: From the solution to Problem 17.54,

$$\mathbf{r}_{B/A} = 0.582\mathbf{i} + 0.489\mathbf{j} \text{ (m)}$$

$$\mathbf{r}_{C/B} = 0.797\mathbf{i} - 0.460\mathbf{j} \text{ (m)}$$

$$\mathbf{v}_B = \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} \quad (\mathbf{v}_A = 0)$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC}\mathbf{k} \times \mathbf{r}_{C/B}$$

We are given

$$\mathbf{v}_C = -0.15\mathbf{i} + 0.42\mathbf{j} + 0\mathbf{k} \text{ (m/s)}.$$

Thus, we know everything in the $\mathbf{v}_C$ equation except ω_{AB} and ω_{BC}.

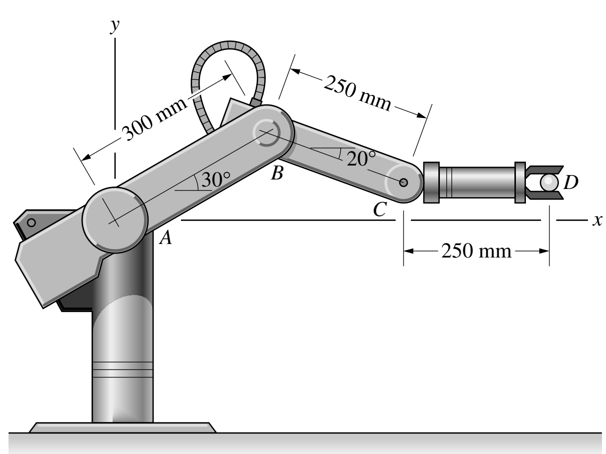
$$\mathbf{v}_C = \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} + \omega_{BC}\mathbf{k} \times \mathbf{r}_{C/B}$$

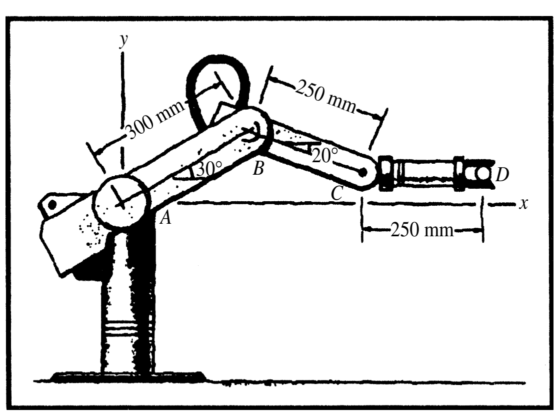
This yields two scalar equations in two unknowns **i** and **j** components. Solving, we get

$$\underline{\boldsymbol{\omega}_{AB} = 0.476\mathbf{k}} \text{ (rad/s)},$$

$$\underline{\boldsymbol{\omega}_{BC} = 0.179\mathbf{k}} \text{ (rad/s)}.$$

Problem 17.56 The link AB of the robot's arm is rotating at 2 rad/s in the counterclockwise direction, the link BC is rotating at 3 rad/s in the clockwise direction, and the link CD is rotating at 4 rad/s in the counterclockwise direction. What is the velocity of point D?





Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A},$$

or $$\mathbf{v}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.3\cos 30^\circ & 0.3\sin 30^\circ & 0 \end{vmatrix}$$

$$= -0.3\mathbf{i} + 0.520\mathbf{j} \text{ (m/s)}.$$

The velocity of C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = -0.3\mathbf{i} + 0.520\mathbf{j} \text{ (m/s)},$$

or $$\mathbf{v}_C = -0.3\mathbf{i} + 0.520\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -3 \\ 0.25\cos 20^\circ & -0.25\sin 20^\circ & 0 \end{vmatrix}$$

$$= -0.557\mathbf{i} - 0.185\mathbf{j} \text{ (m/s)}.$$

The velocity of D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -0.557\mathbf{i} - 0.185\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0.25 & 0 & 0 \end{vmatrix},$$

or $$\mathbf{v}_D = -0.557\mathbf{i} + 0.815\mathbf{j} \text{ (m/s)}.$$

Problem 17.57 Consider the robot shown in Problem 17.56. Link AB is rotating at 2 rad/s in the counterclockwise direction, and link BC is rotating at 3 rad/s in the clockwise direction. If you want the velocity of point D to be parallel to the x axis, what is the necessary angular velocity of link CD? What is the resulting velocity of point D?

Solution: From the solution of Problem 17.56, the velocity of point C is

$$\mathbf{v}_C = -0.557\mathbf{i} - 0.185\mathbf{j} \text{ (m/s)}.$$

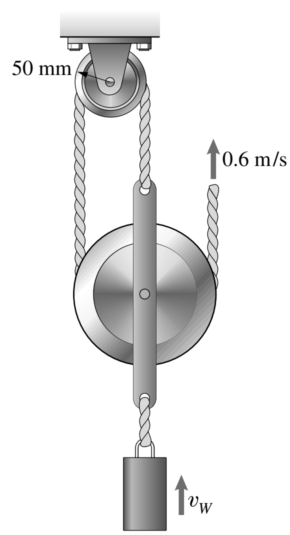
Let ω_{CD} be the counterclockwise angular velocity of link CD. The velocity of D is

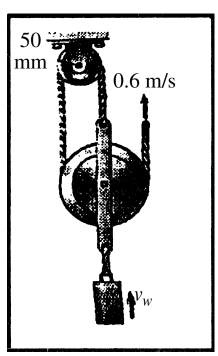
$$v_D\mathbf{i} = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$= -0.557\mathbf{i} - 0.185\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 0.25 & 0 & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, $v_D = -0.557$ m/s and $0 = -0.185 + 0.25\omega_{CD}$ we obtain $\omega_{CD} = 0.741$ rad/s and $v_D = -0.557$ m/s.

Problem 17.58 Determine the velocity v_W and the angular velocity of the small pulley.





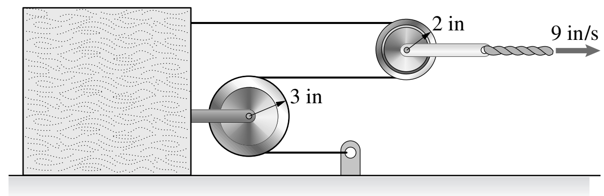
Solution: Since the radius of the bottom pulley is not given, we cannot use Eq (17.6) (or the equivalent). The strategy is to use the fact (derived from elementary principles) that the velocity of the center of a pulley is the mean of the velocities of the extreme edges, where the edges lie on a line normal to the motion, *taking into account the directions of the velocities at the extreme edges*. The center rope from the bottom pulley to the upper pulley moves upward at a velocity of v_W. Since the small pulley is fixed, the velocity of the center is zero, and the rope to the left moves downward at a velocity v_W, from which the left edge of the bottom pulley is moving at a velocity v_W downward. The right edge of the bottom pulley moves upward at a velocity of 0.6 m/s. The velocity of the center of the bottom pulley is the mean of the velocities at the extreme edges, from which $v_W = \dfrac{6 - v_W}{2}$.

Solve: $\boxed{v_W = \dfrac{0.6}{3} = 0.2 \text{ m/s}}$.

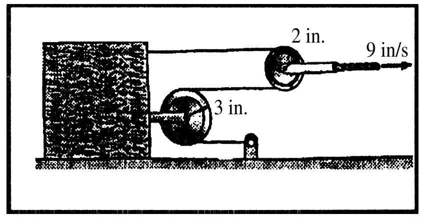
The angular velocity of the small pulley is

$$\boxed{\omega = \frac{v_W}{r} = \frac{0.2}{0.05} = 4 \text{ rad/s}}$$

Problem 17.59 Determine the velocity of the block and the angular velocity of the small pulley.



Solution: Denote the velocity of the block by v_B. The strategy is to determine the velocities of the extreme edges of a pulley by determining the velocity of the element of rope in contact with the pulley. The upper rope is fixed to the block, so that it moves to the right at the velocity of the block, from which the upper edge of the small pulley moves to the right at the velocity of the block. The fixed end of the rope at the bottom is stationary, so that the bottom edge of the large pulley is stationary. The center of the large pulley moves at the velocity of the block, from which the upper edge of the bottom pulley moves at twice the velocity of the block (since the velocity of the center is equal to the mean of the velocities of the extreme edges, one of which is stationary) from which the bottom edge of the small pulley moves at twice the velocity of the block. The center of the small pulley moves to the right at 9 in/s. The velocity of the center of the small pulley is the mean of the velocities at the extreme edges, from which



$$9 = \frac{2v_B + v_B}{2} = \frac{3}{2} v_B,$$

from which

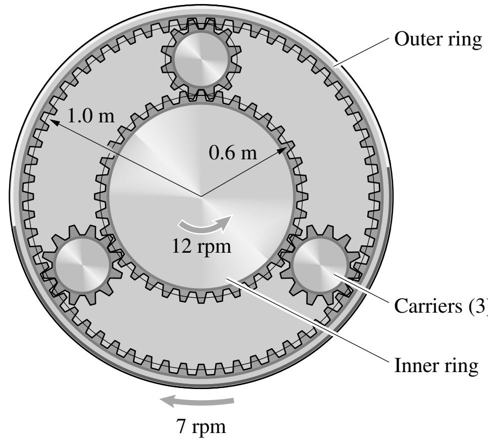
$$\boxed{v_B = \frac{2}{3} 9 = 6 \text{ in/s}}.$$

The angular velocity of small pulley is given by

$$9\mathbf{i} = 2\mathbf{v}_B\mathbf{i} + \boldsymbol{\omega} \times 2\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0 & 2 & 0 \end{bmatrix} = 2v_B\mathbf{i} - 2\omega\mathbf{i},$$

from which $\boxed{\omega = \dfrac{12 - 9}{2} = 15 \text{ rad/s}}$

Problem 17.60 The device shown is used in the semiconductor industry to polish silicon wafers. The wafers are placed on the faces of the carriers. The outer and inner rings are then rotated, causing the wafers to move and rotate against an abrasive surface. If the outer ring rotates in the clockwise direction at 7 rpm and the inner ring rotates in the counterclockwise direction at 12 rpm, what is the angular velocity of the carriers?



Solution: The velocity of pt. B is $\mathbf{v}_B = (1\text{ m})\omega_0\mathbf{i} = \omega_0\mathbf{i}$. The velocity of pt. A is $\mathbf{v}_A = -(0.6\text{ m})\omega_i\mathbf{i}$. Then

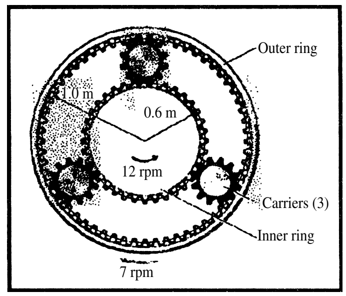
$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_C \times \mathbf{r}_{B/A} : \omega_0\mathbf{i} = -0.6\omega_i\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_C \\ 0 & 0.4 & 0 \end{vmatrix}.$$

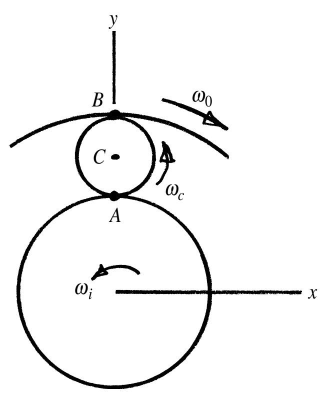
The $\mathbf{i}$ component of this equation is $\omega_0 = -0.6\omega_i - 0.4\omega_C$,

$$\text{so} \quad \omega_C = \frac{-0.6\omega_i - \omega_0}{0.4}$$

$$= \frac{-0.6(12\text{ rpm}) - 7\text{ rpm}}{0.4}$$

$$= -35.5\text{ rpm}.$$





Problem 17.61 In Problem 17.60, suppose that the outer ring rotates in the clockwise direction at 5 rpm and you want the centerpoints of the carriers to remain stationary during the polishing process. What is the necessary angular velocity of the inner ring?

Solution: See the solution of Problem 17.60. The velocity of pt. B is $\mathbf{v}_B = \omega_0\mathbf{i}$ and the angular velocity of the carrier is

$$\omega_C = \frac{-0.6\omega_i - \omega_0}{0.4}.$$

We want the velocity of pt. C to be zero:

$$\mathbf{v}_C = 0 = \mathbf{v}_B + \boldsymbol{\omega}_C \times \mathbf{r}_{C/B} = \omega_0\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_C \\ 0 & -0.2 & 0 \end{vmatrix}.$$

From this equation we see that $\omega_C = -5\omega_0$. Therefore the velocity of pt. A is

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_C \times \mathbf{r}_{A/C}$$

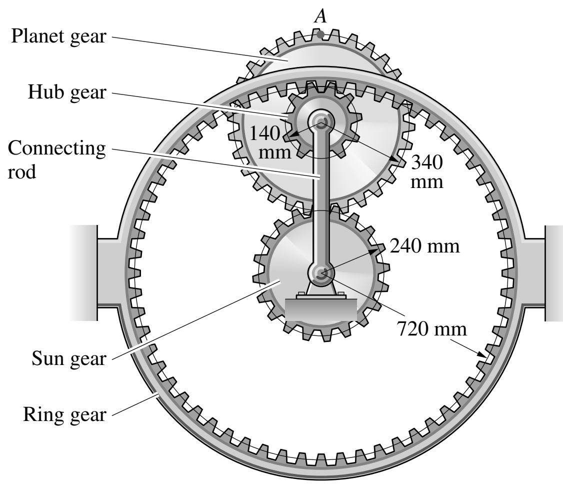
$$= 0 + (-5\omega_0\mathbf{k}) \times (-0.2\mathbf{j})$$

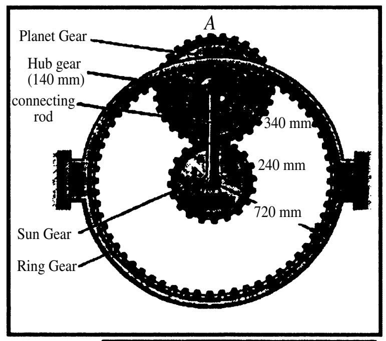
$$= -\omega_0\mathbf{i}$$

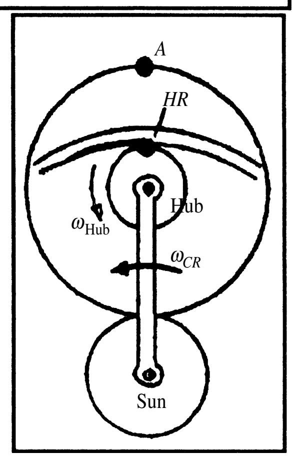
We also know that $\mathbf{v}_A = -(0.6\text{ m})\omega_i\mathbf{i}$,

$$\text{so} \quad \omega_i = \frac{\omega_0}{0.6} = \frac{5\text{ rpm}}{0.6} = 8.33\text{ rpm}.$$

Problem 17.62 The ring gear is fixed and the hub and planet gears are bonded together. The connecting rod rotates in the counterclockwise direction at 60 rpm. Determine the angular velocity of the sun gear and the magnitude of the velocity of point A.







Solution: Denote the centers of the sun, hub and planet gears by the subscripts Sun, Hub, and Planet, respectively. Denote the contact points between the sun gear and the planet gear by the subscript SP and the point of contact between the hub gear and the ring gear by the subscript HR. The angular velocity of the connecting rod is $\omega_{CR} = 6.28$ rad/s. The vector distance from the center of the sun gear to the center of the hub gear is $\mathbf{r}_{\text{Hub/Sun}} = (720 - 140)\mathbf{j} = 580\mathbf{j}$ (mm). The velocity of the center of the hub gear is

$$\mathbf{v}_{\text{Hub}} = \boldsymbol{\omega}_{CR} \times \mathbf{r}_{\text{Hub/Sun}} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2\pi \\ 0 & 580 & 0 \end{bmatrix} = -3644\mathbf{i} \text{ (mm/s)}$$

The angular velocity of the hub gear is found from

$$\mathbf{v}_{HR} = 0 = \mathbf{v}_{\text{Hub}} + \boldsymbol{\omega}_{\text{Hub}} \times 140\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{Hub}} \\ 0 & 140 & 0 \end{bmatrix}$$

$$= -3644\mathbf{i} - 140\omega_{\text{Hub}}\mathbf{i},$$

from which

$$\omega_{\text{Hub}} = -\frac{3644}{140} = -26.03 \text{ rad/s}.$$

This is also the angular velocity of the planet gear. The linear velocity of point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{\text{Hub}} \times (340 - 140)\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -26.03 \\ 0 & 200 & 0 \end{bmatrix}$$

$$\boxed{= 5206\mathbf{i} \text{ (mm/s)}}$$

The velocity of the point of contact with the sun gear is

$$\mathbf{v}_{PS} = \boldsymbol{\omega}_{\text{Hub}} \times (-480\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -26.03 \\ 0 & -480 & 0 \end{bmatrix}$$

$$= -12494.6\mathbf{i} \text{ (mm/s)}.$$

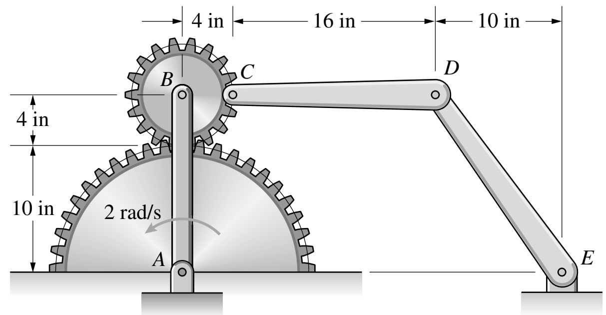
The angular velocity of the sun gear is found from

$$\mathbf{v}_{PS} = -12494.6\mathbf{i} = \boldsymbol{\omega}_{\text{Sun}} \times (240\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{Sun}} \\ 0 & 240 & 0 \end{bmatrix}$$

$$= -240\omega_{\text{Sun}}\mathbf{i},$$

from which $\boxed{\omega_{\text{Sun}} = \dfrac{12494.6}{240} = 52.06 \text{ rad/s}}$

Problem 17.63 The large gear is fixed. Bar AB has a counterclockwise angular velocity of 2 rad/s. What are the angular velocities of bars CD and DE?



Solution: The strategy is to express vector velocity of point D in terms of the unknown angular velocities of CD and DE, and then to solve the resulting vector equations for the unknowns. The vector distance AB is $\mathbf{r}_{B/A} = 14\mathbf{j}$ (in.) The linear velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & 14 & 0 \end{bmatrix} = -28\mathbf{i} \text{ (in/s).}$$

The lower edge of gear B is stationary. The radius vector from the lower edge to B is $\mathbf{r}_B = 4\mathbf{j}$ (in.), The angular velocity of B is

$$\mathbf{v}_B = \boldsymbol{\omega}_B \times \mathbf{r}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_B \\ 0 & 4 & 0 \end{bmatrix} = -4\omega_B\mathbf{i} \text{ (in/s),}$$

from which $\omega_B = -\dfrac{v_B}{4} = 7$ rad/s. The vector distance from B to C is $\mathbf{r}_{C/B} = 4\mathbf{i}$ (in.). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_B \times \mathbf{r}_{C/B} = -28\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 7 \\ 4 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + 28\mathbf{j} \text{ (in/s).}$$

The vector distance from C to D is $\mathbf{r}_{D/C} = 16\mathbf{i}$ (in.), and from E to D is $\mathbf{r}_{D/E} = -10\mathbf{i} + 14\mathbf{j}$ (in.). The linear velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -28\mathbf{i} + 28\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 16 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + (16\omega_{CD} + 28)\mathbf{j} \text{ (in/s).}$$

The velocity of point D is also given by

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -10 & 14 & 0 \end{bmatrix}$$

$$= -14\omega_{DE}\mathbf{i} - 10\omega_{DE}\mathbf{j} \text{ (in/s).}$$

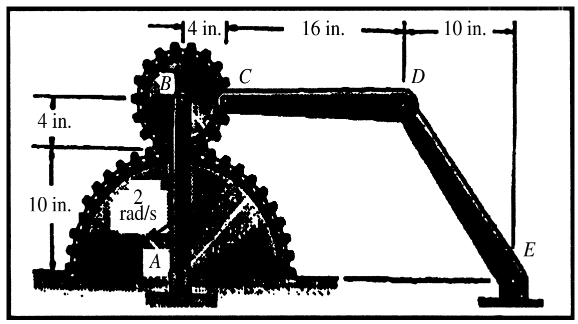
Equate components:

$$(-28 + 14\omega_{DE})\mathbf{i} = 0,$$

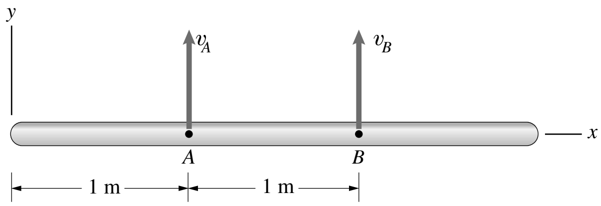
$$(16\omega_{CD} + 28 + 10\omega_{DE})\mathbf{j} = 0.$$

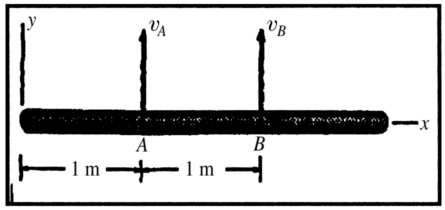
Solve: $\boxed{\omega_{DE} = 2 \text{ rad/s}}$, $\boxed{\omega_{CD} = -3 \text{ rad/s}}$.

The negative sign means a clockwise rotation.



Problem 17.64 If the bar has a clockwise angular velocity of 10 rad/s and $v_A = 20$ m/s, what are the coordinates of its instantaneous center of the bar, and what is the value of v_B?





Solution: Assume that the coordinates of the instantaneous center are (x_C, y_C), $\boldsymbol{\omega} = -\omega\mathbf{k} = -10\mathbf{k}$. The distance to point A is $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity at A is

$$\mathbf{v}_A = 20\mathbf{j} = \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

$$= y_C\omega\mathbf{i} - \omega(1 - x_C)\mathbf{j},$$

from which $y_C\omega\mathbf{i} = 0$, and $(20 + \omega(1 - x_C))\mathbf{j} = 0$.

Substitute $\omega = 10$ rad/s to obtain $y_C = 0$ and $x_C = 3$ m. The coordinates of the instantaneous center are $\boxed{(3, 0) \text{ (m)}}$. The vector distance from C to B is $\mathbf{r}_{B/C} = (2 - 3)\mathbf{i} = -\mathbf{i}$ (m). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -1 & 0 & 0 \end{bmatrix} = -10(-\mathbf{j}) \boxed{= 10\mathbf{j} \text{ (m/s)}}$$

Problem 17.65 In Problem 17.64, if $v_A = 24$ m/s and $v_B = 36$ m/s, what are the coordinates of the instantaneous center of the bar, and what is its angular velocity?

Solution: Let (x_C, y_C) be the coordinates of the instantaneous center. The vectors from the instantaneous center and the points A and B are $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$ (m) and $\mathbf{r}_{B/C} = (2 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity of A is given by

$$\mathbf{v}_A = 24\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

$$= -\omega_{AB}y_C\mathbf{i} + \omega_{AB}(1 - x_C)\mathbf{j} \text{ (m/s)}$$

The velocity of B is

$$\mathbf{v}_B = 36\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 2 - x_C & y_C & 0 \end{bmatrix}$$

$$= -y_C\omega_{AB}\mathbf{i} + \omega_{AB}(2 - x_C)\mathbf{j} \text{ (m/s)}.$$

Separate components:

$$24 - \omega_{AB}(1 - x_C) = 0,$$

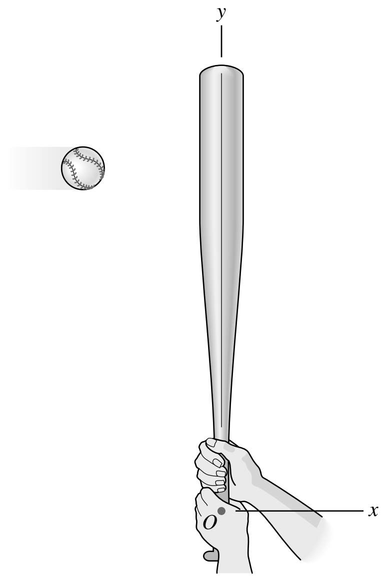
$$36 - \omega_{AB}(2 - x_C) = 0,$$

$$\omega_{AB}y_C = 0.$$

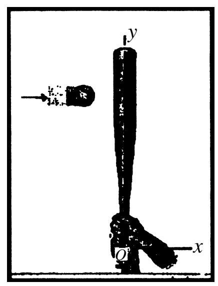
Solve: $\boxed{x_C = -1,}$ $\boxed{y_C = 0,}$

and $\boxed{\omega_{AB} = 12 \text{ rad/s}}$ *counterclockwise*.

Problem 17.66 The velocity of point O of the bat is $\mathbf{v}_O = -6\mathbf{i} - 14\mathbf{j}$ (ft/s), and the bat rotates about the z axis with a counterclockwise angular velocity of 4 rad/s. What are the x and y coordinates of the bat's instantaneous center?



Solution: Let (x_C, y_C) be the coordinates of the instantaneous center. The vector from the instantaneous center to point O is $\mathbf{r}_{O/C} = -x_C\mathbf{i} - y_C\mathbf{j}$ (ft). The velocity of point O is



$$\mathbf{v}_0 = -6\mathbf{i} - 1.4\mathbf{j} = \boldsymbol{\omega} \times \mathbf{r}_{O/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ -x_C & -y_C & 0 \end{bmatrix}$$

$$= y_C\omega\mathbf{i} - x_C\omega\mathbf{j} \text{ (ft/s)}.$$

Equate terms and solve:

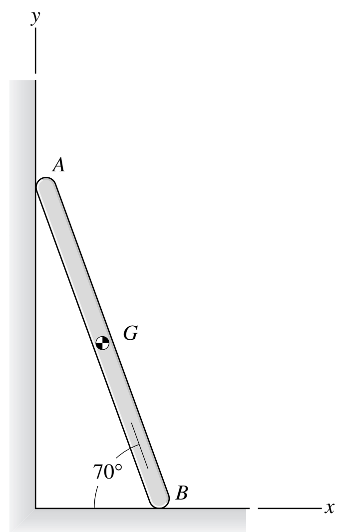
$$y_C = -\frac{6}{\omega} = -\frac{6}{4} = -1.5 \text{ ft},$$

$$x_C = \frac{1.4}{\omega} = \frac{1.4}{4} = 0.35 \text{ ft},$$

from which the coordinates are $\boxed{(0.35, -1.5) \text{ ft.}}$

Problem 17.67 Points A and B of the 1-m bar slide on the plane surfaces. The velocity of B is $\mathbf{v}_B = 2\mathbf{i}$ (m/s).

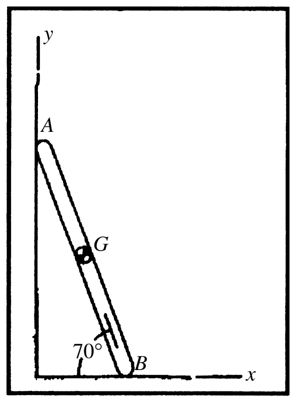
(a) What are the coordinates of the instantaneous center of the bar?
(b) Use the instantaneous center to determine the velocity at A.



Solution:

(a) A is constrained to move parallel to the y axis, and B is constrained to move parallel to the x axis. Draw perpendiculars to the velocity vectors at A and B. From geometry, the perpendiculars intersect at

$$\boxed{(\cos 70^\circ, \sin 70^\circ) = (0.3420, 0.9397)\ \text{m}}.$$



(b) The vector from the instantaneous center to point B is

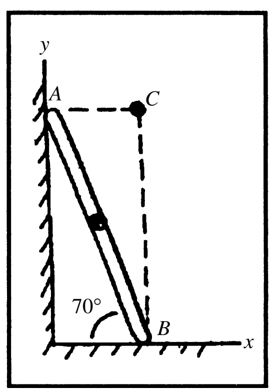
$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 0.3420\mathbf{i} - 0.9397\mathbf{j} = -0.9397\mathbf{j}$$

The angular velocity of bar AB is obtained from

$$\mathbf{v}_B = 2\mathbf{i} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0 & -0.9397 & 0 \end{bmatrix}$$

$$= \omega_{AB}(0.9397)\mathbf{i},$$

from which $\omega_{AB} = \dfrac{2}{0.9397} = 2.13$ rad/s.



The vector from the instantaneous center to point A is $\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -0.3420\mathbf{i}$ (m). The velocity at A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.13 \\ -0.3420 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -0.7279\mathbf{j}\ \text{(m/s)}.}$$

Problem 17.68 In Problem 17.67, use the instantaneous center to determine the velocity of the bar's midpoint G.

Solution: The vector to point G is

$$\mathbf{r}_{G/O} = (1/2)(0.3420\mathbf{i} + 0.9397\mathbf{j}) = 0.1710\mathbf{i} + 0.4698\mathbf{j}\ \text{(m)}.$$

From the solution to Problem 17.67, the vector to the instantaneous center is $\mathbf{r}_C = 0.3420\mathbf{i} + 0.9397\mathbf{j}$ (m), and $\omega_{AB} = 2.13$ rad/s. The vector from the instantaneous center to the point G is

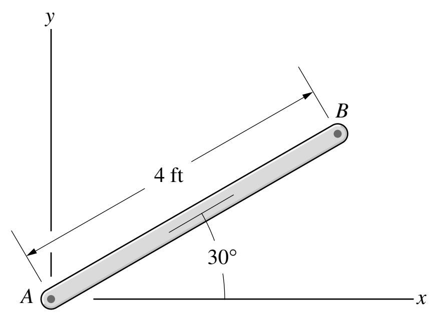
$$\mathbf{r}_{G/C} = \mathbf{r}_G - \mathbf{r}_C = -0.1710\mathbf{i} - 0.4698\mathbf{j}\ \text{(m)}.$$

The velocity of point G is

$$\mathbf{v}_G = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{G/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.13 \\ -0.1710 & -0.4698 & 0 \end{bmatrix}$$

$$\boxed{= i - 0.364\mathbf{j}\ \text{(m/s)}}.$$

Problem 17.69 The bar is in two-dimensional motion in the x-y plane. The velocity of point A is $\mathbf{v}_A = 8\mathbf{i}$ (ft/s), and B is moving in the direction parallel to the bar. Determine the velocity of B (a) by using Eq. (17.6) and (b) by using the instantaneous center of the bar.



Solution:

(a) The unit vector parallel to the bar is

$\mathbf{e}_{AB} = (\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 0.866\mathbf{i} + 0.5\mathbf{j}$.

The vector from A to B is $\mathbf{r}_{B/A} = 4\mathbf{e}_{AB} = 3.46\mathbf{i} + 2\mathbf{j}$ (ft). The velocity of point B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 8\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3.46 & 2 & 0 \end{bmatrix}$$

$\mathbf{v}_B = (8 - 2\omega_{AB})\mathbf{i} + 3.46\omega_{AB}\mathbf{j}$.

But $\mathbf{v}_B$ is also moving parallel to the bar,

$\mathbf{v}_B = v_B\mathbf{e}_{AB} = v_B(0.866\mathbf{i} + 0.5\mathbf{j})$.

Equate, and separate components:

$(8 - 2\omega_{AB} - 0.866v_B)\mathbf{i} = 0$,

$(0.346\omega_{AB} - 0.5v_B)\mathbf{j} = 0$.

Solve: $\omega_{AB} = 1$ rad/s, $v_B = 6.93$ ft/s, from which

$\boxed{\mathbf{v}_B = v_B\mathbf{e}_{AB} = 6\mathbf{i} + 3.46\mathbf{j} \text{ (ft/s)}}$

(b) Let (x_C, y_C) be the coordinates of the instantaneous center. The vector from the center to A is

$\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -\mathbf{r}_C = -x_C\mathbf{i} - y_C\mathbf{j}$ (ft).

The vector from the instantaneous center to B is

$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = (3.46 - x_C)\mathbf{i} + (2 - y_C)\mathbf{j}$.

The velocity of point A is

$$\mathbf{v}_A = 8\mathbf{i} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -x_C & -y_C & 0 \end{bmatrix}$$

$= \omega_{AB}y_C\mathbf{i} - \omega_{AB}x_C\mathbf{j}$ (ft/s).

From which $\boxed{x_C = 0}$, and $\omega_{AB}y_C = 8$. The velocity of point B is

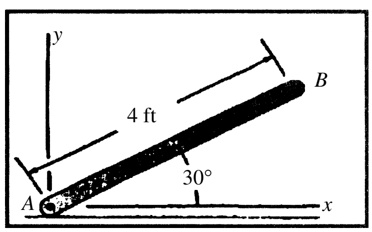
$$\mathbf{v}_B = v_B\mathbf{e}_{AB} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3.46 - x_C & 2 - y_C & 0 \end{bmatrix}$$

$= -\omega_{AB}(2 - y_C)\mathbf{i} + \omega_{AB}(3.46 - x_C)\mathbf{j}$.

Equate terms and substitute

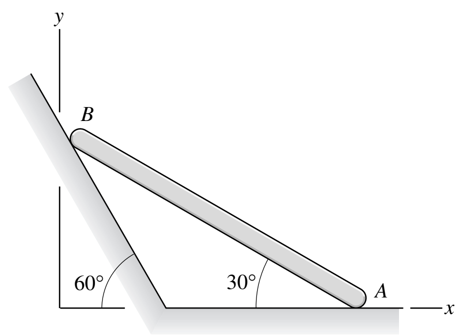
$\omega_{AB}y_C = 8$, and $x_C = 0$, to obtain: $(0.866v_B + 2\omega_{AB} - 8)\mathbf{i} = 0$, and $(0.5v_C - 3.46\omega_{AB})\mathbf{j} = 0$. These equations are algebraically identical with those obtained in Part (a) above (as can be shown by multiplying all terms by -1). Thus $\omega_{AB} = 1$ rad/s, $v_B = 6.93$ (ft/s), and the velocity of B is that obtained in Part (a)

$\boxed{\mathbf{v}_B = v_B\mathbf{e}_{AB} = 6\mathbf{i} + 3.46\mathbf{j} \text{ (ft/s)}}$.

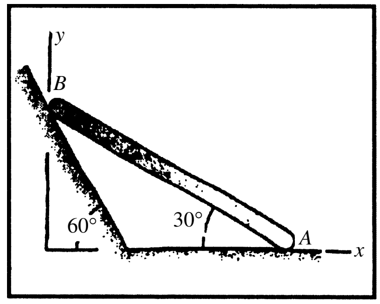


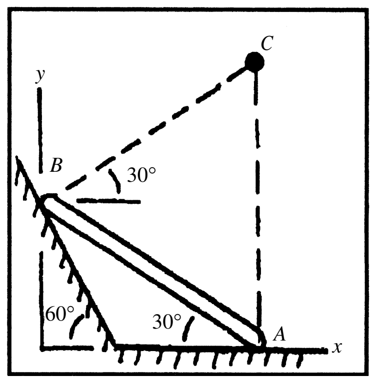
Problem 17.70 Points A and B of the 4-ft bar slide on the plane surfaces. Point B is sliding down the slanted surface at 2 ft/s.

(a) What are the coordinates of the instantaneous center of the bar?
(b) Use the instantaneous center to determine the velocity of A.



Solution:





(a) The strategy is to determine the coordinates of the instantaneous center by finding the intersection of perpendiculars to the motion. The unit vector parallel to the slanting surface in the direction of motion of B is

$$\mathbf{e}_S = \mathbf{i}\cos 60^\circ - \mathbf{j}\sin 60^\circ = 0.5\mathbf{i} - 0.866\mathbf{j}.$$

The vector perpendicular to this motion is $\mathbf{e}_{SP} = 0.866\mathbf{i} + 0.5\mathbf{j}$. A point on the line perpendicular to the velocity of B, from point B, is

$$\mathbf{L}_{PB} = L_B(0.866\mathbf{i} + 0.5\mathbf{j}) + 4\sin 30^\circ\mathbf{j} = 0.866L_B\mathbf{i} + (0.5L_B + 2)\mathbf{j}$$

where L_B is the magnitude of the distance to the point along the line, and the height of B has been added to the y coordinate. The horizontal distance to the intersection of this line with a perpendicular to the motion of A is: $L_B(0.866) = 4\cos 30^\circ = 3.46$ in., from which $L_B = 4$ ft. The vertical height of the intercept of this line and the perpendicular to the motion of A is $0.5L_B + 2 = 4$ ft. The coordinates of the instantaneous center are $\boxed{(3.46, 4)\text{ ft}}$.

(b) The angular velocity of the bar is determined from the known velocity of point B. The vector distance from the instantaneous center is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -3.46\mathbf{i} - (4 - 4\sin 30^\circ)\mathbf{j} = -3.46\mathbf{i} - 2\mathbf{j}\text{ (ft)}$$

The velocity of B is

$$\mathbf{v}_B = 2\mathbf{e}_S = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -3.46 & -2 & 0 \end{bmatrix}$$

$$= \omega_{AB}(2\mathbf{i} - 3.46\mathbf{j}).$$

Equate components: $(1 - 2\omega_{AB})\mathbf{i} = 0$, from which $\omega_{AB} = 0.5$ rad/s. The vector from the instantaneous center to point A is

$$\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = (3.46\mathbf{i} - 3.46\mathbf{i} - 4\mathbf{j}) = -4\mathbf{j}\text{ (ft)}.$$

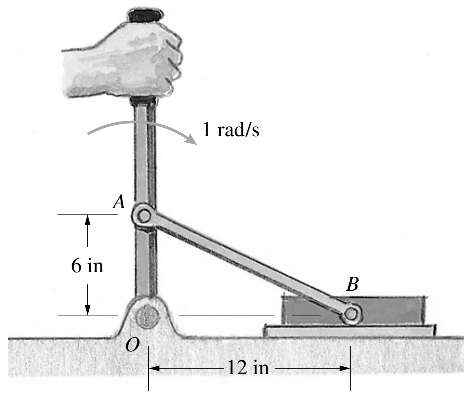
The velocity of point A is

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0 & -4 & 0 \end{bmatrix}$$

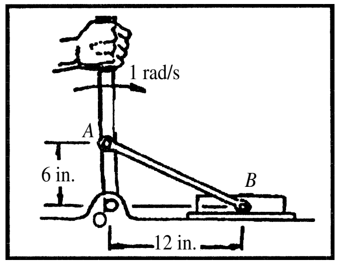
$$= 4\omega_{AB}\mathbf{i}$$

$\boxed{= 2\mathbf{i}\text{ (ft/s)}}$.

Problem 17.71 Use instantaneous centers to determine the horizontal velocity of B.



Solution: The instantaneous center of OA lies at O, by definition, since O is the point of zero velocity, and the velocity at point A is parallel to the x-axis:

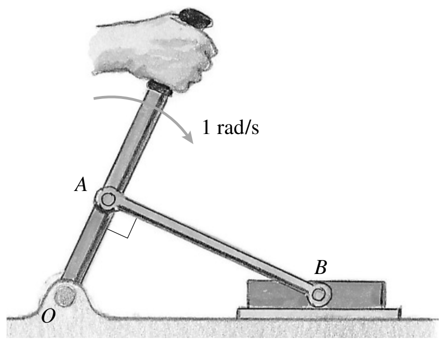


$$\mathbf{v}_A = \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{OA} \\ 0 & 6 & 0 \end{bmatrix} = 6\mathbf{i} \text{ (in/s)}.$$

A line perpendicular to this motion is parallel to the y axis. The point B is constrained to move on the x axis, and a line perpendicular to this motion is also parallel to the y axis. These two lines will not intersect at any finite distance from the origin, hence *at the instant shown the instantaneous center of bar AB is at infinity and the angular velocity of bar AB is zero*. At the instant shown, the bar AB translates only, from which the horizontal velocity of B is the horizontal velocity at A:

$$\boxed{\mathbf{v}_B = \mathbf{v}_A = 6\mathbf{i} \text{ (in/s)}.}$$

Problem 17.72 When the mechanism in Problem 17.71 is in the position shown here, use instantaneous centers to determine the horizontal velocity of B.



Solution: The strategy is to determine the intersection of lines perpendicular to the motions at A and B. The velocity of A is parallel to the bar AB. A line perpendicular to the motion at A will be parallel to the bar OA. From the dimensions given in Problem 17.71, the length of bar AB is $r_{AB} = \sqrt{6^2 + 12^2} = 13.42$ in. Consider the triangle OAB. The interior angle at B is

$$\beta = \tan^{-1}\left(\frac{6}{r_{AB}}\right) = 24.1^\circ,$$

and the interior angle at O is $\theta = 90^\circ - \beta = 65.9^\circ$. The unit vector parallel to the handle OA is $\mathbf{e}_{OA} = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$, and a point on the line is $\mathbf{L}_{OA} = L_{OA}\mathbf{e}_{OA}$, where L_{OA} is the magnitude of the distance of the point from the origin. A line perpendicular to the motion at B is parallel to the y axis. At the intersection of the two lines

$$L_{OA}\cos\theta = \frac{r_{AB}}{\cos\beta},$$

from which $L_{OA} = 36$ in. The coordinates of the instantaneous center are (14.7, 32.9) (in.).

Check: From geometry, the triangle OAB and the triangle formed by the intersecting lines and the base are similar, and thus the interior angles are known for the larger triangle. From the law of sines

$$\frac{L_{OA}}{\sin 90^\circ} = \frac{r_{OB}}{\sin\beta} = \frac{r_{AB}}{\sin\beta\cos\beta} = 36 \text{ in.},$$

and the coordinates follow immediately from $\mathbf{L}_{OA} = L_{OA}\mathbf{e}_{OA}$. *check.* The vector distance from O to A is $\mathbf{r}_{A/O} = 6(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = 2.450\mathbf{i} + 5.478\mathbf{j}$ (in.). The angular velocity of the bar AB is determined from the known linear velocity at A.

$$\mathbf{v}_A = \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 2.450 & 5.477 & 0 \end{bmatrix}$$

$$= 5.48\mathbf{i} - 2.45\mathbf{j} \text{ (in/s)}.$$

The vector from the instantaneous center to point A is

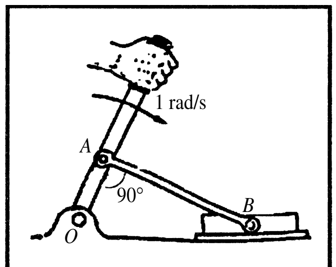
$$\mathbf{r}_{A/C} = \mathbf{r}_{OA} - \mathbf{r}_C = 6\mathbf{e}_{OA} - (14.7\mathbf{i} + 32.86\mathbf{j})$$

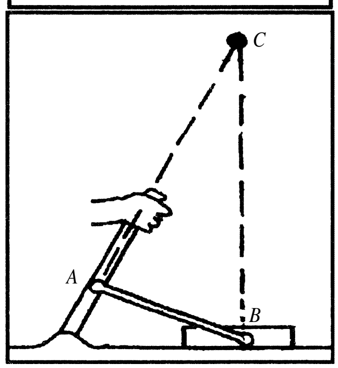
$$= -12.25\mathbf{i} - 27.39\mathbf{j} \text{ (in.)}$$

The velocity at point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -12.25 & -27.39 & 0 \end{bmatrix}$$

$$= \omega_{AB}(27.39\mathbf{i} - 12.25\mathbf{j}) \text{ (ft/s)}.$$





Equate the two expressions for the velocity at point A and separate components, $5.48\mathbf{i} = 27.39\omega_{AB}$, $-2.45\mathbf{j} = -12.25\omega_{AB}\mathbf{j}$ (one of these conditions is superfluous) and solve to obtain $\omega_{AB} = 0.2$ rad/s, counterclockwise.

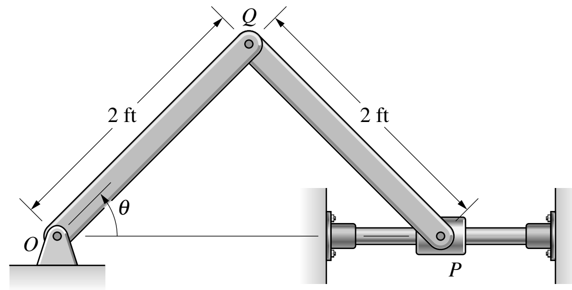
[*Check*: The distance *OA* is 6 in. The magnitude of the velocity at A is $\omega_{OA}(6) = (1)(6) = 6$ in/s. The distance to the instantaneous center from O is $\sqrt{14.7^2 + 32.9^2} = 36$ in., and from C to A is $(36 - 6) = 30$ in. from which $30\omega_{AB} = 6$ in/s, from which $\omega_{AB} = 0.2$ rad/s. *check.*]. The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 14.7\mathbf{i} - (14.7\mathbf{i} + 32.86\mathbf{j} = -32.86\mathbf{j}) \text{ (in.)}$$

The velocity at point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.2 \\ 0 & -32.86 & 0 \end{bmatrix} \boxed{= 6.57\mathbf{i} \text{ (in/s)}}$$

Problem 17.73 The angle $\theta = 45°$, and the bar OQ is rotating in the counterclockwise direction at 0.2 rad/s. Use instantaneous centers to determine the velocity of the sleeve P.



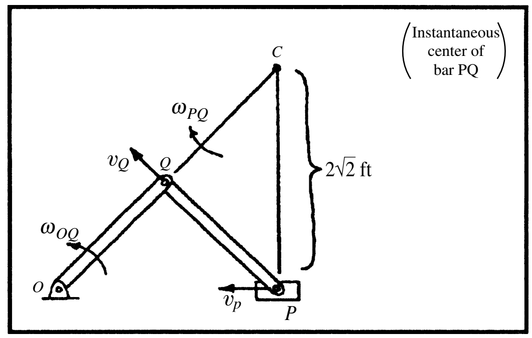
Solution: The velocity of Q is

$v_Q = 2\omega_{0Q} = 2(0.2) = 0.4$ ft/s.

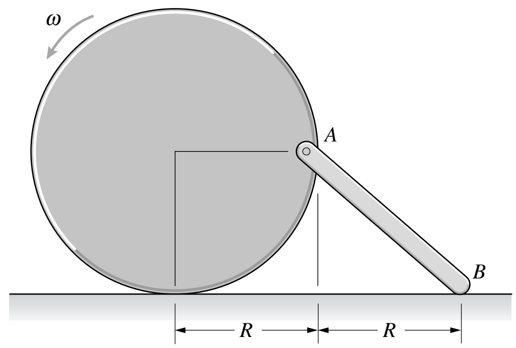
Therefore

$|\overline{\omega}_{PQ}| = \dfrac{v_Q}{2 \text{ ft}} = \dfrac{0.4}{2} = 0.2$ rad/s

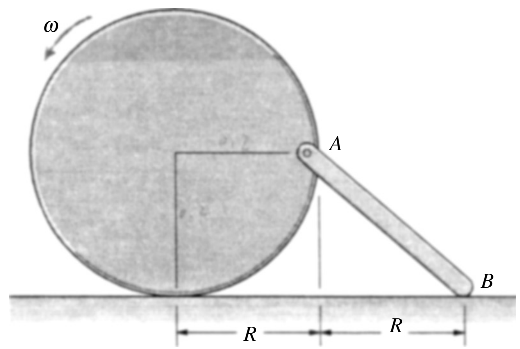
(clockwise) and $|\mathbf{v}_P| = 2\sqrt{2}\omega_{PQ} = 0.566$ ft/s ($\mathbf{v}_P$ is to the left).



Problem 17.74 The radius of the disk is $R = 0.2$ m. The disk is rotating in the counterclockwise direction with angular velocity $\omega = 4$ rad/s. Use instantaneous centers to determine the angular velocity of the bar AB and the velocity of point B.



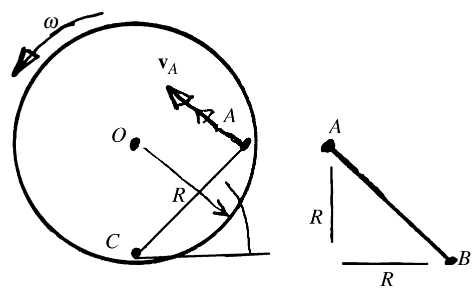
Solution: From the figure, C is the instantaneous center of rotation of the disk. The distance from C to A is $\sqrt{2}R$ $|\mathbf{v}_A| = \sqrt{2}R\omega$ and



$$\mathbf{v}_A = -\sqrt{2}R\omega \cos 45^\circ \mathbf{i} + \sqrt{2}R\omega \sin 45^\circ \mathbf{j}$$

$$= -R\omega\mathbf{i} + R\omega\mathbf{j}$$

Evaluating with $R = 0.2$ m, $\omega = 4$ rad/s $\mathbf{v}_A = -0.8\mathbf{i} - 0.8\mathbf{i}$ m/s
Now consider link AB



$$\ell_1 = \sqrt{2}R$$

$$\ell_2 = 2R$$

$$|\mathbf{v}_A| = \ell_1|\omega_{AB}|$$

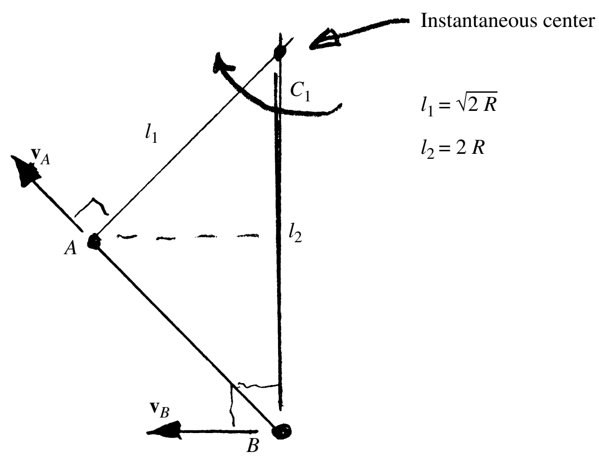
$$\sqrt{2}R\omega = \sqrt{2}R|\omega_{AB}|$$

$$|\omega_{AB}| = \omega = 4 \text{ rad/s (clockwise)}$$

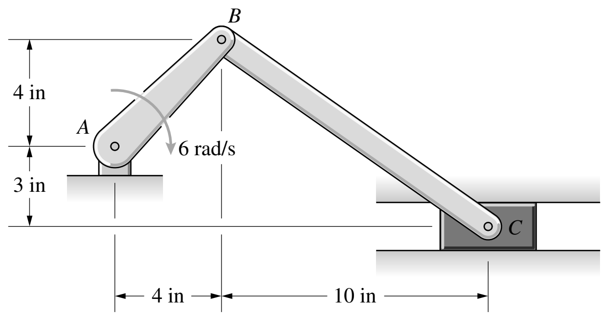
$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C_i}$$

$$= -4\mathbf{k} \times (-RR)\mathbf{j}$$

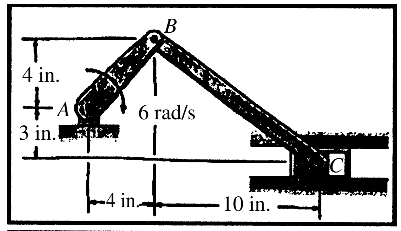
$\underline{\mathbf{v}_B = -1.6\mathbf{i} \text{ (m/s)}}$.



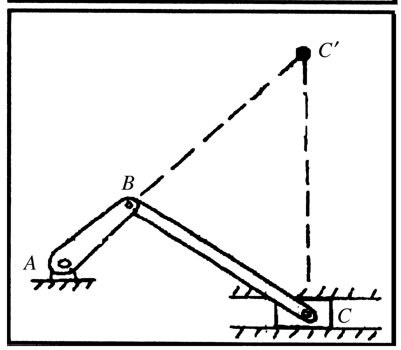
Problem 17.75 Bar AB rotates at 6 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocity of bar BC.



Solution: Choose a coordinate system with origin at A and y axis vertical. Let C' denote the instantaneous center. The instantaneous center for bar AB is the point A, by definition, since A is the point of zero velocity. The vector AB is $\mathbf{r}_{B/A} = 4\mathbf{i} + 4\mathbf{j}$ (in.). The velocity at B is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -6 \\ 4 & 4 & 0 \end{bmatrix} = 24\mathbf{i} - 24\mathbf{j} \text{ (in/s)}.$$



The unit vector parallel to AB is also the unit vector perpendicular to the velocity at B,

$$\mathbf{e}_{AB} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}).$$

The vector location of a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{e}_{AB}$, where L_{AB} is the magnitude of the distance from point A to the point on the line. The vector location of a point on a perpendicular to the velocity at C is $\mathbf{L}_C = (14\mathbf{i} + y\mathbf{j})$ where y is the y-coordinate of the point referenced to an origin at A. When the two lines intersect,

$$\frac{L_{AB}}{\sqrt{2}}\mathbf{i} = 14\mathbf{i},$$

and $y = \dfrac{L_{AB}}{\sqrt{2}} = 14$

from which $L_{AB} = 19.8$ in., and the coordinates of the instantaneous center are (14, 14) (in.).

[*Check*: The line AC' is the hypotenuse of a right triangle with a base of 14 in. and interior angles of 45°, from which the coordinates of C' are (14, 14) in. *check*.]. The angular velocity of bar BC is determined from the known velocity at B. The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 4\mathbf{i} + 4\mathbf{j} - 14\mathbf{i} - 14\mathbf{j} = -10\mathbf{i} - 10\mathbf{j}.$$

The velocity of point B is

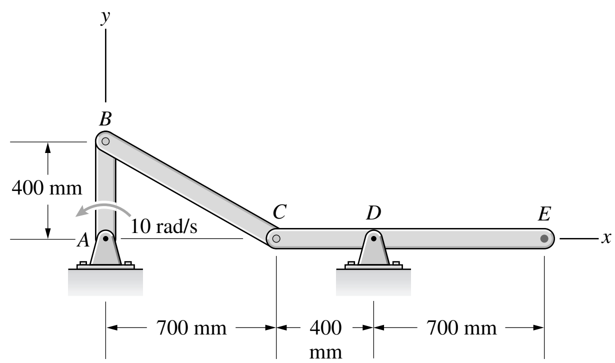
$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & -10 & 0 \end{bmatrix}$$

$$= \omega_{BC}(10\mathbf{i} - 10\mathbf{j}) \text{ (in/s)}.$$

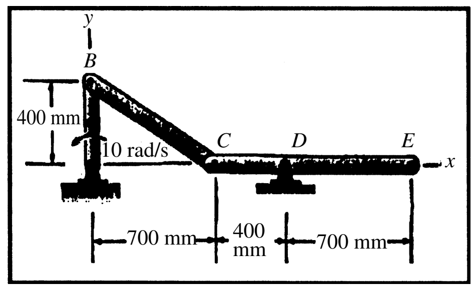
Equate the two expressions for the velocity: $24 = 10\omega_{BC}$, from which

$$\boxed{\omega_{BC} = 2.4 \text{ rad/s}},$$

Problem 17.76 Bar AB rotates at 10 rad/s in the counterclockwise direction. Use instantaneous centers to determine the velocity of point E.



Solution: Choose a coordinate system with origin at A, with the x axis parallel to bar CDE. The instantaneous center of bar AB is point A, and the instantaneous center of bar CDE is point D, by definition, since these are the points with zero velocity. Since AB rotates about A, the velocity of point B will be parallel to the x axis, and a line perpendicular to the velocity of B will be parallel to the y axis. Since CDE rotates about D, the velocity of point C is parallel to the y axis. A line perpendicular to the velocity at C will be parallel to the x axis. From inspection, the intersection of these perpendicular lines will be point A. *Thus point A is the instantaneous center for both bar AB and bar BC*. The velocity of point B, using the known angular velocity of bar AB, is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 400 & 0 \end{bmatrix} = -4000\mathbf{i} \text{ (mm/s)}.$$

The velocity at B, using the unknown angular velocity of BC, and using the point A as the instantaneous center of BC, is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & 400 & 0 \end{bmatrix} = -400\omega_{BC}\mathbf{i} \text{ (mm/s)}.$$

Equate the two expressions for the velocity at B, $-4000 = -400\omega_{BC}$, from which, $\omega_{BC} = \omega_{AB} = 10$ rad/s. The vector distance from A to C is $\mathbf{r}_{C/A} = 700\mathbf{i}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 700 & 0 & 0 \end{bmatrix} = 7000\mathbf{j} \text{ (mm/s)}.$$

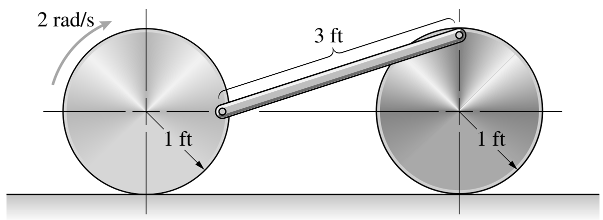
The vector distance from the instantaneous center at D to C is $\mathbf{r}_{C/D} = -400\mathbf{i}$ (mm). The velocity at point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CDE} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CDE} \\ -400 & 0 & 0 \end{bmatrix} = -400\omega_{CDE}\mathbf{j}$$

Equate the expressions for the velocity at C, $7000 = -400\omega_{CDE}$ from which: $\omega_{CDE} = -17.5$ rad/s clockwise. The vector from D to E is $\mathbf{r}_{E/D} = 700\mathbf{i}$ (mm). The velocity of point E is

$$\mathbf{v}_E = \boldsymbol{\omega}_{CDE} \times \mathbf{r}_{E/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -17.5 \\ 700 & 0 & 0 \end{bmatrix} \boxed{= -122.50\mathbf{j} \text{ (mm/s)}}$$

Problem 17.77 The disks roll on the plane surface. The left disk rotates at 2 rad/s in the clockwise direction. Use the instantaneous centers to determine the angular velocities of the bar and the right disk.



Solution: Choose a coordinate system with the origin at the point of contact of the left disk with the surface, and the x axis parallel to the plane surface. Denote the point of attachment of the bar to the left disk by A, and the point of attachment to the right disk by B. The instantaneous center of the left disk is the point of contact with the surface. The vector distance from the point of contact to the point A is $\mathbf{r}_{A/P} = \mathbf{i} + \mathbf{j}$ (ft). The velocity of point A is

$$\mathbf{v}_A = \omega_{LD} \times \mathbf{r}_{A/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 1 & 1 & 0 \end{bmatrix} = 2\mathbf{i} - 2\mathbf{j} \text{ (ft/s)}.$$

The point on a line perpendicular to the velocity at A is $\mathbf{L}_A = L_A(\mathbf{i} + \mathbf{j})$, where L_A is the distance of the point from the origin. The point B is at the top of the right disk, and the velocity is constrained to be parallel to the x axis. A point on a line perpendicular to the velocity at B is $\mathbf{L}_B = (1 + 3\cos\theta)\mathbf{i} + y\mathbf{j}$ (ft), where

$$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ.$$

At the intersection of these two lines $L_A = 1 + 3\cos\theta = 3.83$ ft, and the coordinates of the instantaneous center of the bar are (3.83, 3.83) (ft). The angular velocity of the bar is determined from the known velocity of point A. The vector from the instantaneous center to point A is

$$\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = \mathbf{i} + \mathbf{j} - 3.83\mathbf{i} - 3.83\mathbf{j} = -2.83\mathbf{i} - 2.83\mathbf{j} \text{ (ft)}.$$

The velocity of point A is

$$\mathbf{v}_A = \omega_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2.83 & -2.83 & 0 \end{bmatrix}$$

$$= \omega_{AB}(2.83\mathbf{i} - 2.83\mathbf{j}) \text{ (ft/s)}.$$

Equate the two expressions and solve:

$\boxed{\omega_{AB} = \frac{2}{2.83} = 0.7071 \text{ (rad/s)}}$ counterclockwise.

The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = (1 + 3\cos\theta)\mathbf{i} + 2\mathbf{j} - 3.83\mathbf{i} - 3.83\mathbf{j} = -1.83\mathbf{j}.$$

The velocity of point B is

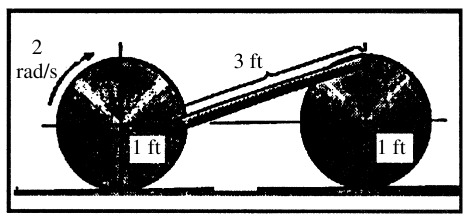
$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.7071 \\ 0 & -1.83 & 0 \end{bmatrix} = 1.294\mathbf{i} \text{ (ft/s)}.$$

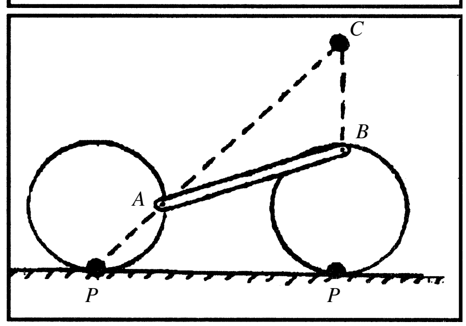
Using the fixed center at point of contact:

$$\mathbf{v}_B = \omega_{RD} \times \mathbf{r}_{B/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{RD} \\ 0 & 2 & 0 \end{bmatrix} = -2\omega_{RD}\mathbf{i} \text{ (ft/s)}.$$

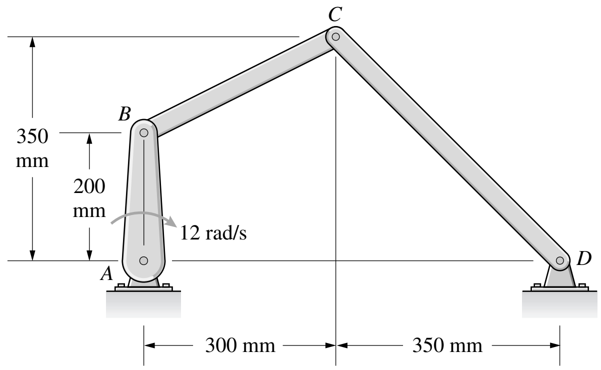
Equate the two expressions for $\mathbf{v}_B$ and solve:

$\boxed{\omega_{RD} = -0.647 \text{ rad/s, clockwise.}}$





Problem 17.78 Bar AB rotates at 12 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocities of bars BC and CD.



Solution: Choose a coordinate system with the origin at A and the x axis parallel to AD. The instantaneous center of bar AB is point A, by definition. The velocity of point B is normal to the bar AB. Using the instantaneous center A and the known angular velocity of bar AB the velocity of B is

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The unit vector perpendicular to the velocity of B is $\mathbf{e}_{AB} = \mathbf{j}$, and a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{j}$ (mm). The instantaneous center of bar CD is point D, by definition. The velocity of point C is constrained to be normal to bar CD. The interior angle at D is 45°, by inspection. The unit vector parallel to DC (and perpendicular to the velocity at C) is

$$\mathbf{e}_{DC} = -\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ = \left(\frac{1}{\sqrt{2}}\right)(-\mathbf{i} + \mathbf{j}).$$

The point on a line parallel to DC is

$$\mathbf{L}_{DC} = \left(650 - \frac{L_{DC}}{\sqrt{2}}\right)\mathbf{i} + \frac{L_{DC}}{\sqrt{2}}\mathbf{j} \text{ (mm)}.$$

At the intersection of these lines $\mathbf{L}_{AB} = \mathbf{L}_{DC}$, from which

$$\left(650 - \frac{L_{DC}}{\sqrt{2}}\right) = 0$$

and $L_{AB} = \dfrac{L_{DC}}{\sqrt{2}}$,

from which $L_{DC} = 919.2$ mm, and $L_{AB} = 650$ mm. The coordinates of the instantaneous center of bar BC are (0, 650) (mm). Denote this center by C'. The vector from C' to point B is

$$\mathbf{r}_{B/C'} = \mathbf{r}_B - \mathbf{r}_{C'} = 200\mathbf{j} - 650\mathbf{j} = -450\mathbf{j}.$$

The vector from C' to point C is

$$\mathbf{r}_{C/C'} = 300\mathbf{i} + 350\mathbf{j} - 650\mathbf{j} = 300\mathbf{i} - 300\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{BC} \times \mathbf{r}_{B/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & -450 & 0 \end{bmatrix} = 450\omega_{BC}\mathbf{i} \text{ (mm/s)}.$$

Equate and solve: $2400 = 450\omega_{BC}$, from which

$$\boxed{\omega_{BC} = \frac{2400}{450} = 5.33 \text{ (rad/s)}}.$$

The angular velocity of bar CD is determined from the known velocity at point C. The velocity at C is

$$\mathbf{v}_C = \omega_{BC} \times \mathbf{r}_{C/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5.33 \\ 300 & -300 & 0 \end{bmatrix}$$

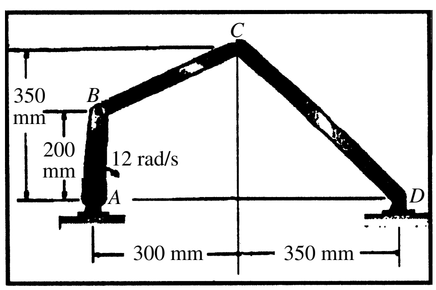
$$= 1600\mathbf{i} + 1600\mathbf{j} \text{ (mm/s)}.$$

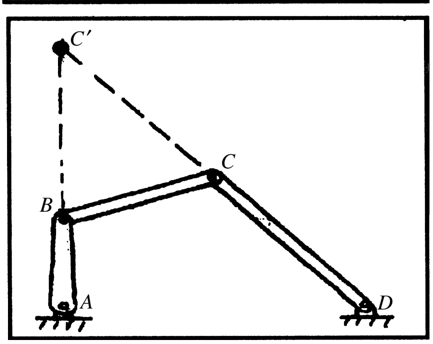
The vector from D to point C is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity at C is

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

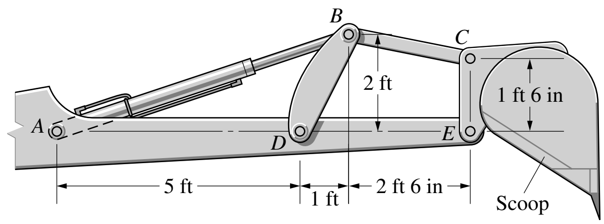
$$= -350\omega_{CD}\mathbf{i} - 350\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate and solve: $\boxed{\omega_{CD} = -4.57 \text{ rad/s}}$ clockwise.





Problem 17.79 The horizontal member *ADE* supporting the scoop is stationary. The link *BD* is rotating in the clockwise direction at 1 rad/s. Use instantaneous centers to determine the angular velocity of the scoop.



Solution: The distance from D to B is $r_{BD} = \sqrt{1^2 + 2^2} = 2.24$ ft. The distance from B to H is

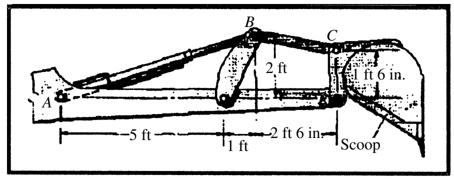
$$r_{BH} = \frac{3.5}{\cos 63.4^\circ} - r_{BD} = 5.59 \text{ ft},$$

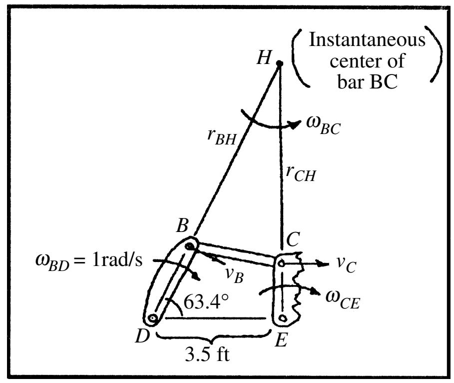
and the distance from C to H is $r_{CH} = 3.5 \tan 63.4^\circ - r_{CE} = 5.5$ ft. The velocity of B is $v_B = r_{BD}\omega_{BD} = (2.24)(1) = 2.24$ ft/s. Therefore

$$\omega_{BC} = \frac{v_B}{r_{BH}} = \frac{2.24}{5.59} = 0.4 \text{ rad/s}.$$

The velocity of C is $v_c = r_{CH}\omega_{BC} = (5.5)(0.4) = 2.2$ ft/s, so the angular velocity of the scoop is

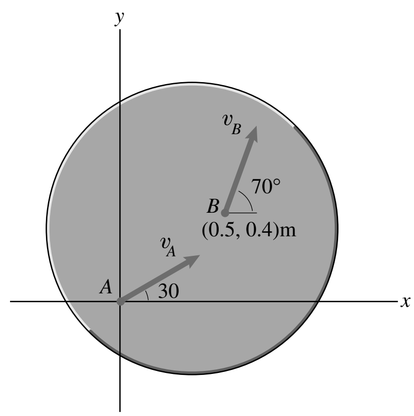
$$\omega_{CE} = \frac{v_C}{r_{CE}} = \frac{2.2}{1.5} = 1.47 \text{ rad/s}$$



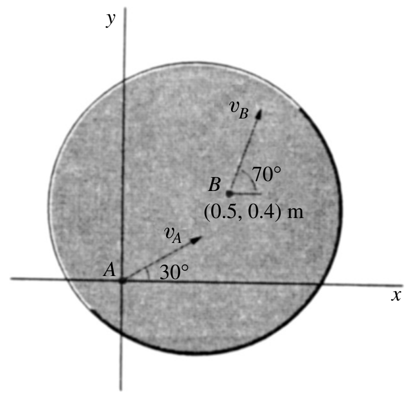


Problem 17.80 The disk is in planar motion. The directions of the velocities of points A and B are shown. The velocity of point A is $v_A = 2$ m/s.

(a) What are the coordinates of the disk's instantaneous center?
(b) Determine the velocity v_B and the disk's angular velocity.



Solution:



$\boldsymbol{\omega} = \omega\mathbf{k}$

$r_{c/A} = x_c\mathbf{i} + y_c\mathbf{j}$

$\mathbf{r}_{c/B} = (x_c - x_B)\mathbf{i} + (y_c - y_B)\mathbf{j}$

The velocity of C, the instantaneous center, is zero.

$\mathbf{v}_c = 0 = \mathbf{v}_A + \omega\mathbf{k} \times \mathbf{r}_{c/A}$

$$\begin{cases} 0 = v_{A_x} - \omega y_c & \mathbf{(1)} \\ 0 = v_{A_y} + \omega x_c & \mathbf{(2)} \end{cases}$$

where $v_{A_x} = v_A \cos 30^\circ = 2\cos 30^\circ$ m/s

$v_{A_y} = v_A \sin 30^\circ = 1$ m/s

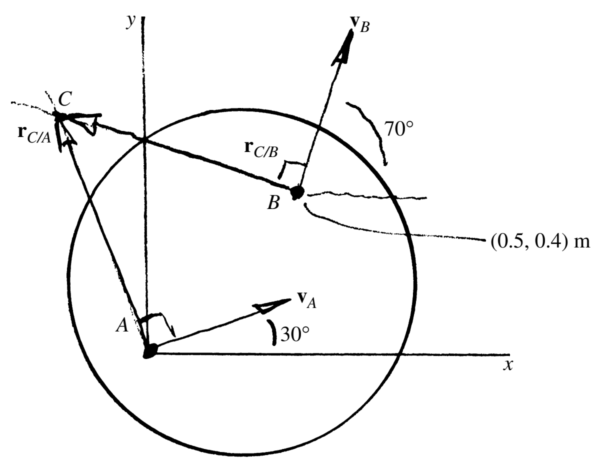
$v_{B_x} = v_B \cos 70^\circ$

$v_{B_y} = v_B \sin 70^\circ$

Also $\mathbf{v}_c = 0 = \mathbf{v}_B + \omega\mathbf{k} \times \mathbf{r}_{c/B}$

$0 = v_B \cos 70^\circ - \omega(y_c - y_B)$ **(3)**

$0 = v_B \sin 70^\circ + \omega(x_c - x_B)$ **(4)**



Eqns (1) → (4) are 4 eqns in the four unknowns ω, v_B, x_c, and y_c.

Solving,

$\omega = 2.351$ rad/s,

$\underline{\boldsymbol{\omega} = 2.351\mathbf{k} \text{ rad/s}}$,

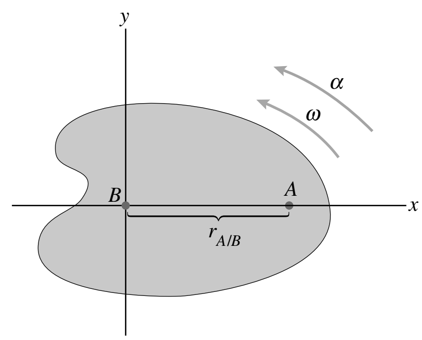
$\underline{v_B = 2.31 \text{ m/s}}$,

$\underline{x_c = -0.425 \text{ m}}$,

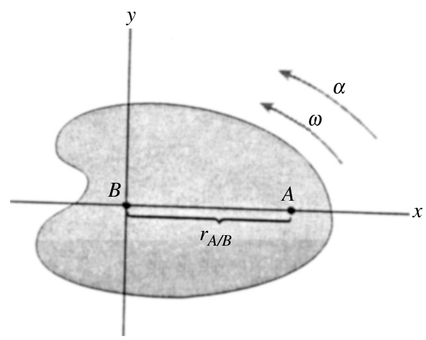
$\underline{y_c = 0.737 \text{ m}}$.

Problem 17.81 The rigid body rotates about the z axis with counterclockwise angular velocity $\omega = 4$ rad/s and counterclockwise angular acceleration $\alpha = 2$ rad/s^2. The distance $r_{A/B} = 0.6$ m.

(a) What are the rigid body's angular velocity and angular acceleration vectors?
(b) Determine the acceleration of point A relative to point B first by using Eq. (17.9) and then by using Eq. (17.10).



Solution:



(a) By definition,

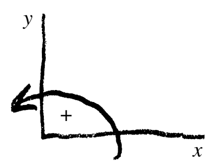
$$\boldsymbol{\omega} = 4\mathbf{k},$$

$$\boldsymbol{\alpha} = 6\mathbf{k}.$$

(b) $\mathbf{a}_{A/B} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) + \boldsymbol{\alpha} \times \mathbf{r}_{A/B}$

$$\mathbf{a}_{A/B} = 4\mathbf{k} \times (4\mathbf{k} \times 0.6\mathbf{i}) + 2\mathbf{k} \times 0.6\mathbf{i}$$

$$\underline{\mathbf{a}_{A/B} = -9.6\mathbf{i} + 1.2\mathbf{j} \text{ (m/s}^2\text{)}}.$$



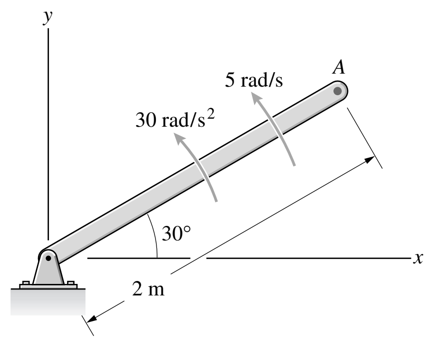
Using Eq. (17.10),

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 r_{A/B}$$

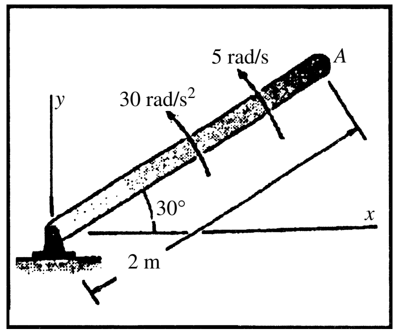
$$= 2\mathbf{k} \times 0.6\mathbf{i} - 16(0.6)\mathbf{j}$$

$$\underline{\mathbf{a}_{A/B} = -9.6\mathbf{i} + 1.2\mathbf{j} \text{ (m/s}^2\text{)}}.$$

Problem 17.82 The bar rotates with a counterclockwise angular velocity of 5 rad/s and a counterclockwise angular acceleration of 30 rad/s^2. Determine the acceleration of A (a) by using Eq. (17.9) and (b) by using Eq. (17.10).



Solution:



(a) Eq. (17.9): $\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$.

Substitute values:

$\mathbf{a}_B = 0. \qquad \boldsymbol{\alpha} = 30\mathbf{k}$ (rad/s^2),

$\mathbf{r}_{A/B} = 2(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.732\mathbf{i} + \mathbf{j}$ (m).

$\boldsymbol{\omega} = 5\mathbf{k}$ (rad/s).

Expand the cross products:

$$\boldsymbol{\alpha} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 1.732 & 1 & 0 \end{bmatrix} = -30\mathbf{i} + 52\mathbf{j} \text{ (m/s}^2\text{)}.$$

$$\boldsymbol{\omega} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 1.732 & 1 & 0 \end{bmatrix} = -5\mathbf{i} + 8.66\mathbf{j} \text{ (m/s)}.$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ -5 & 8.66 & 0 \end{bmatrix} = -43.3\mathbf{i} - 25\mathbf{j} \text{ (m/s}^2\text{)}.$$

Collect terms: $\boxed{\mathbf{a}_A = -73.3\mathbf{i} + 27\mathbf{j} \text{ (m/s}^2\text{)}}$.

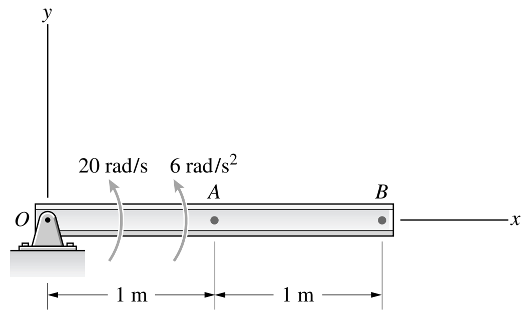
(b) Eq. (17.10): $\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$.

Substitute values, and expand the cross product as in Part (b) to obtain

$$\boxed{\mathbf{a}_A = -30\mathbf{i} + 52\mathbf{j} - (5^2)(1.732\mathbf{i} + \mathbf{j}) = -73.3\mathbf{i} + 27\mathbf{j} \text{ (m/s}^2\text{)}}$$

Problem 17.83 The bar rotates with a counterclockwise angular velocity of 20 rad/s and a counterclockwise angular acceleration of 6 rad/s^2.

(a) By applying Eq. (17.10) to point A and the fixed point O, determine the acceleration of A.
(b) By using the result of part (a) and Eq. (17.10), to points A and B, determine the acceleration point B.



Solution:

(a) $\mathbf{a}_A = \mathbf{a}_0 + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

where

$\boldsymbol{\omega} = 20\mathbf{k}$ rad/s

$\boldsymbol{\alpha} = 6\mathbf{k}$ rad/s^2

$\mathbf{r}_{A/O} = 1\mathbf{i}$, and $\mathbf{a}_A = 0$

$\mathbf{a}_A = O + 6\mathbf{k} \times 1\mathbf{i} - 400(1\mathbf{i})$

$\underline{\mathbf{a}_A = -400\mathbf{i} + 6\mathbf{j} \text{ (m/s}^2\text{)}}$.

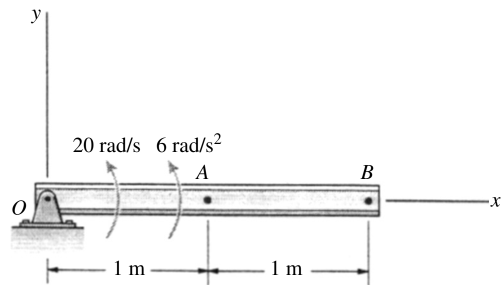
(b) $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

where

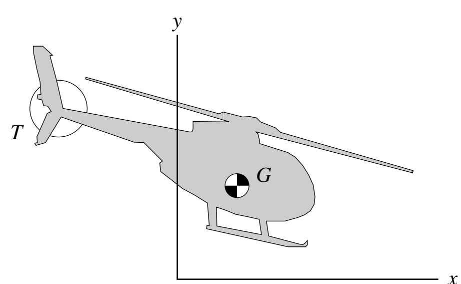
$\mathbf{r}_{B/A} = 1\mathbf{i}$

$\mathbf{a}_B = -400\mathbf{i} + 6\mathbf{j} + 6\mathbf{k} \times 1\mathbf{i} - 400(1\mathbf{i})$

$\underline{\mathbf{a}_B = -800\mathbf{i} + 12\mathbf{j} \text{ (m/s}^2\text{)}}$.



Problem 17.84 The helicopter is in planar motion in the x-y plane. At the instant shown, the position of its center of mass G is $x = 2$ m, $y = 2.5$ m, its velocity is $\mathbf{v}_G = 12\mathbf{i} + 4\mathbf{j}$ (m/s), and its acceleration is $a_G = 2\mathbf{i} + 3\mathbf{j}$ (m/s^2). The position of point T where the tail rotor is mounted is $x = -3.5$ m, $y = 4.5$ m. The helicopter's angular velocity is 0.2 rad/s clockwise, and its angular acceleration is 0.1 rad/s^2 counterclockwise. What is the acceleration of point T?

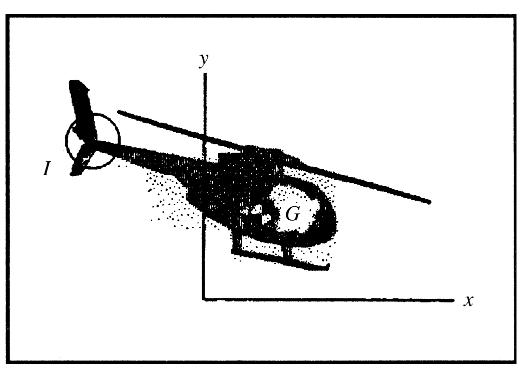


Solution: The acceleration of T is

$\mathbf{a}_T = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{T/G} - \omega^2 \mathbf{r}_{T/G}$;

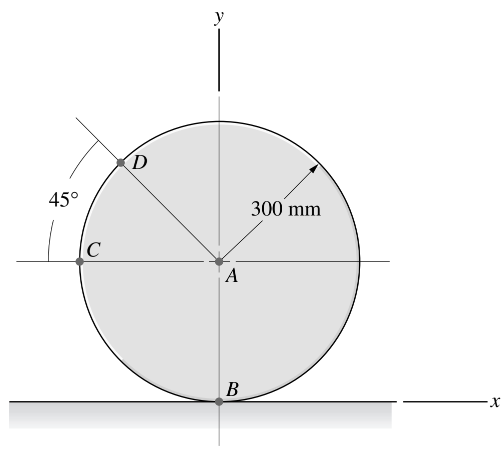
$$\mathbf{a}_T = 2\mathbf{i} + 3\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ -5.5 & 2 & 0 \end{vmatrix} - (0.2)^2(-5.5\mathbf{i} + 2\mathbf{j})$$

$= 2.02\mathbf{i} + 2.37\mathbf{j}$ (m/s^2).



Problem 17.85 The disk rolls on the plane surface. The velocity of point A is 6 m/s to the right, and its acceleration of A is 20 m/s^2 to the right.

(a) What is the angular acceleration vector of the disk?
(b) Determine the accelerations of points B, C and D.



Solution:

(a) The point of contact B between the disk and the surface is stationary. The distance A to B, is $\mathbf{r}_{B/A} = -\mathbf{R} = -R\mathbf{j}(m)$, from which

$$\mathbf{v}_B = 0 = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 6\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0 & -0.3 & 0 \end{bmatrix}$$

$$= (6 + 0.3\omega)\mathbf{i} = 0,$$

from which

$$\omega = -\frac{6}{0.3} = -20 \text{ rad/s}, \boldsymbol{\omega} = -20\mathbf{k}.$$

The instantaneous center of the rolling disk is point B. From $\mathbf{v}_B = 0 = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$, the velocity of point A is $\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{R}$, where $\mathbf{R} = \mathbf{j}R$. Differentiating,

$$\frac{d\mathbf{v}_A}{dt} = \mathbf{a}_A = \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{R} = \boldsymbol{\alpha} \times \mathbf{R}.$$

By definition

$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt} = \frac{d(\omega\mathbf{k})}{dt} = \frac{d\omega}{dt}\mathbf{k} + \omega\frac{d\mathbf{k}}{dt} = \alpha\mathbf{k},$$

since by assumption the disk rolls in a straight line, and $\frac{d\mathbf{k}}{dt} = 0$. From which

$$\mathbf{a}_A = \frac{d\mathbf{v}_A}{dt} = \boldsymbol{\alpha} \times \mathbf{R} = \alpha\mathbf{k} \times \mathbf{R} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 0 & R & 0 \end{bmatrix}$$

$$= -\alpha R\mathbf{i} \text{ (m/s}^2).$$

Substitute values, $-\alpha R\mathbf{i} = 20\mathbf{i}$, from which

$$\alpha = -\frac{20}{0.3} = -66.7 \text{ (m/s}^2), \boxed{\boldsymbol{\alpha} = -66.7\mathbf{k} \text{ (m/s}^2)}.$$

(b) Use Eq. (17.10) to determine the accelerations. The acceleration of point B is

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$

$$= 20\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -66.7 \\ 0 & -3 & 0 \end{bmatrix} - (20)^2(-0.3\mathbf{j}),$$

$$\boxed{\mathbf{a}_B = 20\mathbf{i} - 0.3(66.7)\mathbf{i} + (20^2)(0.3)\mathbf{j} = 120\mathbf{j} \text{ (m/s}^2)}$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{C/A} - \omega^2\mathbf{r}_{C/A}$$

$$= 20\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -66.7 \\ -0.3 & 0 & 0 \end{bmatrix} - (20)^2(-0.3\mathbf{i}),$$

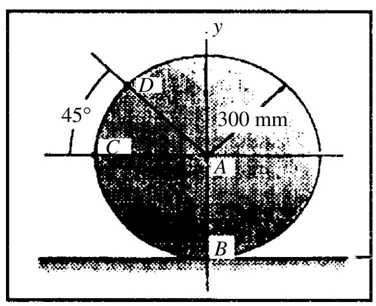
$$\boxed{\mathbf{a}_C = 20\mathbf{i} + 20\mathbf{j} + (20^2)(0.3)\mathbf{i} = 140\mathbf{i} + 20\mathbf{j} \text{ (m/s}^2)}$$

The acceleration of point D is $\mathbf{a}_D = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{D/A} - \omega^2\mathbf{r}_{D/A}$, from which

$$\mathbf{a}_D = 20\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -66.7 \\ -0.3\cos 45^\circ & 0.3\sin 45^\circ & 0 \end{bmatrix}$$

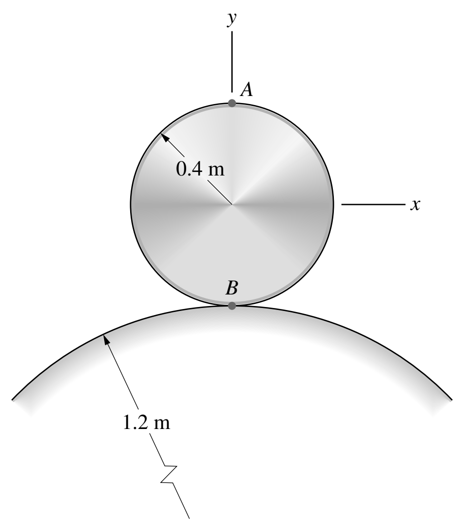
$$- (20^2)(0.3)(-\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ),$$

$$\boxed{\mathbf{a}_D = 119\mathbf{i} - 70.7\mathbf{j} \text{ (m/s}^2)}$$

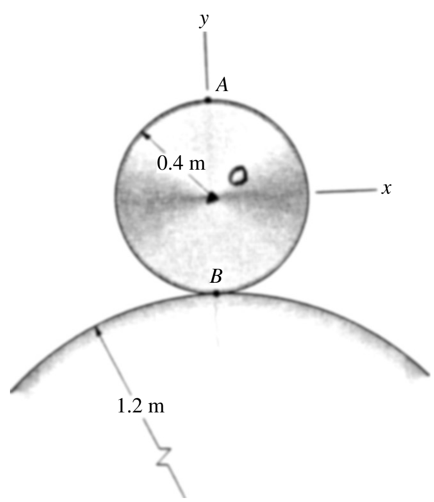


Problem 17.86 The disk rolls on the circular surface with a constant clockwise angular velocity of 1 rad/s. What are the accelerations of points A and B?

Strategy: Begin by determining the acceleration of the center of the disk. Notice that the center moves in a circular path and the magnitude of its velocity is constant.



Solution:



$\mathbf{v}_B = 0$

$\mathbf{v}_0 = \mathbf{v}_B + \omega\mathbf{k} \times \mathbf{r}_{O/B} = (-0.1\mathbf{k}) \times (0.4\mathbf{j})$

$\mathbf{v}_0 = 0.4\ \mathbf{i}$ m/s

Point O moves in a circle at constant speed. The acceleration of O is

$\mathbf{a}_0 = -v_0^2/(R + r)\mathbf{j} = (-0.16)/(1.2 + 0.4)\mathbf{j}$

$\underline{\mathbf{a}_0 = -0.1\mathbf{j}\ (\text{m/s}^2)}$.

$\mathbf{a}_B = \mathbf{a}_0 - \omega^2\mathbf{r}_{B/O} = -0.1\mathbf{j} - (1)^2(-0.4)\mathbf{j}$

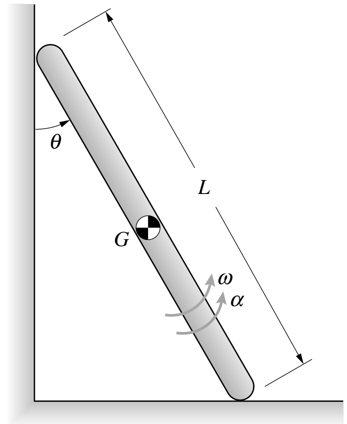
$\underline{\mathbf{a}_B = 0.3\mathbf{j}\ (\text{m/s}^2)}$.

$\mathbf{a}_A = \mathbf{a}_0 - \omega^2\mathbf{r}_{A/O} = -0.1\mathbf{j} - (1)^2(0.4)\mathbf{j}$

$\underline{\mathbf{a}_A = -0.5\mathbf{j}\ (\text{m/s}^2)}$.

Problem 17.87 The endpoints of the bar slide on the plane surfaces. Show that the acceleration of the midpoint G is related to the bar's angular velocity and angular acceleration by

$$\mathbf{a}_G = \tfrac{1}{2}L\left[\left(\alpha\cos\theta - \omega^2\sin\theta\right)\mathbf{i} - \left(\alpha\sin\theta - \omega^2\cos\theta\right)\mathbf{j}\right].$$



Solution: Denote the upper point of contact between rod and the wall by A and the lower point of contact between the rod and the floor by B. The strategy is to use Eq. (17.10) to find the

(a) acceleration of G relative to the points A and B
(b) equate the expressions for the acceleration of G to find the accelerations of A and B and
(c) substitute to find the acceleration of G. The constraint that the motion of A is parallel to the y axis and the motion of B is parallel to the x axis is essential to the solution.

The acceleration of G relative to A is

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}.$$

The acceleration of G relative to B is

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}\times\mathbf{r}_{G/B} - \omega^2\mathbf{r}_{G/B}.$$

Substitute:

$$\mathbf{a}_A = a_A\mathbf{j}, \quad \mathbf{a}_B = a_B\mathbf{i},$$

$$\mathbf{r}_{G/A} = \mathbf{r}_G - \mathbf{r}_A = \frac{L}{2}(\mathbf{i}\sin\theta + \mathbf{j}\cos\theta) - L(\mathbf{j}\cos\theta)$$

$$= \frac{L}{2}(\mathbf{i}\sin\theta - \mathbf{j}\cos\theta),$$

$$\mathbf{r}_{G/B} = \mathbf{r}_G - \mathbf{r}_B = \frac{L}{2}(\mathbf{i}\sin\theta + \mathbf{j}\cos\theta) - L(\mathbf{i}\sin\theta)$$

$$= \frac{L}{2}(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta).$$

From which

$$\mathbf{a}_G = a_A\mathbf{j} + \frac{\alpha L}{2}\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 1\\ \sin\theta & -\cos\theta & 0\end{bmatrix} - \frac{\omega^2 L}{2}(\mathbf{i}\sin\theta - \mathbf{j}\cos\theta)$$

$$\mathbf{a}_G = \left(\frac{L}{2}(\alpha\cos\theta - \omega^2\sin\theta)\right)\mathbf{i} + \left(a_A + \frac{L}{2}(\alpha\sin\theta + \omega^2\cos\theta)\right)\mathbf{j}.$$

The acceleration of G in terms of the acceleration of B,

$$\mathbf{a}_G = a_B\mathbf{i} + \frac{\alpha L}{2}\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 1\\ -\sin\theta & \cos\theta & 0\end{bmatrix} - \frac{\omega^2 L}{2}(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta).$$

$$\mathbf{a}_G = \left(a_B - \frac{L}{2}(\alpha\cos\theta - \omega^2\sin\theta)\right)\mathbf{i} - \left(\frac{L}{2}(\alpha\sin\theta + \omega^2\cos\theta)\right)\mathbf{j}.$$

Equate the expressions for $\mathbf{a}_G$,

$$\left(\frac{L}{2}(\alpha\cos\theta - \omega^2\sin\theta)\right) = \left(a_B - \frac{L}{2}(\alpha\cos\theta - \omega^2\sin\theta)\right).$$

$$\left(a_A + \frac{L}{2}(\alpha\sin\theta + \omega^2\cos\theta)\right) = \left(\frac{L}{2}(\alpha\sin\theta + \omega^2\cos\theta)\right).$$

Solve:

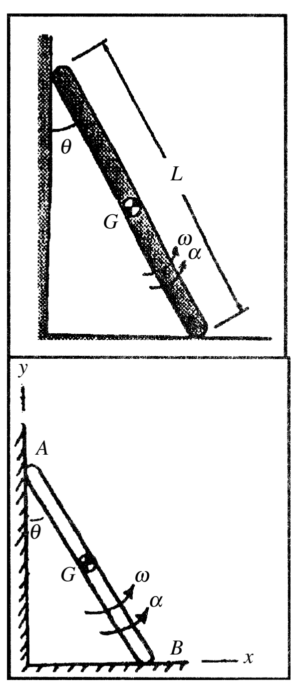
$$a_A = -L(\alpha\sin\theta + \omega^2\cos\theta),$$

$$a_B = L(\alpha\cos\theta - \omega^2\sin\theta).$$

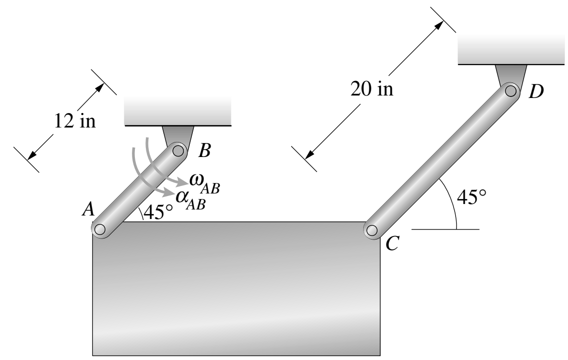
Substitute the expression for the acceleration of the point A into the expression for $\mathbf{a}_G$,

$$\boxed{\mathbf{a}_G = \frac{L}{2}((\alpha\cos\theta - \omega^2\sin\theta)\mathbf{i} - (\alpha\sin\theta + \omega^2\cos\theta)\mathbf{j})}$$

which demonstrates the result.



Problem 17.88 The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 10$ rad/s^2. The dimensions of the rectangular plate are 12 in. $\times$ 24 in. What are the angular velocity and angular acceleration of the rectangular plate?



Solution: The instantaneous center for bar AB is point B, by definition. The instantaneous center for bar CD is point D, by definition. The velocities at points A and C are normal to the bars AB and CD, respectively. However, by inspection these bars are parallel at the instant shown, so that lines perpendicular to the velocities at A and C will never intersect — the instantaneous center of the plate AC is at infinity, hence *the plate only translates at the instant shown*, and $\boxed{\omega_{AC} = 0}$. If the plate is not rotating, the velocity at every point on the plate must be the same, and in particular, the vector velocity at A and C must be identical. The vector A/B is

$$\mathbf{r}_{A/B} = -\mathbf{i}\cos 45^\circ - \mathbf{j}\sin 45^\circ = \left(\frac{-1}{\sqrt{2}}\right)(\mathbf{i}+\mathbf{j}) \text{ (ft).}$$

The velocity at point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \frac{-\omega_{AB}}{\sqrt{2}}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \sqrt{2}(\mathbf{i}-\mathbf{j}) \text{ (ft/s).}$$

The vector C/D is

$$\mathbf{r}_{C/D} = \left(\frac{20}{12}\right)(-\mathbf{i}\cos 45^\circ - \mathbf{j}\sin 45^\circ) = -1.179(\mathbf{i}+\mathbf{j}) \text{ (ft).}$$

The velocity at point C is

$$\mathbf{v}_C = -1.179\omega_{CD}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 1.179\omega_{CD}(\mathbf{i}-\mathbf{j}) \text{ (ft/s).}$$

Equate the velocities $\mathbf{v}_C = \mathbf{v}_A$, separate components and solve: $\omega_{CD} = 1.2$ rad/s. Use Eq. (17.10) to determine the accelerations. The acceleration of point A is

$$\mathbf{a}_A = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B} = -\frac{10}{\sqrt{2}}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \left(\frac{2^2}{\sqrt{2}}\right)(\mathbf{i}+\mathbf{j})$$

$$= 9.9\mathbf{i} - 4.24\mathbf{j} \text{ (ft/s}^2\text{).}$$

The acceleration of point C relative to point A is

$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} = \mathbf{a}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2 & 0 & 0 \end{bmatrix}$$

$$= 9.9\mathbf{i} + (2\alpha_{AC} - 4.24)\mathbf{j} \text{ (ft/s}^2\text{).}$$

The acceleration of point C relative to point D is $\mathbf{a}_C = \mathbf{a}_D + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$. Noting $\mathbf{a}_D = 0$,

$$\mathbf{a}_C = -1.179\alpha_{CD}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + 1.179\omega_{CD}^2(\mathbf{i}+\mathbf{j})$$

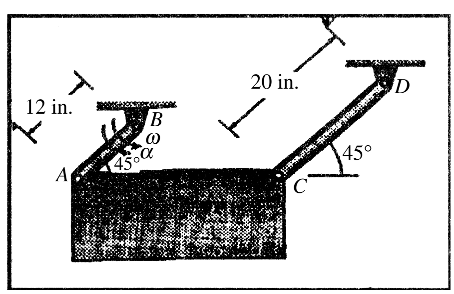
$$= (1.179\alpha_{CD} + 1.697)\mathbf{i} + (-1.179\alpha_{CD} + 1.697)\mathbf{j} \text{ (ft/s}^2\text{).}$$

Equate the two expressions for the acceleration at point C and separate components:

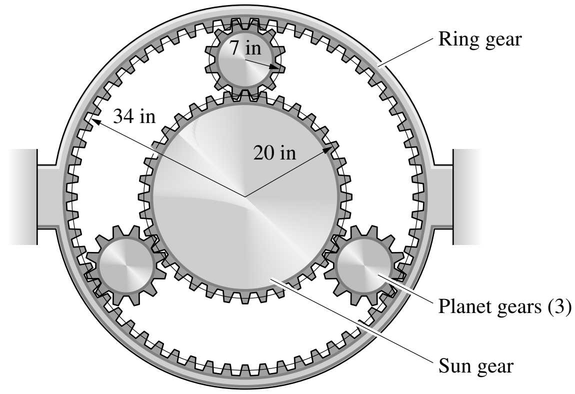
$$(-9.9 + 1.179\alpha_{CD} + 1.697)\mathbf{i} = 0,$$

$$(2\alpha_{AC} - 4.24 + 1.179\alpha_{CD} - 1.697)\mathbf{j} = 0.$$

Solve: $\boxed{\alpha_{AC} = -1.13 \text{ (rad/s}^2\text{)}}$ (clockwise), $\alpha_{CD} = 6.96$ (rad/s^2) (counterclockwise).

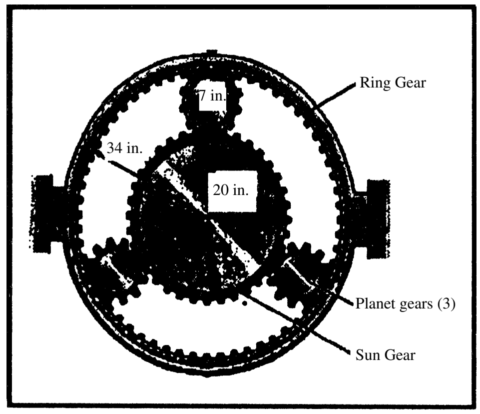


Problem 17.89 The ring gear is stationary, and the sun gear has an angular acceleration of 10 rad/s^2 in the counterclockwise direction. Determine the angular acceleration of the planet gears.



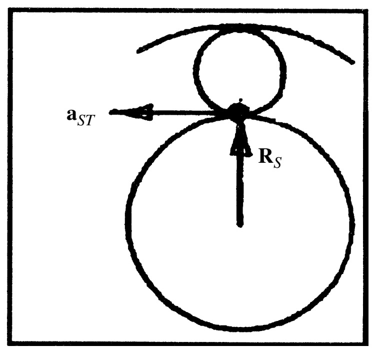
Solution: The strategy is to use the tangential acceleration at the point of contact of the sun and planet gears, together with the constraint that the point of contact of the planet gear and ring gear is stationary, to determine the angular acceleration of the planet gear. The tangential acceleration of the sun gear at the point of contact with the top planet gear is

$$\mathbf{a}_{ST} = \boldsymbol{\alpha} \times \mathbf{r}_S = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 20 & 0 \end{bmatrix} = -200\mathbf{i} \text{ (in/s}^2\text{)}.$$



This is also the tangential acceleration of the planet gear at the point of contact. At the contact with the ring gear, the planet gears are stationary, hence the angular acceleration of the planet gear satisfies

$$\boldsymbol{\alpha}_P \times (-2\mathbf{r}_P) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & -14 & 0 \end{bmatrix} = -200\mathbf{i}$$



from which

$$\boxed{\alpha_P = -\frac{200}{14} = -14.29 \text{ (rad/s}^2\text{)}} \text{ (clockwise).}$$

Problem 17.90 The sun gear in Problem 17.89 has a counterclockwise angular velocity of 4 rad/s and a clockwise angular acceleration of 12 rad/s^2. What is the magnitude of the acceleration of the centerpoints of the planet gears?

Solution: The strategy is to use the tangential velocity and acceleration at the point of contact of the sun and planet gears, together with the constraint that the point of contact of the planet gear and ring gear is stationary, to determine the angular accelerations of the centers of the planet gears. The magnitude of the tangential velocity and tangential acceleration at the point of contact of the sun and a planet gear are, respectively, $v_{SP} = \omega R_S = (4)(20) = 80$ in/s, and $a_t = \alpha R_S = 12(20) = 240$ in/s^2. The point of contact of the planet gear and ring gear is stationary. The magnitude of the velocity of the center of the planet gear is the mean of the sum of the velocities at its extreme edges,

$$v_P = \frac{v_{SP} + 0}{2} = 40 \text{ (in/s)}.$$

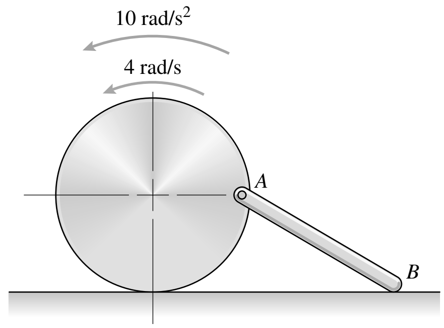
The center of a planet gear moves on a radius of $20 + 7 = 27$ in. so the normal acceleration of the center is

$$a_n = \left(\frac{v_P^2}{27}\right) = 59.26 \text{ in/s}^2.$$

The magnitude of the acceleration of the center of a planet gear is

$$\boxed{a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{120^2 + 59.26^2} = 134 \text{ in/s}^2}$$

Problem 17.91 The 1-m-diameter disk rolls, and point B of the 1-m-long bar slides, on the plane surface. Determine the angular acceleration of the bar and the acceleration of point B



Solution: Choose a coordinate system with the origin at O, the center of the disk, with x axis parallel to the horizontal surface. The point P of contact with the surface is stationary, from which

$$\mathbf{v}_P = 0 = \mathbf{v}_O + \boldsymbol{\omega}_O \times -\mathbf{R} = \mathbf{v}_O + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ 0 & -0.5 & 0 \end{bmatrix} = \mathbf{v}_O + 2\mathbf{i},$$

from which $\mathbf{v}_O = -2\mathbf{i}$ (m/s). The velocity at A is

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_O \times \mathbf{r}_{A/O} = -2\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ 0.5 & 0 & 0 \end{bmatrix} = -2\mathbf{i} + 2\mathbf{j} \text{ (m/s)}$$

The motion at point B is constrained to be parallel to the x axis. The line perpendicular to the velocity of B is parallel to the y axis. The line perpendicular to the velocity at A forms an angle at 45° with the x axis. From geometry, the line from A to the fixed center is the hypotenuse of a right triangle with base $\cos 30^\circ = 0.866$ and interior angles 45°. The coordinates of the fixed center are $(0.5 + 0.866, 0.866) = (1.366, 0.866)$ in. The vector from the instantaneous center to the point A is $\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -0.866\mathbf{i} - 0.866\mathbf{j}$ (m). The angular velocity of the bar AB is obtained from

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -0.866 & -0.866 & 0 \end{bmatrix}$$

$$= 0.866\omega_{AB}\mathbf{i} - 0.866\omega_{AB}\mathbf{j} \text{ (m/s)},$$

from which

$$\omega_{AB} = -\frac{2}{0.866} = -2.31 \text{ (rad/s)}.$$

The acceleration of the center of the rolling disk is $\mathbf{a}_C = -\alpha R\mathbf{i} = -10(0.5)\mathbf{i} = -5\mathbf{i}$ (m/s^2). The acceleration of point A is

$$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\alpha}_O \times \mathbf{r}_{A/O} - \omega_O^2\mathbf{r}_{A/O}$$

$$= -5\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 0.5 & 0 & 0 \end{bmatrix} - 16(0.5)\mathbf{i} \cdot \mathbf{a}_A$$

$$= -13\mathbf{i} + 5\mathbf{j} \text{ (m/s}^2).$$

The vector *B/A* is

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (0.5 + \cos\theta)\mathbf{i} - 0.5\mathbf{j} - 0.5\mathbf{i}$$

$$= 0.866\mathbf{i} - 0.5\mathbf{j} \text{ (m)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B}$$

$$= -13\mathbf{i} + 5\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -\cos\theta & \sin\theta & 0 \end{bmatrix}$$

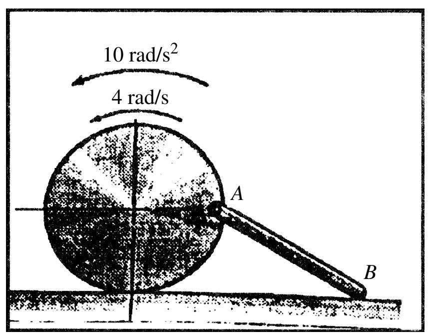
$$- \omega_{AB}^2(-\mathbf{i}\cos\theta + \mathbf{j}\sin\theta).$$

The constraint on B insures that the acceleration of B will be parallel to the x axis. Separate components:

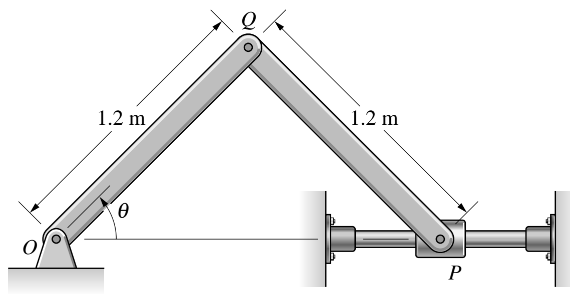
$$a_B = -13 + 0.5\alpha_{AB} - \omega_{AB}^2(0.866),$$

$$0 = 5 + 0.866\alpha_{AB} + 0.5\omega_{AB}^2.$$

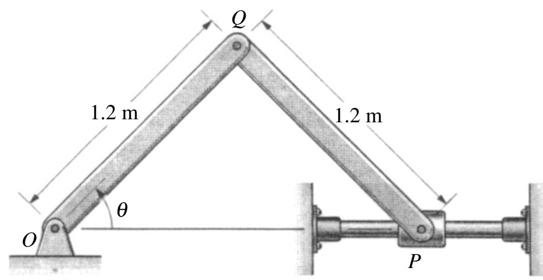
Solve: $\boxed{\alpha_{AB} = -8.85 \text{ (rad/s}^2)}$, where the negative sign means a clockwise rotation. $\boxed{\mathbf{a}_B = -22.04\mathbf{i} \text{ (m/s}^2)}$



Problem 17.92 If $\theta = 45^\circ$ and the sleeve P is moving to the right with a constant velocity of 2 m/s, what are the angular accelerations of the bars OQ and PQ?



Solution:



$\mathbf{v}_0 = \mathbf{a}_0 = 0,\ \mathbf{v}_p = 2\mathbf{i},\ \mathbf{a}_p = 0$

$\mathbf{r}_{Q/o} = 1.2\cos 45^\circ \mathbf{i} + 1.2 \sin 45^\circ \mathbf{j}$ m

$\mathbf{r}_{p/Q} = 1.2\cos 45^\circ \mathbf{i} - 1.2 \sin 45^\circ \mathbf{j}$ m

$\mathbf{v}_Q = \omega_{oQ}\mathbf{k} \times \mathbf{r}_{Q/o} = \omega_{oQ}\mathbf{k} \times (0.848\mathbf{i} + 0.848\mathbf{j})$

$$\begin{cases} v_{Qx} = -0.848\omega_{oQ} & \textbf{(1)} \\ v_{Qy} = 0.848\omega_{oQ} & \textbf{(2)} \end{cases}$$

$\mathbf{v}_p = \mathbf{v}_Q + \omega_{pQ}\mathbf{k} \times (0.848\mathbf{i} - 0.848\mathbf{j})$

$$\begin{cases} Z = v_{Qx} + 0.848\omega_{pQ} & \textbf{(3)} \\ O = v_{Qy} + 0.848\omega_{pQ} & \textbf{(4)} \end{cases}$$

Solving eqns. (1)–(4),

$\omega_{oQ} = -1.179$ rad/s, $\omega_{pQ} = 1.179$ rad/s

$v_{Qx} = 1$ m/s $v_{Qy} = -1$ m/s

$\mathbf{a}_Q = \boldsymbol{\alpha}_{oQ} \times \mathbf{r}_{Q/o} - \omega_{oQ}^2 \mathbf{r}_{Q/o}$

$$\begin{cases} a_{Qx} = -0.848\alpha_{oQ} - 0.848\omega_{oQ}^2 & \textbf{(5)} \\ a_{Qy} = 0.848\alpha_{oQ} - 0.848\omega_{oQ}^2 & \textbf{(6)} \end{cases}$$

Also,

$\mathbf{a}_p = 0 = \mathbf{a}_Q + \alpha_{pQ}\mathbf{k} \times \mathbf{r}_{p/Q} - \omega_{pQ}^2 \mathbf{r}_{p/Q}$

$$\begin{cases} 0 = a_{Qx} + 0.848\alpha_{pQ} - 0.848\omega_{pQ}^2 & \textbf{(7)} \\ 0 = a_{Qy} + 0.848\alpha_{pQ} + 0.848\omega_{pQ}^2 & \textbf{(8)} \end{cases}$$

Solving eqns. (5)–(8), we get

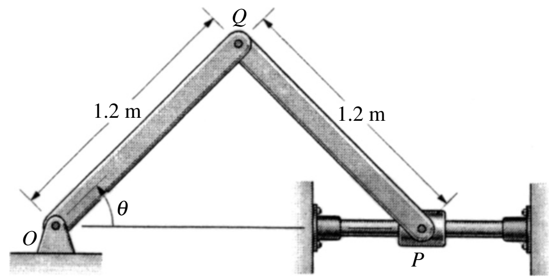
$a_{Qx} = 0, a_{Qy} = 0$

$\underline{\alpha_{oQ} = 1.39 \text{ rad/s}^2}$ (clockwise)

$\underline{\alpha_{pQ} = 1.39 \text{ rad/s}^2}$ (counter clockwise)

Problem 17.93 Consider the system shown in Problem 17.92. If $\theta = 50^\circ$ and bar OQ has a constant clockwise angular velocity of 1 rad/s, what is the acceleration of sleeve P?

Answer: 1.54 m/s to the right.



Solution:

$\boldsymbol{\omega}_{oQ} = -1\mathbf{k}$ rad/s, $\boldsymbol{\alpha}_{oQ} = 0$, $\mathbf{a}_0 = 0$

$$\mathbf{a}_Q = \mathbf{a}_0 + \boldsymbol{\alpha}_{oQ} \times \mathbf{r}_{Q/o} - \omega^2 \mathbf{r}_{Q/o}$$

$$\mathbf{a}_Q = 0 + 0 - (1)^2(1.2\cos 50^\circ \mathbf{l} + 1.2\sin 50^\circ \mathbf{j})$$

$$\mathbf{a}_Q = -0.771\mathbf{i} - 0.919\mathbf{j} \text{ m/s}^2$$

$$\mathbf{a}_p = \mathbf{a}_Q + \alpha_{Qp}^{\mathbf{k}} \times \mathbf{r}_{p/Q} - \omega_{Qp}^2 \mathbf{r}_{p/Q}$$

where $\mathbf{a}_p = a_p \mathbf{i}$

$$\mathbf{r}_{p/Q} = 1.2\cos 50^\circ \mathbf{i} - 1.2\sin 50^\circ \mathbf{j}$$

$$\mathbf{i}: \; a_p = -0.771 + 1.2\alpha_{Qp}\sin 50^\circ - \omega_{Qp}^2(1.2)\cos 50^\circ \quad \textbf{(1)}$$

$$\mathbf{j}: \; 0 = -0.919 + 1.2\alpha_{Qp}\cos 50^\circ + \omega_{Qp}^2(1.2)\sin 50^\circ \quad \textbf{(2)}$$

We have two eqns in three unknowns a_p, α_{Qp}, ω_{Qp}.

We need another eqn. To get it, we use the velocity relationships and determine W_{QP}. Note $\mathbf{v}_p = v_p\mathbf{i}$.

$$\mathbf{v}_Q = \mathbf{v}_0 + \mathbf{w}_{oQ} \times r_{Q/o} \quad \mathbf{v}_0 = 0$$

$$= (-1\mathbf{k}) \times [(1.2\cos 50^\circ)_\mathbf{i} + (1.2\sin 50)_\mathbf{j}]$$

$$= .919\mathbf{i} - 0.771\mathbf{j} \text{ (m/s)}.$$

$$\mathbf{v}_P = \mathbf{v}_Q + \boldsymbol{\omega}_{QP} \times \mathbf{r}_{P/Q}$$

$$= \mathbf{v}_Q + \omega_{QP}\mathbf{k} \times (1.2\cos 50^\circ \mathbf{i} - 1.2\sin 50^\circ \mathbf{j}).$$

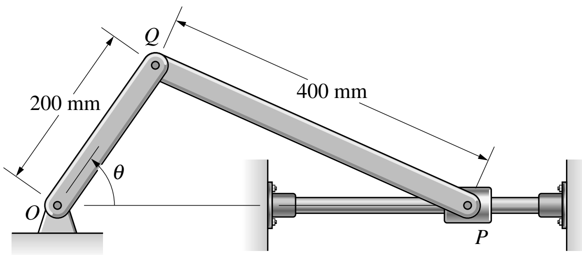
$$\mathbf{i}: \; v_P = 0.919 + 1.2\omega_{QP}\sin 50^\circ$$

$$\mathbf{j}: \; O = -0.771 + 1.2\omega_{QP}\cos 50^\circ$$

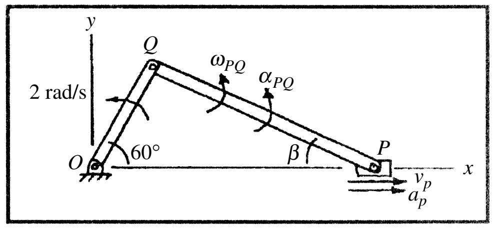
Solving, $v_P = 1.839$ m/s, $\omega_{QP} = 1$ rad/s. Now going back to eqns. (1) and (2), we solve to get

$$\frac{a_P = -1.54 \text{ m/s}^2}{\alpha_{QP} = 0}, \quad \text{(to the left)}$$

Problem 17.94 The angle $\theta = 60°$, and bar OQ has a constant counterclockwise angular velocity of 2 rad/s. What is the angular acceleration of the bar PQ?



Solution: By applying the law of sines, the angle $\beta = 25.7°$ The velocity of Q is $\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{0Q} \times \mathbf{r}_{Q/O}$



$$\mathbf{v}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.2\cos 60° & 0.2\sin 60° & 0 \end{vmatrix}$$

$$= -0.4\sin 60°\mathbf{i} + 0.4\cos 60°\mathbf{j}.$$

The velocity of P is

$$v_P\mathbf{i} = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$$

$$= -0.4\sin 60°\mathbf{i} + 0.4\cos 60°\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{j}$ components, we get $0 = 0.4\cos 60° + 0.4\omega_{PQ}\cos\beta$, and obtain $\omega_{PQ} = -0.555$ rad/s. The acceleration of Q is

$$\mathbf{a}_Q = \mathbf{a}_0 + \alpha_{0Q} \times \mathbf{r}_{Q/0} - \omega_{0Q}^2\mathbf{r}_{Q/0},$$

$$\text{or } \mathbf{a}_Q = 0 + 0 - (2)^2(0.2\cos 60°\mathbf{i} + 0.2\sin 60°\mathbf{j})$$

$$= -0.8\cos 60°\mathbf{i} - 0.8\sin 60°\mathbf{j}.$$

The acceleration of P is

$$a_P\mathbf{i} = \mathbf{a}_Q + \boldsymbol{\alpha}_{PQ} \times \mathbf{r}_{P/Q} - \omega_{PQ}^2\mathbf{r}_{P/Q}$$

$$= -0.8\cos 60°\mathbf{i} - 0.8\sin 60°\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}$$

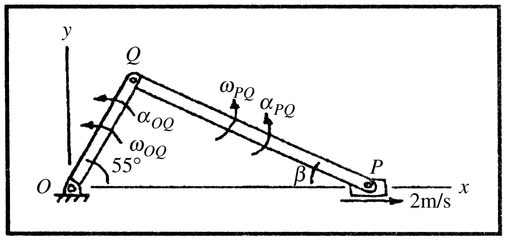
$$-(-0.555)^2(0.4\cos\beta\mathbf{i} - 0.4\sin\beta\mathbf{j}).$$

Equating $\mathbf{j}$ components

$$0 = -0.8\sin 60° + 0.4\alpha_{PQ}\cos\beta + (0.555)^2 0.4\sin\beta.$$

Solving, we obtain $\alpha_{PQ} = 1.77$ rad/s^2.

Problem 17.95 Consider the system shown in Problem 17.94. If $\theta = 55^\circ$ and sleeve P is moving to the right with a constant velocity of 2 m/s, what are the angular accelerations of the bars OQ and PQ?



Solution: By applying the law of sines, the angle $\beta = 24.2^\circ$. The velocity of Q is $\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{0Q} \times \mathbf{r}_{Q/0}$, or

$$\mathbf{v}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{0Q} \\ 0.2\cos 55^\circ & 0.2\sin 55^\circ & 0 \end{vmatrix}$$

$$= -0.2\omega_{0Q}\sin 55^\circ\mathbf{i} + 0.2\omega_{0Q}\cos 55^\circ\mathbf{j}.$$

The velocity of P is

$$\mathbf{v}_P = 2\mathbf{i} = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$$

$$= -0.2\omega_{0Q}\sin 55^\circ\mathbf{i} + 0.2\omega_{0Q}\cos 55^\circ\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$$2 = -0.2\omega_{0Q}\sin 55^\circ + 0.4\omega_{PQ}\sin\beta,$$

and $0 = 0.2\omega_{0Q}\cos 55^\circ + 0.4\omega_{PQ}\cos\beta,$

Solving, we obtain

$$\omega_{0Q} = -9.29 \text{ rad/s}$$

and $\omega_{PQ} = 2.92$ rad/s.

The acceleration of Q is

$$\mathbf{a}_Q = \mathbf{a}_0 + \boldsymbol{\alpha}_{0Q} \times \mathbf{r}_{Q/0} - \omega_{0Q}^2\mathbf{r}_{Q/0}$$

$$\mathbf{a}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{0Q} \\ 0.2\cos 55^\circ & 0.2\sin 55^\circ & 0 \end{vmatrix}$$

$$-(9.29)^2(0.2\cos 55^\circ\mathbf{i} + 0.2\sin 55^\circ\mathbf{j}), \text{ or}$$

$$\mathbf{a}_Q = [-0.2\alpha_{0Q}\sin 55^\circ - (9.29)^2 0.2\cos 55^\circ]\mathbf{i}$$

$$+ [0.2\alpha_{0Q}\cos 55^\circ - (9.29)^2 0.2\sin 55^\circ]\mathbf{j}.$$

The acceleration P is $\mathbf{a}_P = 0 = \mathbf{a}_Q + \boldsymbol{\alpha}_{PQ} \times \mathbf{r}_{P/Q} - \omega_{PQ}^2\mathbf{r}_{P/Q}$,

or $\mathbf{a}_P = \mathbf{a}_Q + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}$

$$-(2.92)^2(0.4\cos\beta\mathbf{i} - 0.4\sin\beta\mathbf{j}).$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components to zero,

$$0 = -0.2\alpha_{0Q}\sin 55^\circ - (9.29)^2 0.2\cos 55^\circ$$

$$+ 0.4\alpha_{PQ}\sin\beta - (2.92)^2 0.4\cos\beta,$$

and $0 = 0.2\alpha_{0Q}\cos 55^\circ - (9.29)^2 0.2\sin 55^\circ$

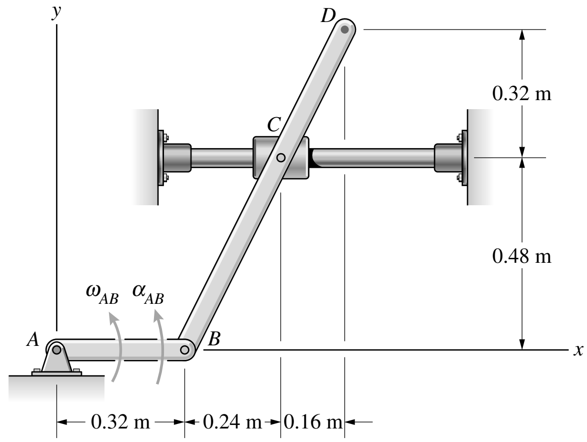
$$+ 0.4\alpha_{PQ}\cos\beta + (2.92)^2 0.4\sin\beta.$$

Solving, we obtain

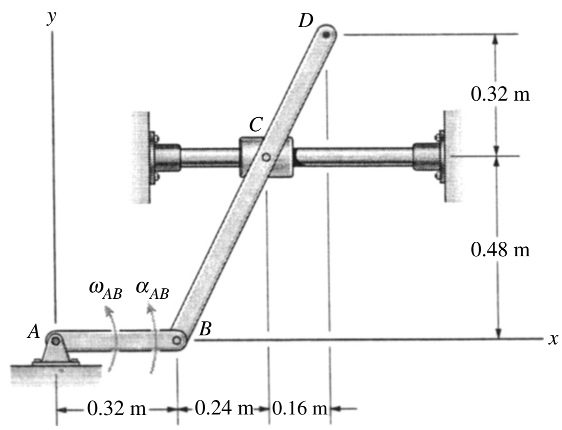
$$\alpha_{0Q} = -33.8 \text{ rad/s}^2$$

$$\alpha_{PQ} = 45.5 \text{ rad/s}^2$$

Problem 17.96 The angular velocity and acceleration of bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 6$ rad/s^2. Determine the angular velocity and angular acceleration of bar BD.



Solution:



$\mathbf{v}_A = 0, \mathbf{a}_A = 0, \mathbf{v}_c = v_{cx}\mathbf{i}, \mathbf{a}_c = a_{cx}\mathbf{i}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times 0.32\mathbf{i}$

$= 0.64\mathbf{j}$ (m/s).

$\mathbf{a}_B = \mathbf{a}_A^0 + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$

$= 6\mathbf{k} \times 0.32\mathbf{i} - 4(0.32\mathbf{i})$

$= -1.28\mathbf{i} + 1.92\mathbf{j}$ (m/s^2).

$\mathbf{v}_c = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$:

$v_{cx}\mathbf{i} = 0.64\mathbf{j} + \omega_{BC}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j})$

$$\begin{cases} v_{cx} = -0.48\omega_{BC} \\ 0 = 0.64 + 0.24\omega_{BC} \end{cases}$$

Solving,

$v_{C_x} = 1.28$ m/s

$\omega_{BC} = -2.67$ rad/s (clockwise)

$\boldsymbol{\omega}_{BD} = \underline{\boldsymbol{\omega}_{BC} = -2.67\mathbf{k} \text{ (rad/s)}}.$

$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$:

$a_{Cx}\mathbf{i} = \mathbf{a}_B + \alpha_{BC}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j})$

$- \omega_{BC}^2(0.24\mathbf{i} + 0.48\mathbf{j})$

$$\begin{cases} a_{Cx} = a_{B_x} - 0.48\alpha_{BC} - 0.24\omega_{BC}^2 \\ O = a_{B_y} + 0.24\alpha_{BC} - 0.48\omega_{BC}^2 \end{cases}$$

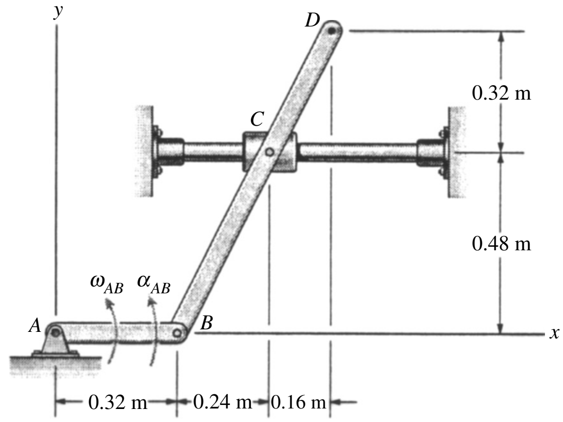
Solving,

$a_{Cx} = -5.97$ m/s^2,

$\alpha_{BC} = 6.22$ rad/s^2. (counterclockwise)

$\boldsymbol{\alpha}_{BD} = \underline{\boldsymbol{\alpha}_{BC} = 6.22\mathbf{k} \text{ (rad/s}^2)}.$

Problem 17.97 Consider the system shown in Problem 17.96. If $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = -10$ rad/s^2, what is the acceleration of point D?



Solution:

$$\mathbf{v}_A = 0, \mathbf{a}_A = 0, \mathbf{v}_C = v_{Cx}\mathbf{i}, \mathbf{a}_C = a_{Cx}\mathbf{i}$$

$$\mathbf{v}_B = \cancel{\mathbf{v}_A}^0 + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times 0.32\mathbf{i}$$

$$= 0.64\mathbf{j} \text{ (m/s)}.$$

$$\mathbf{a}_B = \cancel{\mathbf{a}_A}^0 + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= -10\mathbf{k} \times 0.32\mathbf{i} - 4(0.32\mathbf{i})$$

$$= -1.28\mathbf{i} - 3.20\mathbf{j} \text{ (m/s}^2\text{)}.$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} :$$

$$v_{Cx}\mathbf{i} = 0.64\mathbf{j} + \omega_{BC}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j})$$

$$\begin{cases} v_{Cx} = -0.48\omega_{BC} \\ v_{Cy} = 0.64 + 0.24\omega_{BC} \end{cases}$$

Solving,

$$v_{Cx} = 1.28 \text{ m/s},$$

$$\boldsymbol{\omega}_{BC} = -2.67\mathbf{k} \text{ (rad/s)}.$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B} :$$

$$a_{Cx}\mathbf{i} = \mathbf{a}_B + \alpha_{BC}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j})$$

$$- \omega_{BC}^2(0.24\mathbf{i} + 0.48\mathbf{j})$$

$$\begin{cases} a_{Cx} = a_{B_x} - 0.48\alpha_{BC} - 0.24\omega_{BC}^2 \\ O = a_{B_y} + 0.24\alpha_{BC} - 0.48\omega_{BC}^2 \end{cases}.$$

Solving, we get

$$a_{Cx} = -16.21 \text{ m/s}^2$$

$$\boldsymbol{\alpha}_{BC} = 27.56\mathbf{k} \text{ (rad/s}^2\text{)}.$$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{D/B} - \omega_{BC}^2 \mathbf{r}_{D/B}$$

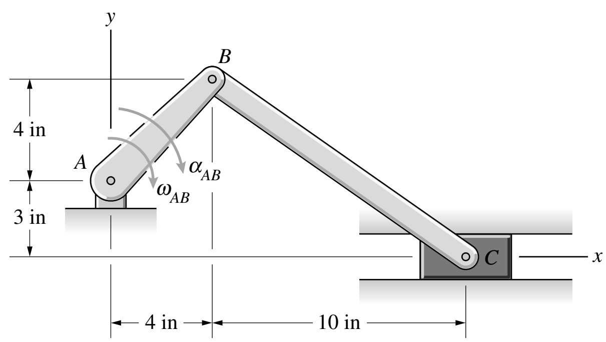
We know everything on the right hand side of this eqn.

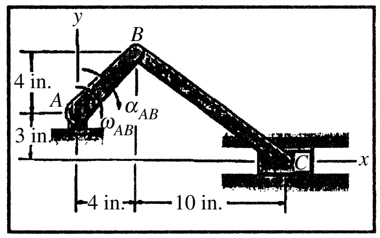
$(\mathbf{r}_{D/B} = 0.4\mathbf{i} + 0.8\mathbf{j}m)$

Solving,

$$\underline{\mathbf{a}_D = -26.17\mathbf{i} + 2.13\mathbf{j} \text{ (m/s}^2\text{)}}.$$

Problem 17.98 If $\omega_{AB} = 6$ rad/s and $\alpha_{AB} = 20$ rad/s^2, what are the velocity and acceleration of point C?





Solution: The vector B/A is $\mathbf{r_{B/A}} = \mathbf{r_B} - \mathbf{r_A} = 4\mathbf{i} + 4\mathbf{j}$ (in.). The velocity of point B is

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BA} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -6 \\ 4 & 4 & 0 \end{bmatrix} = -24(-\mathbf{i} + \mathbf{j}) \text{ (in/s)}.$$

The vector $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = (14\mathbf{i}) - (4\mathbf{i} + 7\mathbf{j}) = 10\mathbf{i} - 7\mathbf{j}$ (in.) The velocity of point C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B} = 24(\mathbf{i} - \mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 10 & -7 & 0 \end{bmatrix}$$

$$= 24(\mathbf{i} - \mathbf{j}) + \omega_{BC}(7\mathbf{i} + 10\mathbf{j}).$$

The velocity at point C is constrained to be parallel to the x axis. Separate components:

$$\mathbf{v}_C = 24 + 7\omega_{BC},$$

$$0 = -24 + 10\omega_{BC},$$

from which $\boxed{\mathbf{v}_C = 40.8\mathbf{i} \text{ (in/s)}}$, $\omega_{BC} = 2.4$ rad/s.
The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{AB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -20 \\ 4 & 4 & 0 \end{bmatrix} - 36(4\mathbf{i} + 4\mathbf{j}).$$

$$= 80\mathbf{i} - 80\mathbf{j} - 144\mathbf{i} - 144\mathbf{j}.$$

$$\mathbf{a}_B = -64\mathbf{i} - 224\mathbf{j} \text{ (in/s}^2)$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -64\mathbf{i} - 224\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 10 & -7 & 0 \end{bmatrix} - (\omega_{BC}^2)(10\mathbf{i} - 7\mathbf{j}).$$

The acceleration of C is constrained to be parallel to the x axis. Separate components:

$$a_C = -64 + 7\alpha_{BC} - 10\omega_{BC}^2,$$

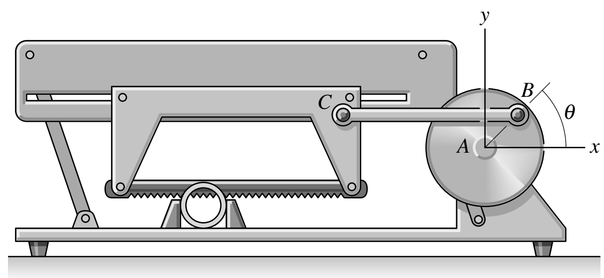
$$0 = -224 + 10\alpha_{BC} + 7\omega_{BC}^2.$$

Substitute $\omega_{BC} = 2.4$ rad/s and solve:

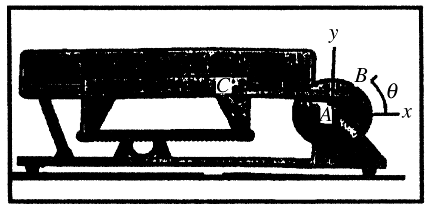
$\boxed{\mathbf{a}_C = 6.98\mathbf{i} \text{ (in/s}^2)}$,

$\alpha_{BC} = 18.4$ rad/s^2 (counterclockwise).

Problem 17.99 A motor rotates the circular disk mounted at A, moving the saw back and forth. (The saw is supported by horizontal slot so that point C moves horizontally.) The radius AB is 4 in., and the link BC is 14 in. long. In the position shown, $\theta = 45^\circ$ and the link BC is horizontal. If the disk has a constant angular velocity of one revolution per second counterclockwise, what is the acceleration of the saw?



Solution: The angular velocity of the disk is $\omega_{AB} = 2\pi$ rad/s. The vector from A to B is $\mathbf{r}_{B/A} = 4(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)$ (in.). The velocity of point B is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2\pi \\ 2\sqrt{2} & 2\sqrt{2} & 0 \end{bmatrix} = 2\pi(-2.83\mathbf{i} + 2.83\mathbf{j}).$$

$\mathbf{v}_B = -17.8(\mathbf{i} - \mathbf{j})$ (in/s).

The vector from B to C is $\mathbf{r}_{C/B} = -14\mathbf{i}$ (in.) The velocity of point C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -14 & 0 & 0 \end{bmatrix}$$

$$= -17.8\mathbf{i} + 17.8\mathbf{j} - 14\omega_{BC}\mathbf{j} \text{ (in/s)}.$$

The velocity of C is constrained to be parallel to the x axis. Separate components and solve: $\mathbf{v}_C = -17.8\mathbf{i}$. $\omega_{BC} = 1.27$ (rad/s). The acceleration of B is

$$\mathbf{a}_B = -\omega_{AB}^2(2.83\mathbf{i} + 2.83\mathbf{j}) = -111.7(\mathbf{i} + \mathbf{j}) \text{ (in/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration at B:

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= -111.7(\mathbf{i} + \mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -14 & 0 & 0 \end{bmatrix} - (1.27^2)(-14\mathbf{i}).$$

The acceleration of C is constrained to lie parallel to the x axis. Separate components: $a_C = -111.7 + 14\omega_{BC}^2$, $0 = -111.7 - 14\alpha_{BC}$.

Solve: $\boxed{\mathbf{a}_C = -89\mathbf{i} \text{ (in/s}^2\text{)}}$

Problem 17.100 In Problem 17.99, if the disk has a constant angular velocity of one revolution per second counterclockwise and $\theta = 180°$, what is the acceleration of the saw?

Solution: The angular velocity of the disk is $\omega_{AB} = 2\pi$ rad/s. The vector location of B is $\mathbf{r}_B = -4\mathbf{i}$ (in.). The bar BC is level when the angle is $\theta = 45°$, from which the vector location of point C when $\theta = 180°$ is $\mathbf{r}_C = -(14\cos\alpha + 4)\mathbf{i} + (4\sin 45°)\mathbf{j} = -17.7\mathbf{i} + 2.83\mathbf{j}$ (in.), where the angle

$$\alpha = \sin^{-1}\left(\frac{4\sin 45°}{14}\right) = 11.66°.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2\pi \\ -4 & 0 & 0 \end{bmatrix} = -8\pi\mathbf{j} = -251\mathbf{j} \text{ (in/s).}$$

The vector $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = -13.7\mathbf{i} + 2.83\mathbf{j}$ (in.), The velocity of point C in terms of the velocity of B is

$$\boldsymbol{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -13.7 & 2.83 & 0 \end{bmatrix}$$

$$= -25.1\mathbf{j} - 2.83\omega_{BC}\mathbf{i} - 13.7\omega_{BC}\mathbf{j} \text{ (in/s).}$$

The velocity of C is constrained to be parallel to the x axis. Separate components and solve: $v_C = 2.83\omega_{BC}$, $0 = -25.1 - 13.7\omega_{BC}$, from which $\mathbf{v}_C = -5.18\mathbf{i}$ (in/s), $\omega_{BC} = -1.833$ rad/s. The acceleration of B is $\mathbf{a}_B = -\omega_{AB}^2(-4\mathbf{i}) = 157.9\mathbf{i}$ (in/s^2). The acceleration of point C in terms of the acceleration at B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= 157.9\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -13.7 & 2.83 & 0 \end{bmatrix} - \omega_{BC}^2(-13.7\mathbf{i} + 2.83\mathbf{j})$$

$$\mathbf{a}_C = 157.9\mathbf{i} + \alpha_{BC}(-2.83\mathbf{i} - 13.7\mathbf{j}) - \omega_{BC}^2(-13.7\mathbf{i} + 2.83\mathbf{j}).$$

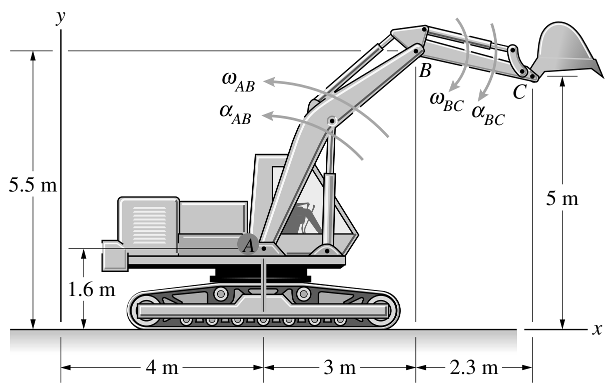
The acceleration of C is constrained to lie parallel to the x axis. Separate components:

$$a_C = 157.9 - 2.83\alpha_{BC} + 13.7\omega_{BC}^2,$$

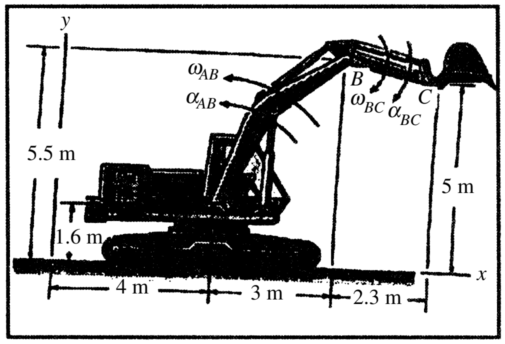
$$0 = -13.7\alpha_{BC} - 2.83\omega_{BC}^2.$$

Solve: $\boxed{\mathbf{a}_C = 206\mathbf{i} \text{ (in/s}^2)}$

Problem 17.101 If $\omega_{AB} = 2$ rad/s, $\alpha_{AB} = 2$ rad/s^2, $\omega_{BC} = -1$ rad/s, and $\alpha_{BC} = -4$ rad/s^2, what is the acceleration of point C where the scoop of the excavator is attached?



Solution: The vector locations of points A, B, C are



$\mathbf{r}_A = 4\mathbf{i} + 1.6\mathbf{j}$ (m),

$\mathbf{r}_B = 7\mathbf{i} + 5.5\mathbf{j}$ (m).

$\mathbf{r}_C = 9.3\mathbf{i} + 5\mathbf{j}$ (m).

The vectors

$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 3\mathbf{i} + 3.9\mathbf{j}$ (m),

$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 2.3\mathbf{i} - 0.5\mathbf{j}$ (m).

The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}.$$

$$\mathbf{a}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 3 & 3.9 & 0 \end{bmatrix} - (2^2)(3.0\mathbf{i} + 3.9\mathbf{j}),$$

$$\mathbf{a}_B = +2(-3.9\mathbf{i} + 3\mathbf{j}) - (4)(3\mathbf{i} + 3.9\mathbf{j})$$

$$= -19.8\mathbf{i} - 9.6\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration at point B is

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2(\mathbf{r}_{C/B})$$

$$= -19.8\mathbf{i} - 9.6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -4 \\ 2.3 & -0.5 & 0 \end{bmatrix} - 1^2(2.3\mathbf{i} - 0.5\mathbf{j}),$$

$$\mathbf{a}_C = -19.8\mathbf{i} - 9.6\mathbf{j} - 2\mathbf{i} - 9.2\mathbf{j} - 2.3\mathbf{i} + 0.5\mathbf{j}$$

$$\boxed{= -24.1\mathbf{i} - 18.3\mathbf{j} \text{ (m/s}^2\text{)}}$$

Problem 17.102 If the velocity of point C of the excavator in Problem 17.101 is $\mathbf{v}_C = 4\mathbf{i}$ (m/s) and is constant, what are ω_{AB}, α_{AB}, ω_{BC}, α_{BC}?

Solution: The strategy is to determine the angular velocities ω_{AB}, ω_{BC} from the known velocity at point C, and the angular velocities α_{AB}, α_{BC} from the data that the linear acceleration at point C is constant.

The angular velocities: The vector locations of points A, B, C are

$$\mathbf{r}_A = 4\mathbf{i} + 1.6\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_B = 7\mathbf{i} + 5.5\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_C = 9.3\mathbf{i} + 5\mathbf{j} \text{ (m)}.$$

The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 3\mathbf{i} + 3.9\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 2.3\mathbf{i} - 0.5\mathbf{j} \text{ (m)}.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3 & 3.9 & 0 \end{bmatrix} = -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j}.$$

The velocity of C in terms of the velocity of B

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$= -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix},$$

$$\mathbf{v}_C = -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j} - 0.5\omega_{BC}\mathbf{i} - 2.3\omega_{BC}\mathbf{j} \text{ (m/s)}.$$

Substitute $\mathbf{v}_C = 4\mathbf{i}$ (m/s), and separate components:

$$4 = -3.9\omega_{AB} - 0.5\omega_{BC},$$

$$0 = 3\omega_{AB} - 2.3\omega_{BC}.$$

Solve: $\boxed{\omega_{AB} = -0.8787 \text{ rad/s}}$, $\boxed{\omega_{BC} = -1.146 \text{ rad/s}}$.

The angular accelerations: The acceleration of point B is

$$\mathbf{a}_B = \mathbf{a}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 3 & 3.9 & 0 \end{bmatrix} - \left(\omega_{AB}^2\right)(3\mathbf{i} + 3.9\mathbf{j}),$$

$$\mathbf{a}_B = -3.9\alpha_{AB}\mathbf{i} + 3\alpha_{AB}\mathbf{j} - 3\omega_{AB}^2\mathbf{i} - 3.9\omega_{AB}^2\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of C in terms of the acceleration of B is

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\alpha_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix} - \omega_{BC}^2 (2.3\mathbf{i} - 0.5\mathbf{j})$$

$$\mathbf{a}_C = \left(-3.9\alpha_{AB} - 3\omega_{AB}^2\right)\mathbf{i} + \left(3\alpha_{AB} - 3.9\omega_{AB}^2\right)\mathbf{j}$$

$$+ (-0.5\alpha_{BC} - 2.3\omega_{BC}^2)\mathbf{i} + \left(-2.3\alpha_{BC} + 0.5\omega_{BC}^2\right)\mathbf{j}.$$

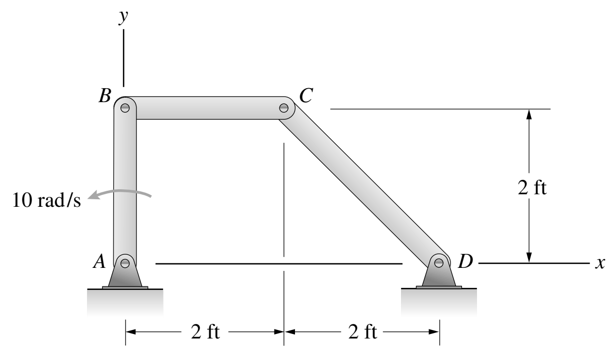
Substitute $\mathbf{a}_C = 0$ from the conditions of the problem, and separate components:

$$0 = -3.9\alpha_{AB} - 0.5\alpha_{BC} - 3\omega_{AB}^2 - 2.3\omega_{BC}^2,$$

$$0 = 3\alpha_{AB} - 2.3\alpha_{BC} - 3.9\omega_{AB}^2 + 0.5\omega_{BC}^2.$$

Solve: $\boxed{\alpha_{BC} = -2.406 \text{ rad/s}^2}$, $\boxed{\alpha_{AB} = -1.06 \text{ rad/s}^2.}$

Problem 17.103 Bar AB rotates in the counterclockwise direction with a constant angular velocity of 10 rad/s. What are the angular accelerations of BC and CD?



Solution: The vector locations of A, B, C and D are: $\mathbf{r}_A = 0$, $\mathbf{r}_B = 2\mathbf{j}$ (ft), $\mathbf{r}_C = 2\mathbf{i} + 2\mathbf{j}$ (ft), $\mathbf{r}_D = 4\mathbf{i}$ (ft). The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 2\mathbf{j} \text{ (ft)}.$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 2\mathbf{i} \text{ (ft)},$$

$$\mathbf{r}_{C/D} = \mathbf{r}_D - \mathbf{r}_C = -2\mathbf{i} + 2\mathbf{j} \text{ (ft)}.$$

(a) *Get the angular velocities ω_{BC}, ω_{CD}*. The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 2 & 0 \end{bmatrix} = -20\mathbf{i} \text{ (ft/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$= -20\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2 & 0 & 0 \end{bmatrix} = -20\mathbf{i} + 2\omega_{BC}\mathbf{j} \text{ (ft/s)}.$$

The velocity of C in terms of the velocity of point D

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -2 & 2 & 0 \end{bmatrix} = 2\omega_{CD}(-\mathbf{i} - \mathbf{j}) \text{ (ft/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components: $-20 = 2\omega_{CD}$, $2\omega_{BC} = -2\omega_{CD}$. Solve: $\omega_{BC} = -10$ rad/s, $\omega_{CD} = 10$ rad/s.

(b) *Get the angular accelerations*. The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} = -\omega_{AB}^2(2\mathbf{j}) = -200\mathbf{j} \text{ (ft/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -200\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 2 & 0 & 0 \end{bmatrix} - \omega_{BC}^2(2\mathbf{i}).$$

$$\mathbf{a}_C = -200\mathbf{j} + 2\alpha_{BC}\mathbf{j} - 200\mathbf{i} \text{ (ft/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2 \mathbf{r}_{C/D}$$

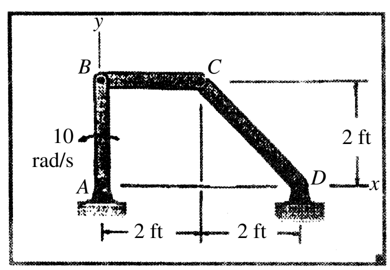
$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -2 & -2 & 0 \end{bmatrix} - \omega_{CD}^2(-2\mathbf{i} + 2\mathbf{j}).$$

$$\mathbf{a}_C = 2\alpha_{CD}\mathbf{i} - 2\alpha_{CD}\mathbf{j} + 200\mathbf{i} - 200\mathbf{j} \text{ (ft/s}^2\text{)}.$$

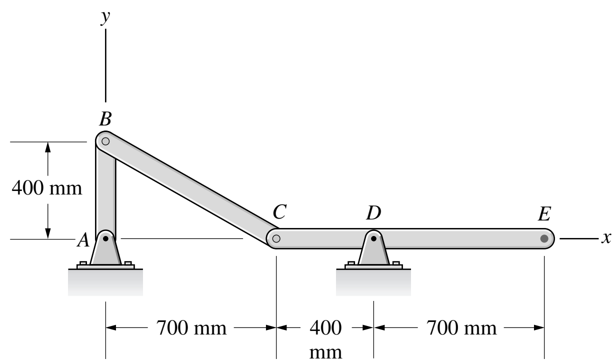
Equate the expressions and separate components: $-200 = -2\alpha_{CD} + 200$, $-200 + 2\alpha_{BC} = -200 - 2\alpha_{CD}$. Solve:

$\boxed{\alpha_{BC} = -200 \text{ rad/s}^2}$, $\boxed{\alpha_{CD} = 200 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.



Problem 17.104 At the instant shown, bar AB has no angular velocity but has a counterclockwise angular acceleration of 10 rad/s^2. Determine the acceleration of point E.



Solution: The vector locations of A, B, C and D are: $\mathbf{r}_A = 0$, $\mathbf{r}_B = 400\mathbf{j}$ (mm), $\mathbf{r}_C = 700\mathbf{i}$ (mm), $\mathbf{r}_D = 1100\mathbf{i}$ (mm). $\mathbf{r}_E = 1800\mathbf{i}$ (mm) The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 400\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 700\mathbf{i} - 400\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = \mathbf{r}_D - \mathbf{r}_C = -400\mathbf{i} \text{ (mm)} \cdot \mathbf{r}_{E/D} = 700\mathbf{i} \text{ (mm)}$$

(a) *Get the angular velocities* ω_{BC}, ω_{CD}. The velocity of point B is zero. The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 700 & -400 & 0 \end{bmatrix}$$

$$= +400\omega_{BC}\mathbf{i} + 700\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

$$= -400\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components: $400\omega_{BC} = 0$, $700\omega_{BC} = -400\omega_{CD}$. Solve: $\omega_{BC} = 0$ rad/s, $\omega_{CD} = 0$ rad/s.

(b) *Get the angular accelerations*. The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 400 & 0 \end{bmatrix}$$

$$= -4000\mathbf{i} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= -4000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 700 & -400 & 0 \end{bmatrix}.$$

$$\mathbf{a}_C = -4000\mathbf{i} + 400\alpha_{BC}\mathbf{i} + 700\alpha_{BC}\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

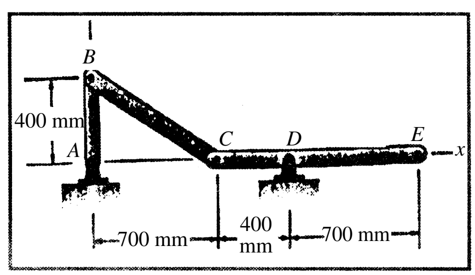
$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

$$= -400\alpha_{CD}\mathbf{j} \text{ (mm/s}^2\text{)}.$$

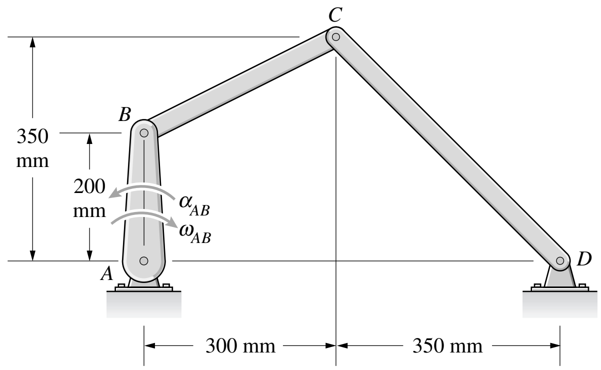
Equate the expressions and separate components: $-4000 + 400\alpha_{CD} = 0$, $700\alpha_{BC} = -400\alpha_{CD}$.
Solve: $\alpha_{BC} = 10$ rad/s^2, $\alpha_{CD} = -17.5$ rad/s^2, The acceleration of point E in terms of the acceleration of point D is

$$\boldsymbol{a}_E = \alpha_{CD} \times \mathbf{r}_{E/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -17.5 \\ 700 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -12300\mathbf{j} \text{ (mm/s}^2\text{)}} \text{ (clockwise)}$$



Problem 17.105 If $\omega_{AB} = 12$ rad/s and $\alpha_{AB} = 100$ rad/s^2, what are the angular accelerations of bars BC and CD?



Solution: The vector locations of A, B, C and D are: $\mathbf{r}_A = 0$, $\mathbf{r}_B = 200\mathbf{j}$ (mm), $\mathbf{r}_C = 300\mathbf{i} + 350\mathbf{j}$ (mm), $\mathbf{r}_D = 650\mathbf{i}$ (mm). The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 200\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 300\mathbf{i} + 150\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = \mathbf{r}_D - \mathbf{r}_C = -350\mathbf{i} + 350\mathbf{j} \text{ (mm)} \cdot \mathbf{r}_{E/D} = 700\mathbf{i} \text{ (mm)}$$

(a) *Get the angular velocities ω_{BC}, ω_{CD}.* The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 300 & 150 & 0 \end{bmatrix}$$

$$= 2400\mathbf{i} - 150\omega_{BC}\mathbf{i} + 300\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}\mathbf{i} - 350\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components: $2400 - 150\omega_{BC} = -350\omega_{CD}$, $300\omega_{BC} = -350\omega_{CD}$. Solve: $\omega_{BC} = 5.33$ rad/s, $\omega_{CD} = -4.57$ rad/s.

(b) *Get the angular accelerations*. The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 100 \\ 0 & 200 & 0 \end{bmatrix}$$

$$- \omega_{AB}^2(200\mathbf{j})$$

$$= -20{,}000\mathbf{i} - 28{,}800\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 300 & 150 & 0 \end{bmatrix} - \omega_{BC}^2(300\mathbf{i} + 150\mathbf{j}).$$

$$\mathbf{a}_C = (-20{,}000 - 150\alpha_{BC} - 300\omega_{BC}^2)\mathbf{i}$$

$$+ (-28{,}800 + 300\alpha_{BC} - 150\omega_{BC}^2)\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -350 & 350 & 0 \end{bmatrix} - \omega_{CD}^2(-350\mathbf{i} + 350\mathbf{j})$$

$$\mathbf{a}_C = -350\alpha_{CD}\mathbf{i} - 350\alpha_{CD}\mathbf{j} + 350\omega_{CD}^2\mathbf{i} - 350\omega_{CD}^2\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Equate the expressions and separate components:

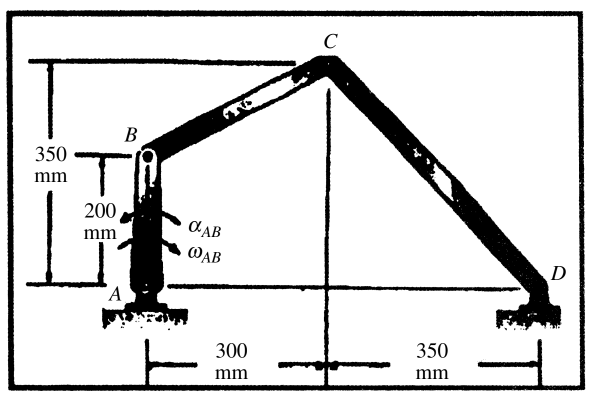
$$-20{,}000 - 150\alpha_{BC} - 300\omega_{BC}^2 = -350\alpha_{CD} + 350\omega_{CD}^2,$$

$$-28{,}800 + 300\alpha_{BC} - 150\omega_{BC}^2 = -350\alpha_{CD} - 350\omega_{CD}^2.$$

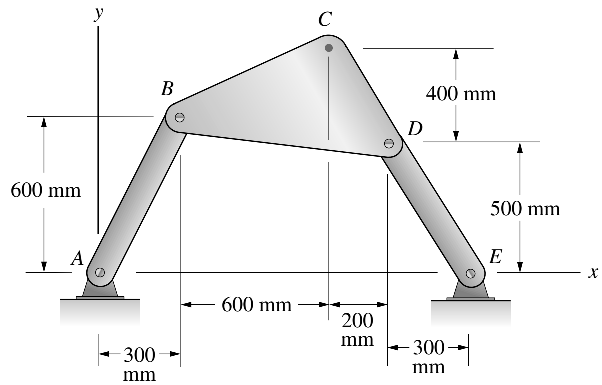
Solve: $\boxed{\alpha_{BC} = -22.43 \text{ rad/s}^2}$,

$\boxed{\alpha_{CD} = 92.8 \text{ rad/s}^2}$,

where the negative sign means a clockwise acceleration.



Problem 17.106 If $\omega_{AB} = 4$ rad/s counterclockwise and $\alpha_{AB} = 12$ rad/s^2 counterclockwise, what is the acceleration of point C?



Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.3 & 0.6 & 0 \end{vmatrix}$$

$$= -0.6\omega_{AB}\mathbf{i} + 0.3\omega_{AB}\mathbf{j}.$$

The velocity of D is

$$\mathbf{v}_D = \mathbf{v}_B + \omega_{BD} \times \mathbf{r}_{D/B}$$

$$= -0.6\omega_{AB}\mathbf{i} + 0.3\omega_{AB}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0.8 & -0.1 & 0 \end{vmatrix}. \quad \textbf{(1)}$$

We can also express the velocity of D as

$$\mathbf{v}_D = \mathbf{v}_E + \omega_{DE} \times \mathbf{r}_{D/E} = \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -0.3 & 0.5 & 0 \end{vmatrix}. \quad \textbf{(2)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Eqns. (1) and (2), we obtain

$$-0.6\omega_{AB} + 0.1\omega_{BD} = -0.5\omega_{DE}, \quad \textbf{(3)}$$

$$0.3\omega_{AB} + 0.8\omega_{BD} = -0.3\omega_{DE}. \quad \textbf{(4)}$$

Solving these two eqns with $\omega_{AB} = 4$ rad/s, we obtain

$$\omega_{BD} = -3.57 \text{ rad/s}, \quad \omega_{DE} = 5.51 \text{ rad/s}.$$

The acceleration of B is

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.3 & 0.6 & 0 \end{vmatrix} - \omega_{AB}^2(0.3\mathbf{i} + 0.6\mathbf{j})$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}.$$

The acceleration of D is

$$\mathbf{a}_D = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0.8 & -0.1 & 0 \end{vmatrix} - \omega_{BD}^2(0.8\mathbf{i} - 0.1\mathbf{j}). \quad \textbf{(5)}$$

We can also express the acceleration of D as

$$\mathbf{a}_D = \mathbf{a}_E + \alpha_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2\mathbf{r}_{D/E}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{DE} \\ -0.3 & 0.5 & 0 \end{vmatrix} - \omega_{DE}^2(-0.3\mathbf{i} + 0.5\mathbf{j}). \quad \textbf{(6)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Eqns. (5) and (6), we obtain

$$-0.6\alpha_{AB} - 0.3\omega_{AB}^2 + 0.1\alpha_{BD} - 0.8\omega_{BD}^2$$

$$= -0.5\alpha_{DE} + 0.3\omega_{DE}^2, \quad \textbf{(7)}$$

$$0.3\alpha_{AB} - 0.6\omega_{AB}^2 + 0.8\alpha_{BD} + 0.1\omega_{BD}^2$$

$$= -0.3\alpha_{DE} - 0.5\omega_{DE}^2. \quad \textbf{(8)}$$

Solving these two eqns with $\alpha_{AB} = 12$ rad/s^2, we obtain

$$\alpha_{BD} = -39.5 \text{ rad/s}^2.$$

The acceleration of C is

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{C/B} - \omega_{BD}^2\mathbf{r}_{C/B}$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}$$

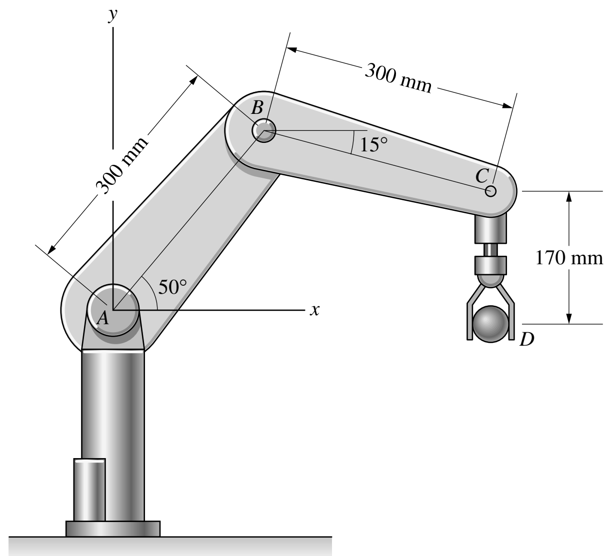
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0.6 & 0.3 & 0 \end{vmatrix} - \omega_{BD}^2(0.6\mathbf{i} + 0.3\mathbf{j}). \quad \textbf{(9)}$$

$$\mathbf{a}_C = -7.78\mathbf{i} - 33.5\mathbf{j} \text{ (m/s}^2\text{)}.$$

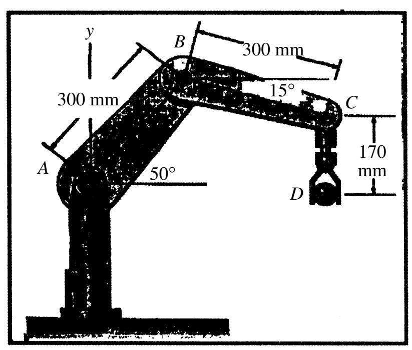
Problem 17.107 In Problem 17.106, if $\omega_{AB} = 6$ rad/s clockwise and $\alpha_{DE} = 0$, what is the acceleration of point C?

Solution: See the solution of Problem 17.106. Solving Eqns. (3) and (4) with $\omega_{AB} = -6$ rad/s, we obtain $\omega_{BD} = 5.35$ rad/s, $\omega_{DE} = -8.27$ rad/s. Then solving Eqs. (7) and (8) with $\alpha_{DE} = 0$, we obtain $\alpha_{AB} = -88.1$ rad/s^2, $\alpha_{BD} = 13.7$ rad/s^2. Then from Eq. (9), $\mathbf{a}_c = 20.8\mathbf{i} - 48.4\mathbf{j}$ (m/s^2).

Problem 17.108 If arm AB has a constant clockwise angular velocity of 0.8 rad/s, arm BC has a constant angular velocity of 0.2 rad/s, and arm CD remains vertical, what is the acceleration of part D?



Solution: The constraint that the arm CD remain vertical means that the angular velocity of arm CD is zero. This implies that arm CD translates only, and in a translating, non-rotating element the velocity and acceleration at any point is the same, and the velocity and acceleration of arm CD is the velocity and acceleration of point C. The vectors:



$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B} = -\omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j}) \text{ (mm/s}^2\text{)},$$

since $\alpha_{AB} = 0$. $\mathbf{a}_B = -123.4\mathbf{i} - 147.1\mathbf{j}$ (mm/s). The acceleration of C in terms of the acceleration of B is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= -123.4\mathbf{i} - 147.1\mathbf{j} - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}),$$

since $\boldsymbol{\alpha}_{BC} = 0$. $\mathbf{a}_C = -135\mathbf{i} - 144\mathbf{j}$ (mm/s^2). Since CD is translating:

$$\boxed{\mathbf{a}_D = \mathbf{a}_C = -135\mathbf{i} - 144\mathbf{j} \text{ (mm/s}^2\text{)}}$$

Problem 17.109 In Problem 17.108, if arm AB has a constant clockwise angular velocity of 0.8 rad/s and you want D to have zero velocity and acceleration, what are the necessary angular velocities and angular accelerations of arms BC and CD?

Solution: Except for numerical values, the solution follows the same strategy as the solution strategies for Problems 17.103 and 17.105. The vectors:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = 170\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.8 \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$= 183.8\mathbf{i} - 154.3\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = 183.9\mathbf{i} - 154.3\mathbf{j} + \omega_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D:

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 0 & 170 & 0 \end{bmatrix} = -170\omega_{CD}\mathbf{i} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components:

$$183.9 + 77.6\omega_{BC} = -170\omega_{CD},$$

$$-154.3 + 289.8\omega_{BC} = 0.$$

Solve: $\boxed{\omega_{BC} = 0.532 \text{ rad/s}}$, $\boxed{\omega_{CD} = -1.325 \text{ rad/s}}$.

Get the angular accelerations. The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} = -\omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j})$$

$$= -123.4\mathbf{i} - 147.1\mathbf{j} \text{ (mm/s}^2).$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B} = \mathbf{a}_B - \omega_{BC}^2 \mathbf{r}_{C/B}.$$

$$\mathbf{a}_C = -123.4\mathbf{i} - 147.1\mathbf{j} + 77.6\alpha_{BC}\mathbf{i} + 289.8\alpha_{BC}\mathbf{j} - 289.8\omega_{BC}^2\mathbf{i}$$

$$+ 77.6\omega_{BC}^2\mathbf{j} \text{ (mm/s}^2).$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2 \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 0 & 170 & 0 \end{bmatrix} - \omega_{CD}^2(170\mathbf{j}).$$

$$\mathbf{a}_C = -170\alpha_{CD}\mathbf{i} - 170\omega_{CD}^2\mathbf{j} \text{ (mm/s}^2).$$

Equate the expressions and separate components:

$$-123.4 + 77.6\alpha_{BC} - 289.8\omega_{BC}^2 = -170\alpha_{CD},$$

$$-147.1 + 289.8\alpha_{BC} + 77.6\omega_{BC}^2 = -170\omega_{CD}^2.$$

Solve:

$\boxed{\alpha_{BC} = -0.598 \text{ rad/s}^2}$, $\boxed{\alpha_{CD} = 1.482 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.

Problem 17.110 In Problem 17.108, if you want arm CD to remain vertical and you want part D to have velocity $\mathbf{v}_D = 1.0\mathbf{i}$ (m/s) and zero acceleration, what are the necessary angular velocities and angular accelerations of arms AB and BC?

Solution: The constraint that CD remain vertical with zero acceleration means that every point on arm CD is translating, without rotation, at a velocity of 1 m/s. This means that the velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s), and the acceleration of point C is zero. The vectors:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)}.$$

The angular velocities of AB and BC: The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-229.8\mathbf{i} + 192.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = -229.8\omega_{AB}\mathbf{i} + 192.8\omega_{AB}\mathbf{j} + \omega_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C is known, $\mathbf{v}_C = 1000\mathbf{i}$ (mm/s). Equate the expressions for $\mathbf{v}_C$ and separate components: $1000 = -229.8\omega_{AB} + 77.6\omega_{BC}$, $0 = 192.8\omega_{AB} + 289.8\omega_{BC}$. Solve:

$\boxed{\omega_{AB} = -3.55 \text{ rad/s}}$, $\boxed{\omega_{BC} = 2.36 \text{ rad/s}}$,

where the negative sign means a clockwise angular velocity.

The accelerations of AB and BC: The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$- \omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm/s}^2).$$

$$\mathbf{a}_B = \alpha_{AB}(-229.8\mathbf{i} + 192.8\mathbf{j}) - \omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j}) \text{ (mm/s}^2)$$

The acceleration of C in terms of the acceleration of B is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix} - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}),$$

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}) \text{ (mm/s}^2).$$

The acceleration of point C is known to be zero. Substitute this value for $\mathbf{a}_C$, and separate components:

$$-229.8\alpha_{AB} - 192.8\omega_{AB}^2 + 77.6\alpha_{BC} - 289.8\omega_{BC}^2 = 0,$$

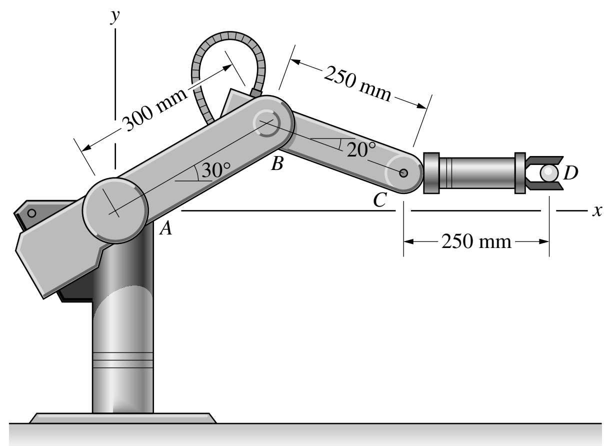
$$192.8\alpha_{AB} - 229.8\omega_{AB}^2 + 289.8\alpha_{BC} + 77.6\omega_{BC}^2 = 0.$$

Solve:

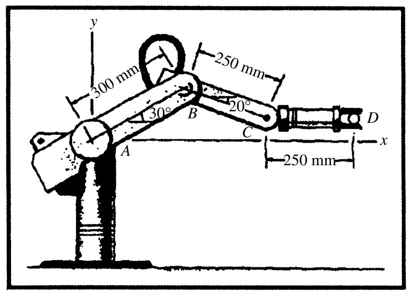
$\boxed{\alpha_{AB} = -12.1 \text{ rad/s}^2}$, $\boxed{\alpha_{BC} = 16.5 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.

Problem 17.111 Link AB of the robot's arm is rotating with a constant counterclockwise angular velocity of 2 rad/s, and link BC is rotating with a constant clockwise angular velocity of 3 rad/s. Link CD is rotating at 4 rad/s in the counterclockwise direction and has a counterclockwise angular acceleration of 6 rad/s^2. What is the acceleration of point D?



Solution: The acceleration of B is $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$. Evaluating, we get



$$\mathbf{a}_B = 0 + 0 - (2)^2(0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j})$$

$$= -1.039\mathbf{i} - 0.600\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of C is $\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$. Evaluating, we get

$$\mathbf{a}_C = -1.039\mathbf{i} - 0.600\mathbf{j} - (3)^2(0.25\cos 20°\mathbf{i} - 0.25\sin 20°\mathbf{j})$$

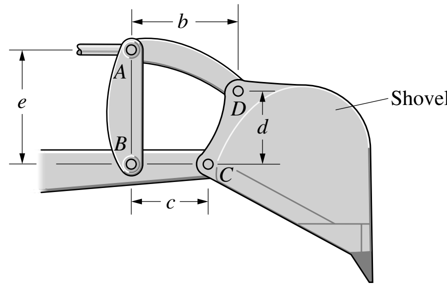
$$= -3.154\mathbf{i} + 0.170\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of D is $\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2 \mathbf{r}_{D/C}$. Evaluating, we get

$$\mathbf{a}_D = -3.154\mathbf{i} + 0.170\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 0.25 & 0 & 0 \end{vmatrix} - (4)^2(0.25\mathbf{i})$$

$$= -7.154\mathbf{i} + 1.67\mathbf{j} \text{ (m/s}^2\text{)}$$

Problem 17.112 At the instant shown, the dimensions are $b = 300$ mm, $c = 200$ mm, $d = 150$ mm, and $e = 260$ mm. If link AB has a clockwise angular velocity of 0.2 rad/s and a clockwise angular acceleration of 0.06 rad/s^2, what is the angular acceleration of the excavator's shovel?



Solution:

$\mathbf{r}_{A/B} = e\mathbf{j} = 0.26\mathbf{j}$ (m),

$\mathbf{r}_{D/A} = b\mathbf{i} - (e - d)\mathbf{j}$ (m)

$= 0.3\mathbf{i} - 0.11\mathbf{j}$(m),

$\mathbf{r}_{D/C} = (b - c)\mathbf{i} + d\mathbf{j}$ (m)

$= 0.1\mathbf{i} + 0.15\mathbf{j}$ (m),

$\mathbf{v}_B = \mathbf{v}_C = \mathbf{a}_B = \mathbf{a}_C = 0,$

$\boldsymbol{\omega}_{AB} = -0.2\mathbf{k}$ (rad/s).

$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$

Solving,

$\mathbf{v}_A = 0.052\mathbf{i}$ (m/s).

$\mathbf{v}_D = \mathbf{v}_A + \omega_{AD}\mathbf{k} \times \mathbf{r}_{D/A}$:

$$\begin{cases} v_{D_x} = 0.052 + 0.11\omega_{AD}, \\ v_{D_y} = 0 + 0.3\omega_{AD} \end{cases}.$$

We also have another relationship for $\mathbf{v}_D$.

$\mathbf{v}_D = \cancel{\mathbf{v}_c}^{0} + \omega_{CD}\mathbf{k} \times \mathbf{r}_{D/C}$:

$$\begin{cases} \mathbf{v}_{D_x} = -0.15\ \omega_{CD}, \\ \mathbf{v}_{D_y} = +0.1\ \omega_{CD}. \end{cases}$$

Setting components of $\mathbf{v}_D$ equal to each other, we get

$$\begin{cases} 0.052 + 0.11\ \omega_{AD} = -0.15\ \omega_{CD} \\ 0.3\ \omega_{AD} = 0.1\ \omega_{CD} \end{cases}$$

Two eqns, two unknowns.

$\boldsymbol{\omega}_{AD} = -0.0928\ \mathbf{k}$ (rad/s)

$\boldsymbol{\omega}_{CD} = -0.279\ \mathbf{k}$ (rad/s)

Now we can calculate accelerations.

$\mathbf{a}_A = \cancel{a_B} + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$

Solving, we get

$\mathbf{a}_A = 0.0156\mathbf{i} - 0.0104\mathbf{j}$ (m/s^2)

Now let us find $\mathbf{a}_D$ starting at A and at C. This will provide the equations we need.

$\mathbf{a}_D = a_A + \alpha_{AD}\ \mathbf{k} \times \mathbf{r}_{D/A} - \omega_{AD}^2\mathbf{r}_{D/A}$:

$$\begin{cases} a_{D_x} = a_{Ax} + 0.11\alpha_{AD} - (0.3)\omega_{AD}^2 & \textbf{(1)} \\ a_{D_y} = a_{Ay} + 0.3\alpha_{AD} + (0.11)\omega_{AD}^2 & \textbf{(2)} \end{cases}$$

(We know a_{Ax}, a_{Ay}, ω_{AD})

Also $\mathbf{a}_D = \cancel{\mathbf{a}_C}^{0} + \alpha_{CD}\ \mathbf{k} \times \mathbf{r}_{D/C} - \omega_{CD}^2\mathbf{r}_{D/C}$

$$\begin{cases} a_{D_x} = -0.15\ \alpha_{CD} - (0.1)\ \omega_{CD}^2 & \textbf{(3)} \\ a_{D_y} = 0.1\ \alpha_{CD} - (0.15)\ \omega_{CD}^2 & \textbf{(4)} \end{cases}$$

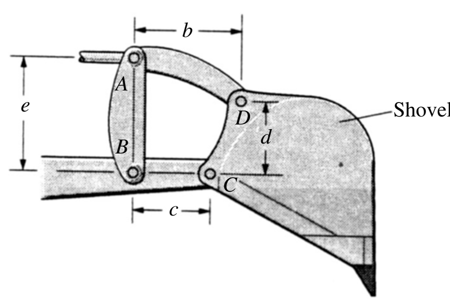
(we know ω_{CD})

We have 4 eqns in 4 unknowns (α_{CD}, α_{AD}, a_{D_x}, a_{D_y}) Solving,

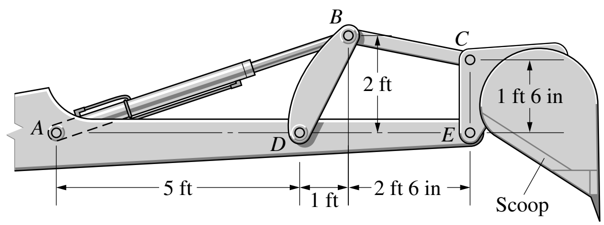
$\underline{\alpha_{CD} = -0.107 \text{ rad/s}}$ (clockwise)

$\alpha_{AD} = -0.0430$ rad/s

$\mathbf{a}_D = 0.00829\mathbf{i} - 0.0223\mathbf{j}$ (m/s^2)



Problem 17.113 The horizontal member ADE supporting the scoop is stationary. If the link BD has a clockwise angular velocity of 1 rad/s and a counterclockwise angular acceleration of 2 rad/s^2, what is the angular acceleration of the scoop?



Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$= 2\mathbf{i} - \mathbf{j} \text{ (ft/s)}.$$

The velocity of C is

$$\mathbf{v}_c = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 2\mathbf{i} - \mathbf{j} + 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2.5 & -.5 & 0 \end{vmatrix} \quad \textbf{(1)}$$

We can also express $\mathbf{v}_c$ as

$$\mathbf{v}_c = \mathbf{v}_E + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{C/E} = 0 + (\omega_{CE}\mathbf{k}) \times (1.5\mathbf{j}) = -1.5\omega_{CE}\mathbf{i}. \quad \textbf{(2)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (1) and (2) we get $2 + 0.5\omega_{BC} = -1.5\omega_{CE}$, and $-1 + 2.5\omega_{BC} = 0$. Solving, we obtain $\omega_{BC} = 0.400$ rad/s and $\omega_{CE} = -1.467$ rad/s.

The acceleration of B is

$$\mathbf{a}_B = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2 \mathbf{r}_{B/D},$$

or $$\mathbf{a}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} - (1)^2(\mathbf{i} + 2\mathbf{j})$$

$$= -5\mathbf{i} \text{ (ft/s}^2\text{)}.$$

The acceleration of C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$\mathbf{a}_C = -5\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 2.5 & -0.5 & 0 \end{vmatrix} - (0.4)^2(2.5\mathbf{i} - 0.5\mathbf{j}). \quad \textbf{(3)}$$

We can also express $\mathbf{a}_C$ as

$$\mathbf{a}_C = \mathbf{a}_E + \boldsymbol{\alpha}_{CE} \times \mathbf{r}_{C/E} - \omega_{CE}^2 \mathbf{r}_{C/E} = 0 + (\alpha_{CE}\mathbf{k})$$

$$\times (1.5j) - (-1.467)^2(1.5\mathbf{j})$$

$$= -1.5\alpha_{CE}\mathbf{i} - 3.23\mathbf{j}. \quad \textbf{(4)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (3) and (4), we get

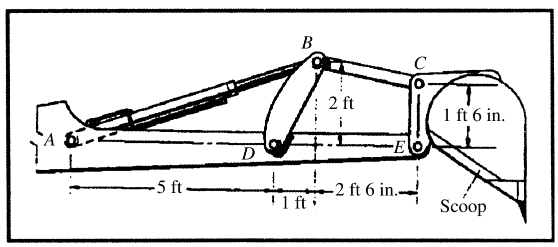
$$-5 + 0.5\alpha_{BC} - (0.4)^2(2.5) = -1.5\alpha_{CE},$$

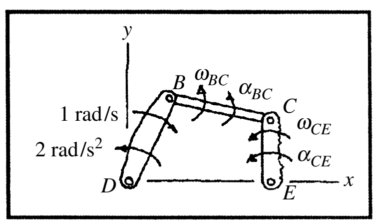
and $$2.5\alpha_{BC} + (0.4)^2(0.5) = -3.23.$$

Solving, we obtain

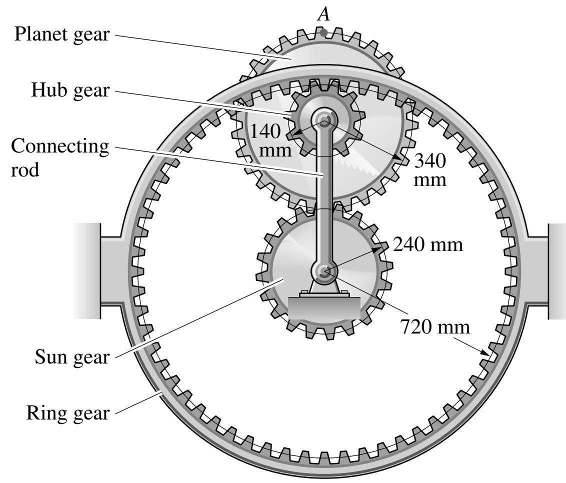
$$\alpha_{BC} = -1.32 \text{ rad/s}^2$$

$$\alpha_{CE} = 4.04 \text{ rad/s}^2.$$

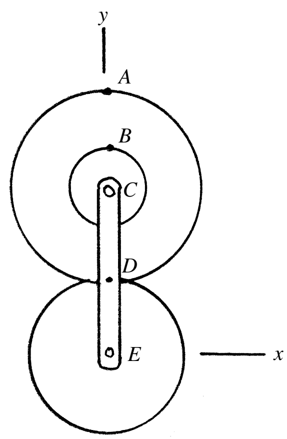




Problem 17.114 The ring gear is fixed, and the hub and planet gears are bonded together. The connecting rod has a counterclockwise angular acceleration of 10 rad/s^2. Determine the angular acceleration of the planet and sun gears.



Solution: The x components of the accelerations of pts B and C are



$a_{Bx} = 0,$

$$a_{Cx} = -(10 \text{ rad/s}^2)(0.58 \text{ m})$$

$$= -5.8 \text{ m/s}^2.$$

Let α_P and α_S be the angular accelerations of the planet and sun gears.

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{B/C} - w_P^2 \mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & 0.14 & 0 \end{vmatrix} - w_P^2(0.14\mathbf{j}).$$

The $\mathbf{i}$ component of this equation is

$$0 = -5.8 - 0.14\alpha_P.$$

We obtain

$$\underline{\alpha_P = -41.4 \text{ rad/s}^2}.$$

Also, $\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{D/C} - \omega_P^2 \mathbf{r}_{D/C}$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -41.4 \\ 0 & -0.34 & 0 \end{vmatrix} - \omega_P^2(-0.34\mathbf{j}).$$

The $\mathbf{i}$ component of this equation is

$$a_{Dx} = -5.8 - (41.4)(0.34) = -19.9 \text{ m/s}^2.$$

Therefore

$$\underline{\alpha_S = \frac{19.9}{0.24} = 82.9 \text{ rad/s}^2}.$$

Problem 17.115 The connecting rod in Problem 17.114 has a counterclockwise angular velocity of 4 rad/s and a clockwise angular acceleration of 12 rad/s^2. Determine the magnitude of the acceleration at point A.

Solution: See the solution of Problem 17.114. The velocities of pts B and C are

$$\mathbf{v}_B = \mathbf{O}, \mathbf{v}_C = -(4)(0.58)\mathbf{i} = -2.32\mathbf{i} \text{ (m/s)}.$$

Let ω_P and ω_S be the angular velocities of the planet and sun gears.

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{B/C}:$$

$$\mathbf{O} = -2.32\mathbf{i} + (\omega_P\mathbf{k}) \times (0.14\mathbf{j})$$

$$= (-2.32 - 0.14\omega_P)\mathbf{i}.$$

We see that $\omega_P = -16.6$ rad/s. Also,

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{D/C}$$

$$= -2.32\mathbf{i} + (-16.6\mathbf{k}) \times (-0.34\mathbf{j})$$

$$= -7.95\mathbf{i} \text{ (m/s)},$$

So $\quad \omega_S = \dfrac{7.95}{0.24} = 33.1$ rad/s.

The x components of the accelerations of pts B and C are

$$a_{Bx} = 0,$$

$$a_{Cx} = (12 \text{ rad/s}^2)(0.58 \text{ m})$$

$$= 6.96 \text{ m/s}^2.$$

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{B/C} - \omega_P^2\mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & 0.14 & 0 \end{vmatrix} - \omega_P^2(0.14\mathbf{j}).$$

The $\mathbf{i}$ component is

$$0 = 6.96 - 0.14\alpha_P,$$

so $\quad \alpha_P = 49.7$ rad/s^2.

The acceleration of C is

$$\mathbf{a}_C = \mathbf{a}_E + (-12\mathbf{k}) \times \mathbf{r}_{C/E} - (4)^2\mathbf{r}_{C/E}$$

$$= 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 0.58 & 0 \end{vmatrix} - (4)^2(0.58\mathbf{j})$$

$$= 6.96\mathbf{i} - 9.28\mathbf{j} \text{ (m/s}^2).$$

Then the acceleration of A is

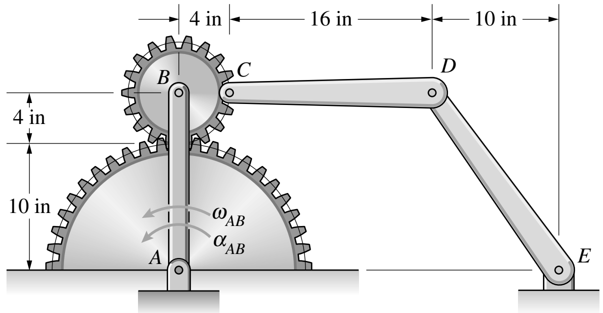
$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{A/C} - \omega_P^2\mathbf{r}_{A/C}$$

$$= 6.96\mathbf{i} - 9.28\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 49.7 \\ 0 & 0.34 & 0 \end{vmatrix} - (-16.6)^2(0.34\mathbf{j})$$

$$= -9.94\mathbf{i} - 102.65\mathbf{j} \text{ (m/s}^2).$$

$\underline{|\mathbf{a}_A| = 103 \text{ m/s}^2}$.

Problem 17.116 The large gear is fixed. The angular velocity and angular acceleration of the bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 4$ rad/s^2. Determine the angular acceleration of the bars CD and DE.



Solution: The strategy is to express vector velocity of point D in terms of the unknown angular velocities and accelerations of CD and DE, and then to solve the resulting vector equations for the unknowns. *The angular velocities* ω_{CD} and ω_{DE}. (See solution to Problem 17.51). The linear velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & 14 & 0 \end{bmatrix} = -28\mathbf{i} \text{ (in/s)}.$$

The lower edge of gear B is stationary. The velocity of B is also

$$\mathbf{v}_B = \boldsymbol{\omega}_B \times \mathbf{r}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_B \\ 0 & 4 & 0 \end{bmatrix} = -4\omega_B\mathbf{i} \text{ (in/s)}.$$

Equate the velocities $\mathbf{v}_B$ to obtain the angular velocity of B:

$$\omega_B = -\frac{v_B}{4} = 7 \text{ rad/s}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_B \times \mathbf{r}_{BC} = -28\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 7 \\ 4 & 0 & 0 \end{bmatrix} = -28\mathbf{i} + 28\mathbf{j} \text{ (in/s)}.$$

The velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{CD} = -28\mathbf{i} + 28\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 16 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + (16\omega_{CD} + 28)\mathbf{j} \text{ (in/s)}.$$

The velocity of point D is also given by

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{ED} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -10 & 14 & 0 \end{bmatrix}$$

$$= -14\omega_{DE}\mathbf{i} - 10\omega_{DE}\mathbf{j} \text{ (in/s)}.$$

Equate and separate components:

$$(-28 + 14\omega_{DE})\mathbf{i} = 0, \quad (16\omega_{CD} + 28 + 10\omega_{DE})\mathbf{j} = 0.$$

Solve: $\omega_{DE} = 2$ rad/s,

$\omega_{CD} = -3$ rad/s.

The negative sign means a clockwise rotation. *The angular accelerations*. The tangential acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 14 & 0 \end{bmatrix} = -56\mathbf{i} \text{ (in/s}^2\text{)}.$$

The tangential acceleration at the point of contact between the gears A and B is zero, from which

$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times 4\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 0 & 4 & 0 \end{bmatrix} = -4\alpha_{BC}\mathbf{i} \text{ (in/s}^2\text{)},$$

from which $\alpha_{BC} = 14$ rad/s^2. The acceleration of point C in terms of the acceleration of point B is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times 4\mathbf{i} - \omega_B^2(4\mathbf{i}) = -56\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 14 \\ 4 & 0 & 0 \end{bmatrix} - 49(4\mathbf{i})$$

$$= -252\mathbf{i} + 56\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point D in terms of the acceleration of point C is

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times 16\mathbf{i} - \omega_{CD}^2(16\mathbf{i})$$

$$= \mathbf{a}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 16 & 0 & 0 \end{bmatrix} - \omega_{CD}^2(16\mathbf{i}),$$

$$\mathbf{a}_D = -396\mathbf{i} + (16\alpha_{CD} + 56)\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point D in terms of the acceleration of point E is

$$\mathbf{a}_D = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{DE} \\ -10 & 14 & 0 \end{bmatrix} - \omega_{DE}^2(-10\mathbf{i} + 14\mathbf{j})$$

$$= (40 - 14\alpha_{DE})\mathbf{i} - (10\alpha_{DE} + 56)\mathbf{j} \text{ (in/s}^2\text{)}$$

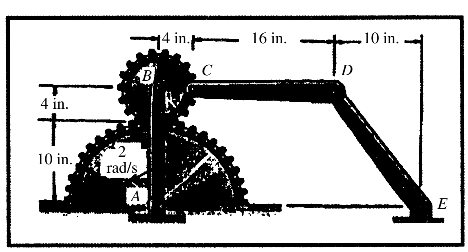
Equate the expressions for $\mathbf{a}_D$ and separate components:

$$-396 = 40 - 14\alpha_{DE}, \quad 16\alpha_{CD} + 56 = -10\alpha_{DE} - 56.$$

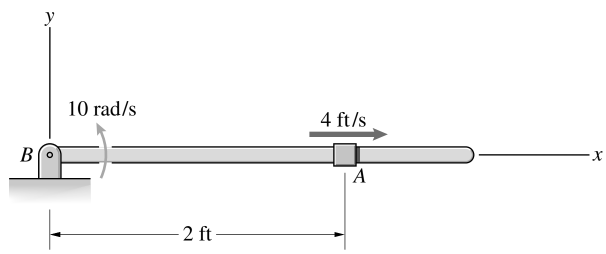
Solve: $\boxed{\alpha_{DE} = 31.1 \text{ rad/s}^2}$,

$\boxed{\alpha_{CD} = -26.5 \text{ rad/s}^2}$,

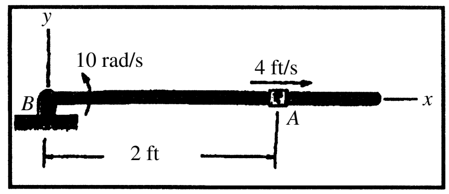
where the negative sign means a clockwise angular acceleration.



Problem 17.117 The bar rotates with a constant counterclockwise angular velocity of 10 rad/s and the sleeve A slides at 4 ft/s relative to the bar. Use Eq. (17.11) and the body-fixed coordinate system shown to determine the velocity of A.



Solution: Eq. (17.11) is $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$. Substitute:



$$\mathbf{v}_A = 0 + 4\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 2 & 0 & 0 \end{bmatrix} \boxed{= 4\mathbf{i} + 20\mathbf{j} \text{ (ft/s)}}$$

Problem 17.118 Sleeve A in Problem 17.117 slides relative to the bar at a constant velocity of 4 ft/s. Use Eq. (17.15) to determine the acceleration of A.

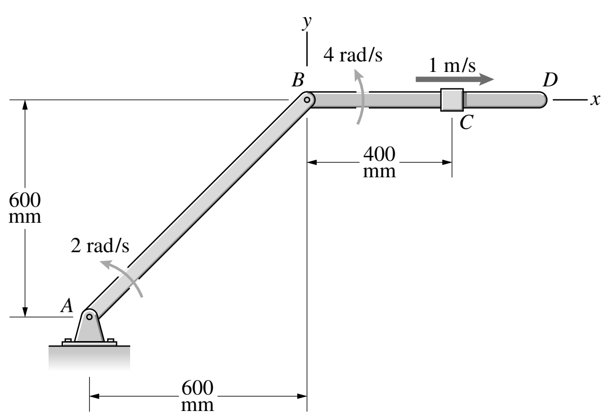
Solution: Eq. (17.15) is

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\omega \times \mathbf{v}_{A\text{rel}} + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}.$$

Substitute: $$\mathbf{a}_A = 0 + 0 + 2 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 4 & 0 & 0 \end{bmatrix} + 0 - 100(2\mathbf{i})$$

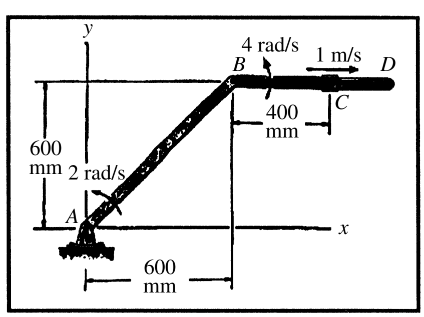
$$\boxed{= -200\mathbf{i} + 80\mathbf{j} \text{ (ft/s}^2\text{)}}$$

Problem 17.119 Sleeve C slides at 1 m/s relative to bar BD. Use the body-fixed coordinate system shown to determine the velocity of C.



Solution: The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 600 & 600 & 0 \end{bmatrix} = -1200(\mathbf{i} - \mathbf{j}) \text{ (mm/s)}.$$



Use Eq. (17.11). The velocity of sleeve C is

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{C/B}.$$

$$\mathbf{v}_C = -1200\mathbf{i} + 1200\mathbf{j} + 1000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 400 & 0 & 0 \end{bmatrix}.$$

$$\boxed{\mathbf{v}_C = -200\mathbf{i} + 2800\mathbf{j} \text{ (mm/s)}}.$$

Problem 17.120 In Problem 17.119, the angular accelerations of the two bars are zero and the sleeve C slides at a constant velocity of 1 m/s relative to bar BD. What is the acceleration of C?

Solution: From Problem 17.119, $\omega_{AB} = 2$ rad/s, $\omega_{BC} = 4$ rad/s. The acceleration of point B is

$$\mathbf{a}_B = -\omega_{AB}^2 \mathbf{r}_{B/A} = -4(600\mathbf{i} + 600\mathbf{j})$$

$$= -2400\mathbf{i} - 2400\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Use Eq. (17.15). The acceleration of C is

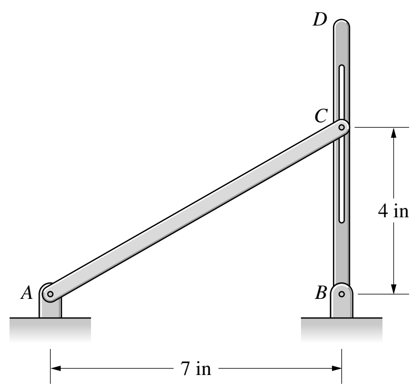
$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BD} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{C/B} - \omega_{BD}^2 \mathbf{r}_{C/B}.$$

$$\mathbf{a}_C = -2400\mathbf{i} - 2400\mathbf{j} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 1000 & 0 & 0 \end{bmatrix} - \omega_{BD}^2(400\mathbf{i}),$$

$$\boxed{\mathbf{a}_C = -8800\mathbf{i} + 5600\mathbf{j} \text{ (mm/s}^2\text{)}}$$

Problem 17.121 Bar AC has an angular velocity of 2 rad/s in the counterclockwise direction that is decreasing at 4 rad/s^2. The pin at C slides in the slot in bar BD.

(a) Determine the angular velocity of bar BD and the velocity of the pin relative to the slot.
(b) Determine the angular acceleration of bar BD and the acceleration of the pin relative to the slot.



Solution: The coordinate system is fixed with respect to the vertical bar.

(a) $$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AC} \times \boldsymbol{r}_{C/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ 7 & 4 & 0 \end{vmatrix}. \quad \textbf{(1)}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C\text{rel}} + \boldsymbol{\omega}_{BD} \times \boldsymbol{r}_{C/B}$$

$$= \mathbf{0} + v_{C\text{rel}}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0 & 4 & 0 \end{vmatrix}. \quad \textbf{(2)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Eqs. (1) and (2),

$$-4\omega_{AC} = -4\omega_{BD}, \quad \textbf{(3)}$$

$$7\omega_{AC} = v_{C\text{rel}}, \quad \textbf{(4)}$$

We obtain $\underline{\omega_{BD} = 2 \text{ rad/s}}$, $\underline{v_{C\text{rel}} = 14 \text{ in/s}}$.

(b) $$\boldsymbol{a}_C = \boldsymbol{a}_A + \boldsymbol{\alpha}_{AC} \times \boldsymbol{r}_{C/A} - \omega_{AC}^2 \boldsymbol{r}_{C/A}$$

$$= \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 7 & 4 & 0 \end{vmatrix} - \omega_{AC}^2(7\mathbf{i} + 4\mathbf{j}). \quad \textbf{(5)}$$

$$\boldsymbol{a}_C = \boldsymbol{a}_B + \boldsymbol{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BD} \times \mathbf{v}_{C\text{rel}}$$

$$+ \boldsymbol{\alpha}_{BD} \times \boldsymbol{r}_{C/B} - \omega_{BD}^2 \boldsymbol{r}_{C/B}$$

$$= \mathbf{0} + a_{C\text{rel}}\mathbf{j} + 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0 & v_{C\text{rel}} & 0 \end{vmatrix}$$

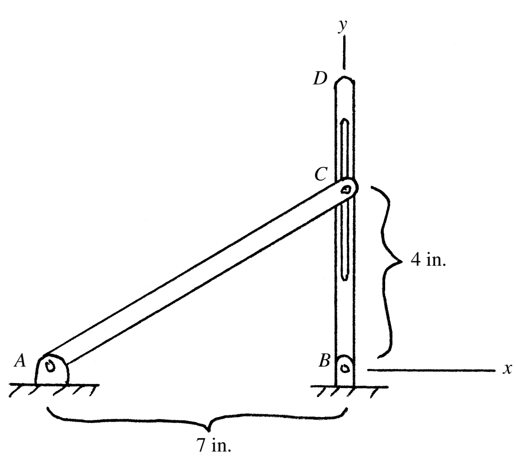
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0 & 4 & 0 \end{vmatrix} - \omega_{BD}^2(4\mathbf{j}). \quad \textbf{(6)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Eqs. (5) and (6),

$$-4\alpha_{AC} - 7\omega_{AC}^2 = -2\omega_{BD} v_{C\text{rel}} - 4\alpha_{BD}, \quad \textbf{(7)}$$

$$7\alpha_{AC} - 4\omega_{AC}^2 = a_{C\text{rel}} - 4\omega_{BD}^2, \quad \textbf{(8)}$$

We obtain $\underline{\alpha_{BD} = -11 \text{ rad/s}^2}$, $\underline{a_{C\text{rel}} = -28 \text{ in/s}^2}$.



Problem 17.122 In the system shown in Problem 17.121, the velocity of the pin C relative to the slot is 21 in./s upward and is decreasing at 42 in./s^2. What are the angular velocity and acceleration of bar AC?

Solution: See the solution of Problem 17.121. Solving Eqs. (3), (4), (7), and (8) with $v_{C\text{rel}} = 21$ in/s and $a_{C\text{rel}} = -42$ in/s^2, we obtain

$\underline{\omega_{AC} = 3 \text{ rad/s}}$,

$\underline{\alpha_{AC} = -6 \text{ rad/s}^2}$.

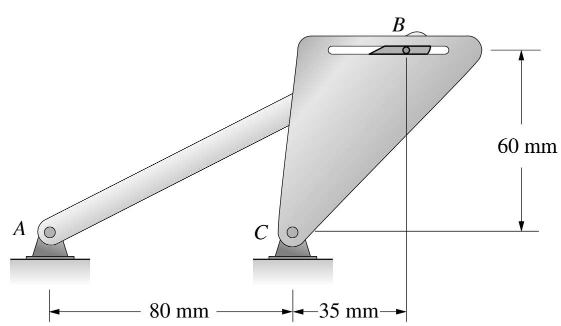
Problem 17.123 In the system shown in Problem 17.121, what should the angular velocity and acceleration of bar AC be if you want the angular velocity and acceleration of bar BD to be 4 rad/s counterclockwise and 24 rad/s^2 counterclockwise, respectively?

Solution: See the solution of Problem 17.121. Solving Eqs. (3), (4), (7), and (8) with $\omega_{BD} = 4$ rad/s^2 and $\alpha_{BD} = 24$ rad/s^2, we obtain

$\omega_{AC} = 4$ rad/s,

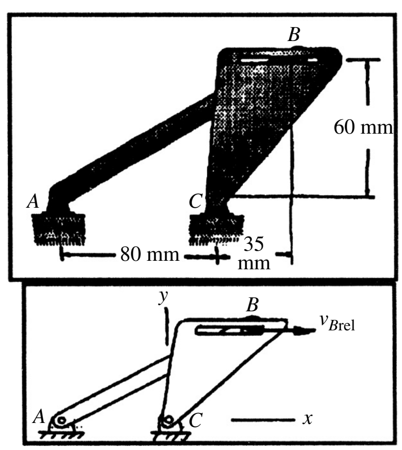
$\alpha_{AC} = 52$ rad/s^2.

Problem 17.124 Bar AB has an angular velocity of 4 rad/s in the clockwise direction. What is the velocity of pin B relative to the slot?



Solution: The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{AB} \\ 115 & 60 & 0 \end{bmatrix} = 240\mathbf{i} - 460\mathbf{j} \text{ (mm/s)}.$$



The velocity of point B is also determined from bar CB

$$\mathbf{v}_B = \mathbf{v}_{B\text{rel}} + \boldsymbol{\omega}_{CB} \times (35\mathbf{i} + 60\mathbf{j}),$$

$$\mathbf{v}_B = \mathbf{v}_{B\text{rel}} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CB} \\ 35 & 60 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{i} - 60\omega_{CB}\mathbf{i} + 35\omega_{CB}\mathbf{j} \text{ (mm/s)}.$$

Equate like terms: $240 = v_{B\text{rel}} - 60\omega_{CB}$, $-460 = 35\omega_{CB}$ from which

$\omega_{BC} = -13.14$ rad/s, $\boxed{v_{B\text{rel}} = -548.6 \text{ mm/s}}$

Problem 17.125 In the system shown in Problem 17.124, the bar AB has an angular velocity of 4 rad/s in the clockwise direction and an angular acceleration of 10 rad/s^2 in the counterclockwise direction. What is the acceleration of pin B relative to the slot?

Solution: Use the solution to Problem 17.124, from which $\omega_{BC} = -13.14$ rad/s, $v_{B\text{rel}} = -548.6$ mm/s. *The angular acceleration and the relative acceleration*. The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 115 & 60 & 0 \end{bmatrix} - (16)(115\mathbf{i} + 60\mathbf{j}) \text{ (mm/s}^2\text{)},$$

$$\mathbf{a}_B = -600\mathbf{i} + 1150\mathbf{j} - 1840\mathbf{i} - 960\mathbf{j} = -2440\mathbf{i} + 190\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of pin B in terms of bar BC is

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C},$$

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -13.14 \\ -548.6 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 35 & 60 & 0 \end{bmatrix} - (13.14^2)(35\mathbf{i} + 60\mathbf{j}).$$

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 14{,}419.5\mathbf{j} + 35\alpha_{BC}\mathbf{j} - 60\alpha_{BC}\mathbf{i}$$

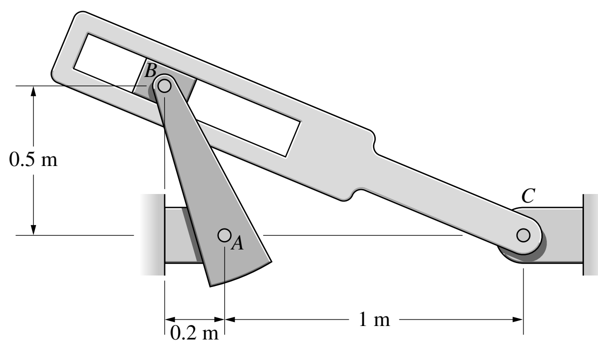
$$- 6045.7\mathbf{i} - 10,364.1\mathbf{j}.$$

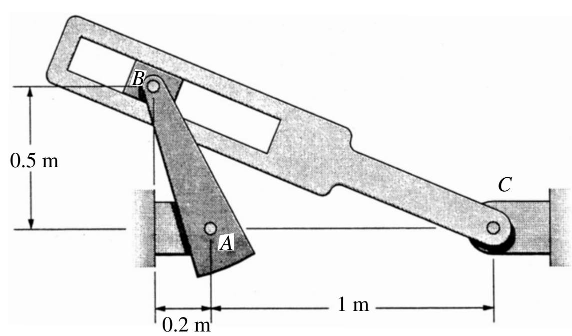
Equate expressions for $\mathbf{a}_B$ and separate components: $-2440 = a_{B\text{rel}} - 60\alpha_{BC} - 6045.7$, $190 = 14{,}419.6 + 35\alpha_{BC} - 10364.1$. Solve:

$\boxed{\mathbf{a}_{B\text{rel}} = -3021\mathbf{i} \text{ (mm/s}^2)}$, $\alpha_{BC} = -110.4$ rad/s^2.

Problem 17.126 Arm AB is rotating at 4 rad/s in the clockwise direction.

(a) What is the angular velocity of arm BC?
(b) What is the velocity of point B relative to the slot in arm BC?





Solution:

Arm AB:

$\mathbf{v}_A = 0$, $\boldsymbol{\omega}_{AB} = -4\mathbf{k}$, $\mathbf{r}_{B/A} = -0.2\mathbf{i} + 0.5\mathbf{j}$ (m).

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times r_{B/A}$$

$$= 2.0\mathbf{i} + 0.8\mathbf{j} \text{ (m/s)}.$$

Arm BC:

$\boldsymbol{\omega}_{BC} = \omega_{BC}\mathbf{k}$, $\mathbf{r}_{B/C} = -1.2\mathbf{i} + 0.5\mathbf{j}$ m

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B_{\text{rel}}} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$\mathbf{v}_{B_{\text{rel}}} = -v_{B_{\text{rel}}} \cos\theta\mathbf{i} + v_{B_{\text{rel}}} \sin\theta\mathbf{j}$$

where $\tan\theta = \dfrac{0.5}{1.2}$ $\theta = 22.62°$ $\mathbf{v}_C = 0$. Thus

$$\mathbf{v}_B = -v_{B_{\text{rel}}} \cos\theta\mathbf{i} + v_{B_{\text{rel}}} \sin\theta\mathbf{j}$$

$$+ \omega_{BC}\mathbf{k} \times (-1.2\mathbf{i} + 0.5\mathbf{j})$$

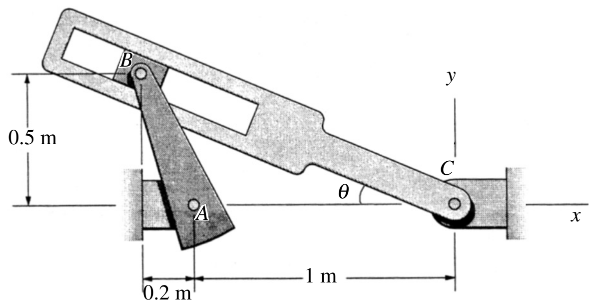
$$\begin{cases} v_{B_x} = 2.0 = -v_{B_{\text{rel}}} \cos\theta - 0.5\omega_{BC} \\ v_{B_y} = 0.8 = v_{B_{\text{rel}}} \sin\theta - 1.2\omega_{BC} \end{cases}$$

Solving, $\underline{\omega_{BC} = -1.16 \text{ rad/s}}$ (clockwise)

$\underline{v_{B_{\text{rel}}} = -1.54 \text{ m/s}}$ (toward C)

Problem 17.127 The angular acceleration of arm AB in Problem 17.126 is zero.

(a) What is the angular acceleration of arm BC?
(b) What is the acceleration of point B relative to the slot in arm BC?



Solution: From the solution to Problem 17.126, we know

$\mathbf{v}_A = 0 \quad \theta = 22.62°$

$\mathbf{v}_B = 2.0\mathbf{i} + 0.8\mathbf{j}$ (m/s)

$\mathbf{v}_C = 0$

$v_{B_{\text{rel}}} = -1.54$ m/s (toward C)

$\mathbf{r}_{B/A} = -0.2\mathbf{i} + 0.5\mathbf{j}$ (m)

$\mathbf{r}_{B/C} = -1.2\mathbf{i} + 0.5\mathbf{j}$ (m)

$\boldsymbol{\omega}_{AB} = -4\mathbf{k}$ (rad/s)

$\boldsymbol{\omega}_{BC} = -1.16\mathbf{k}$ (rad/s)

we also know $\mathbf{a}_A = \mathbf{a}_C = 0$ and $\boldsymbol{\alpha}_{AB} = 0$.

$\boldsymbol{\alpha}_{AB} = 0$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= 0 + 0 - \omega_{AB}^2(-0.2\mathbf{i} + 0.5\mathbf{j}).$$

$a_{B_x} = +(16)(0.2) = +3.2 \text{ m/s}^2$

$a_{B_y} = -(16)(0.5) = -8 \text{ m/s}^2$

Also, $\mathbf{a}_B = \mathbf{a}_C^0 + \mathbf{a}_{B_{\text{rel}}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$

$$= 0 + \mathbf{a}_{B_{\text{rel}}} + \alpha_{BC}\mathbf{k} \times (-1.2\mathbf{i} + 0.5\mathbf{j})$$

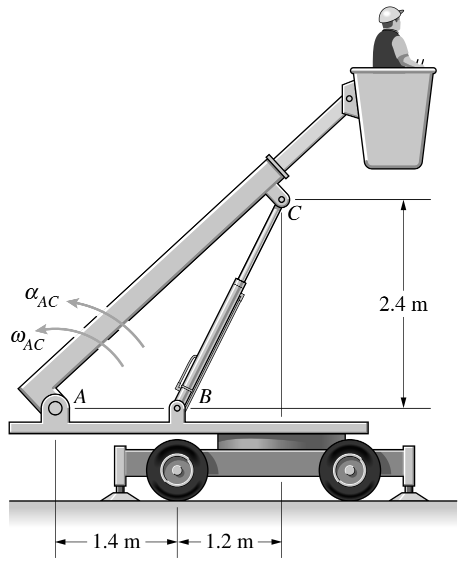
$$- \omega_{BC}^2(-1.2\mathbf{i} + 0.5\mathbf{j}).$$

$$\begin{cases} a_{B_x} = +3.2 = a_{B_{\text{rel}x}} - 0.5\alpha_{BC} + 1.2\omega_{BC}^2 \\ a_{B_y} = -8 = a_{B_{\text{rel}y}} - 1.2\alpha_{BC} - 0.5\omega_{BC}^2 \\ a_{B_{\text{rel}}} = -a_{B_{\text{rel}}}\cos\theta\mathbf{i} + a_{B_{\text{rel}}}\sin\theta\mathbf{j} \end{cases}$$

Solving, $a_{B_{\text{rel}}} = -4.28 \text{ m/s}^2$ toward C

$\alpha_{BC} = 4.73 \text{ m/s}^2$ (count clockwise)

Problem 17.128 The angular velocity $\omega_{AC} = 5^\circ$ per second. Determine the angular velocity of the hydraulic actuator BC and the rate at which the actuator is extending.



Solution: The point C effectively slides in a slot in the arm BC. The angular velocity of

$$\omega_{AC} = 5\left(\frac{\pi}{180}\right) = 0.0873 \text{ rad/s}.$$

The velocity of point C with respect to arm AC is

$$\mathbf{v}_C = \omega_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ 2.6 & 2.4 & 0 \end{bmatrix}$$

$$= -0.2094\mathbf{i} + 0.2269\mathbf{j} \text{ (m/s)},$$

The unit vector parallel to the actuator BC is

$$\mathbf{e} = \frac{1.2\mathbf{i} + 2.4\mathbf{j}}{\sqrt{1.2^2 + 2.4^2}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The velocity of point C in terms of the velocity of the actuator is

$$\mathbf{v}_C = v_{C\text{rel}}\mathbf{e} + \omega_{BC} \times \mathbf{r}_{C/B}.$$

$$\mathbf{v}_C = v_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 1.2 & 2.4 & 0 \end{bmatrix}$$

$$\mathbf{v}_C = v_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + \omega_{BC}(-2.4\mathbf{i} + 1.2\mathbf{j}).$$

Equate like terms in the two expressions:

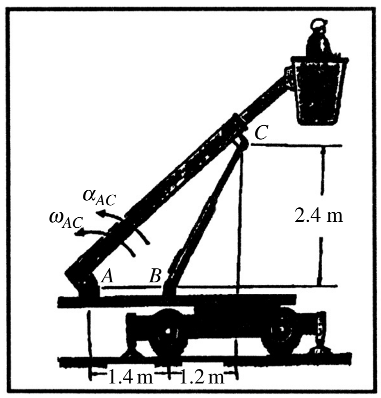
$$-0.2094 = 0.4472v_{C\text{rel}} - 2.4\omega_{BC},$$

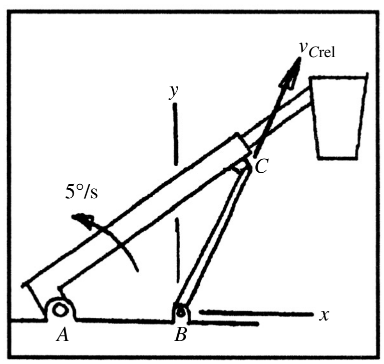
$$0.2269 = 0.8944v_{C\text{rel}} + 1.2\omega_{BC}.$$

$$\boxed{\omega_{BC} = 0.1076 \text{ rad/s} = 6.17 \text{ deg/s}},$$

$$\boxed{v_{C\text{rel}} = 0.109 \text{ (m/s)}},$$

which is also the velocity of extension of the actuator.





Problem 17.129 In Problem 17.128, if the angular velocity $\omega_{AC} = 5^\circ$ per second and the angular acceleration $\alpha_{AC} = -2^\circ$ per second squared, determine the angular acceleration of the hydraulic actuator BC and the rate of change of the actuator's rate of extension.

Solution: Use the solution to Problem 17.128 for the velocities:

$\omega_{BC} = 0.1076$ rad/s,

$v_{C\text{rel}} = 0.1093$ (m/s).

The angular acceleration

$$\alpha_{AC} = -2\left(\frac{\pi}{180}\right) = -0.03491 \text{ rad/s}^2.$$

The acceleration of point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} - \omega_{AC}^2 \mathbf{r}_{C/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2.6 & 2.4 & 0 \end{bmatrix} - \omega_{AC}^2(2.6\mathbf{i} + 2.4\mathbf{j}),$$

$$\mathbf{a}_C = \alpha_{AC}(-2.4\mathbf{i} + 2.6\mathbf{j}) - \omega_{AC}^2(2.6\mathbf{i} + 2.4\mathbf{j})$$

$$= 0.064\mathbf{i} - 0.109\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point C in terms of the hydraulic actuator is

$$\mathbf{a}_C = a_{C\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{\mathbf{BC}}^2 \mathbf{r}_{C/B},$$

$$\mathbf{a}_C = a_{C\text{rel}}\mathbf{e} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.4472 v_{C\text{rel}} & 0.8944 v_{C\text{rel}} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 1.2 & 2.4 & 0 \end{bmatrix} - \omega_{BC}^2(1.2\mathbf{i} + 2.4\mathbf{j})$$

$$\mathbf{a}_C = a_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + 2\omega_{BC}(-0.0977\mathbf{i} + 0.0489\mathbf{j})$$

$$+ \alpha_{BC}(-2.4\mathbf{i} + 1.2\mathbf{j}) - \omega_{BC}^2(1.2\mathbf{i} + 2.4\mathbf{j}).$$

Equate like terms in the two expressions for $\mathbf{a}_C$.

$$0.0640 = 0.4472 a_{C\text{rel}} - 0.0139 - 2.4\alpha_{BC} - 0.0210,$$

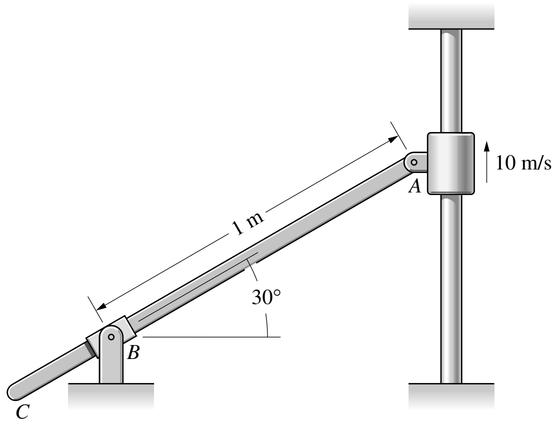
$$-0.1090 = 0.8944 a_{C\text{rel}} - 0.0278 + 1.2\alpha_{BC} + 0.0105.$$

Solve: $\boxed{a_{C\text{rel}} = -0.0378 \text{ (m/s}^2)}$,

which is the rate of change of the rate of extension of the actuator, and

$$\boxed{\alpha_{BC} = -0.0483 \text{ (rad/s}^2) = -2.77 \text{ deg/s}^2}$$

Problem 17.130 The sleeve at A slides upward at a constant velocity of 10 m/s. Bar AC slides through the sleeve at B. Determine the angular velocity of bar AC and the velocity at which the bar slides relative to the sleeve at B.



Solution: The velocity of the sleeve at A is given to be $\mathbf{v}_A = 10\mathbf{j}$ (m/s). The unit vector parallel to the bar (toward A) is

$$\mathbf{e} = 1(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 0.866\mathbf{i} + 0.5\mathbf{j}.$$

Choose a coordinate system with origin at B that rotates with the bar. The velocity at A is

$$\mathbf{v}_A = \mathbf{v}_B + v_{A\mathrm{rel}}\mathbf{e} + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{A/B}$$

$$= 0 + v_{A\mathrm{rel}}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

$$\mathbf{v}_A = (0.866\mathbf{i} + 0.5\mathbf{j})v_{A\mathrm{rel}} + \omega_{AC}(-0.5\mathbf{i} + 0.866\mathbf{j}) \text{ (m/s)}.$$

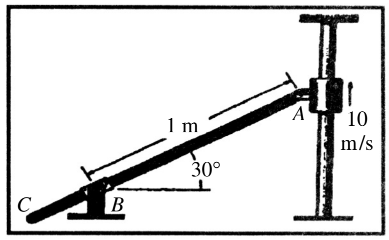
The given velocity is $\mathbf{v}_A = 10\mathbf{j}$ (m/s). Equate like components in the two expressions for $\mathbf{v}_A$:

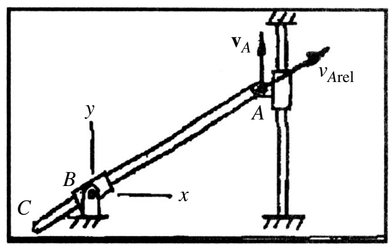
$$0 = 0.866 v_{A\mathrm{rel}} - 0.5\omega_{AC},$$

$$10 = 0.5 v_{A\mathrm{rel}} + 0.866\omega_{AC}.$$

Solve: $\boxed{\omega_{AC} = 8.66 \text{ rad/s}}$ (counterclockwise),

$\boxed{v_{A\mathrm{rel}} = 5 \text{ m/s}}$ from B toward A.





Problem 17.131 In Problem 17.130, the sleeve at A slides upward at a constant velocity of 10 m/s. Determine the angular acceleration of the bar AC and the rate of change of the velocity at which it slides relative to the sleeve at B.

Solution: Use the solution of Problem 17.130:

$$\mathbf{e} = 0.866\mathbf{i} + 0.5\mathbf{j},$$

$$\omega_{AB} = 8.66 \text{ rad/s},$$

$$v_{A\mathrm{rel}} = 5 \text{ m/s}.$$

The acceleration of the sleeve at A is given to be zero. The acceleration in terms of the motion of the arm is

$$\mathbf{a}_A = 0 = a_{A\mathrm{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{AB} \times v_{A\mathrm{rel}}\mathbf{e} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B}.$$

$$\mathbf{a}_A = 0 = a_{A\mathrm{rel}}\mathbf{e} + 2v_{A\mathrm{rel}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix} - \omega_{AB}^2(0.866\mathbf{i} + 0.5\mathbf{j})$$

$$0 = (0.866\mathbf{i} + 0.5\mathbf{j})a_{A\mathrm{rel}} - 43.3\mathbf{i} + 75\mathbf{j}$$

$$+ \alpha_{\mathbf{AB}}(-0.5\mathbf{i} + 0.866\mathbf{j}) - 64.95\mathbf{i} - 37.5\mathbf{j}.$$

Separate components:

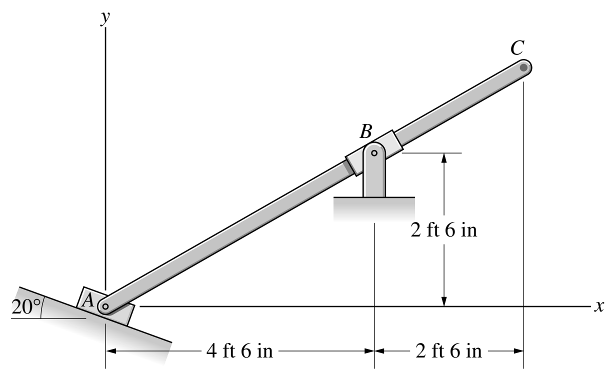
$$0 = 0.866 a_{A\mathrm{rel}} - 43.3 - 0.5\alpha_{AB} - 64.95,$$

$$0 = 0.5 a_{A\mathrm{rel}} + 75 + 0.866\alpha_{AB} - 37.5.$$

Solve: $\boxed{a_{A\mathrm{rel}} = 75 \text{ (m/s}^2)}$ (toward A).

$\boxed{\alpha_{AB} = -86.6 \text{ rad/s}^2}$, (clockwise).

Problem 17.132 Block A slides up the inclined surface at 2 ft/s. Determine the angular velocity of bar AC and the velocity of point C.



Solution: The velocity at A is given to be

$$\mathbf{v}_A = 2(-\mathbf{i}\cos 20^\circ + \mathbf{j}\sin 20^\circ) = -1.879\mathbf{i} + 0.6840\mathbf{j} \text{ (ft/s)}.$$

From geometry, the coordinates of point C are

$$\left(7, 2.5\left(\frac{7}{4.5}\right)\right) = (7, 3.89) \text{ (ft)}.$$

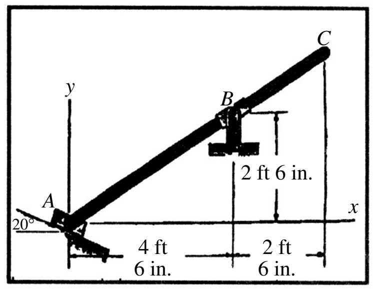
The unit vector parallel to the bar (toward A) is

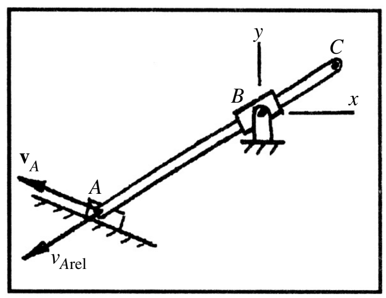
$$\mathbf{e} = (7^2 + 3.89^2)^{-1/2}(-7\mathbf{i} - 3.89\mathbf{j}) = -0.8742\mathbf{i} - 0.4856\mathbf{j}.$$

The velocity at A in terms of the motion of the bar is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = v_{A\text{rel}}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix},$$

$$\mathbf{v_A} = -0.8742 v_{A\text{rel}}\mathbf{i} - 0.4856 v_{A\text{rel}}\mathbf{j} + 2.5\omega_{AC}\mathbf{i} - 4.5\omega_{AC}\mathbf{j} \text{ (ft/s)}.$$





Equate the two expressions for $\mathbf{v}_A$ and separate components:

$$-1.879 = -0.8742 v_{A\text{rel}} + 2.5\omega_{AC},$$

$$0.6840 = -0.4856 v_{A\text{rel}} - 4.5\omega_{AC}.$$

Solve: $v_{A\text{rel}} = 1.311$ ft/s,

$\boxed{\omega_{AC} = -0.293 \text{ rad/s}}$ (clockwise).

Noting that $\mathbf{v}_A = 2$ ft/s, the velocity at point C is

$$\mathbf{v}_C = v_A(-0.8742\mathbf{i} - 0.4856\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.293 \\ 2.5 & 3.89 - 2.5 & 0 \end{bmatrix},$$

$\boxed{\mathbf{v}_C = -0.738\mathbf{i} - 1.37\mathbf{j} \text{ (ft/s)}}$.

Problem 17.133 In Problem 17.132, the block A slides up the inclined surface at a constant velocity of 2 ft/s. Determine the angular acceleration of bar AC and the acceleration of point C.

Solution: *The velocities*: The velocity at A is given to be

$$\mathbf{v}_A = 2(-\mathbf{i}\cos 20^\circ + \mathbf{j}\sin 20^\circ) = -1.879\mathbf{i} + 0.6840\mathbf{j} \text{ (ft/s)}.$$

From geometry, the coordinates of point C are

$$\left(7, 2.5\left(\frac{7}{4.5}\right)\right) = (7, 3.89) \text{ (ft)}.$$

The unit vector parallel to the bar (toward A) is

$$\mathbf{e} = \frac{-7\mathbf{i} - 3.89\mathbf{j}}{\sqrt{7^2 + 3.89^2}} = -0.8742\mathbf{i} - 0.4856\mathbf{j}.$$

The velocity at A in terms of the motion of the bar is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e} + \boldsymbol{\omega}_{\mathbf{AC}} \times \mathbf{r}_{A/B} = v_{A\text{rel}}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = -0.8742 v_{A\text{rel}}\mathbf{i} - 0.4856 v_{A\text{rel}}\mathbf{j} + 2.5\omega_{AC}\mathbf{i} - 4.5\omega_{AC}\mathbf{j} \text{ (ft/s)}.$$

Equate the two expressions and separate components:

$$-1.879 = -0.8742 v_{A\text{rel}} + 2.5\omega_{AC},$$

$$0.6842 = -0.4856 v_{B\text{rel}} - 4.5\omega_{AC}.$$

Solve: $v_{A\text{rel}} = 1.311$ ft/s, $\omega_{AC} = -0.293$ rad/s (clockwise).

The accelerations: The acceleration of block A is given to be zero. In terms of the bar AC, the acceleration of A is

$$\mathbf{a}_A = 0 = a_{A\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{AC} \times v_{A\text{rel}}\mathbf{e} + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{A/B} - \omega_{AC}^2 \mathbf{r}_{A/B}.$$

$$0 = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -0.8742 & -0.4856 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix} - \omega_{AC}^2(-4.5\mathbf{i} - 2.5\mathbf{j}).$$

$$0 = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}}(-e_y\mathbf{i} + e_x\mathbf{j}) + \alpha_{AC}(2.5\mathbf{i} - 4.5\mathbf{j})$$

$$- \omega_{AC}^2(-4.5\mathbf{i} - 2.5\mathbf{j}).$$

Separate components to obtain:

$$0 = -0.8742 a_{A\text{rel}} - 0.3736 + 2.5\alpha_{AC} + 0.3875,$$

$$0 = -0.4856 a_{A\text{rel}} + 0.6742 - 4.5\alpha_{AC} + 0.2153.$$

Solve: $a_{A\text{rel}} = 0.4433$ (ft/s^2) (toward A).

$\boxed{\alpha_{AC} = 0.1494 \text{ rad/s}^2}$ (counterclockwise).

The acceleration of point C is

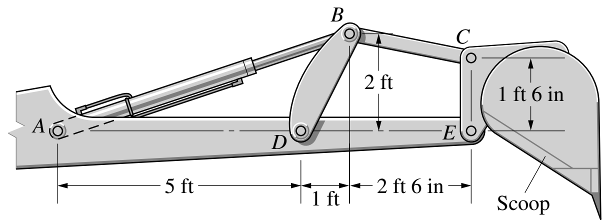
$$\mathbf{a}_C = a_{A\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{AC} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/B} - \omega_{AC}^2 \mathbf{r}_{C/B}$$

$$\mathbf{a}_C = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ e_x & e_y & 0 \end{bmatrix}$$

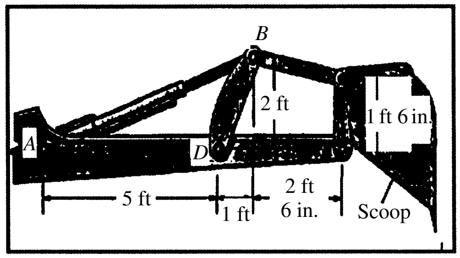
$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2.5 & 3.89 - 2.5 & 0 \end{bmatrix} - \omega_{AC}^2(2.5\mathbf{i} + (3.89 - 2.5)\mathbf{j}).$$

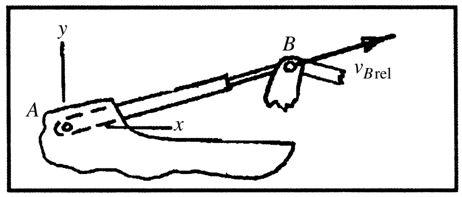
Substitute numerical values: $\boxed{\mathbf{a}_C = -1.184\mathbf{i} + 0.711\mathbf{j} \text{ (ft/s}^2)}$

Problem 17.134 The angular velocity of the scoop is 1.0 rad/s clockwise. Determine the rate at which the hydraulic actuator AB is extending.



Solution: The point B slides in the arm AB. The velocity of point C is





$$\mathbf{v}_C = \boldsymbol{\omega}_{\text{scoop}} \times (1.5\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 15 & 0 \end{bmatrix} = 1.5\mathbf{i} \text{ (ft/s)}.$$

Point B is constrained to move normally to the arm DB: The unit vector parallel to DB is

$$\mathbf{e}_{DB} = \frac{1\mathbf{i} + 2\mathbf{j}}{\sqrt{1^2 + 2^2}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The unit vector normal to $\mathbf{e}_{DB}$ is $\mathbf{e}_{NDB} = 0.8944\mathbf{i} - 0.4472\mathbf{j}$, from which the velocity of C in terms of BC is

$$\mathbf{v}_C = v_B \mathbf{e}_{NBD} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= v_B(0.8944\mathbf{i} - 0.4472\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2.5 & -0.5 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = v_B(0.8944\mathbf{i} - 0.4472\mathbf{j}) + \omega_{BC}(0.5\mathbf{i} + 2.5\mathbf{j}).$$

Equate terms in $\mathbf{v}_C$, $1.5 = 0.8944 v_B + 0.5\omega_{BC}$, $O = -0.4472 v_B + 2.5\omega_{BC}$. Solve: $\omega_{BC} = 0.2727$ rad/s, $v_B = 1.525$ ft/s, from which $\mathbf{v}_B = v_B \mathbf{e}_{NDB} = 1.364\mathbf{i} - 0.6818\mathbf{j}$ (ft/s).

The unit vector parallel to the arm AB is

$$\mathbf{e}_{AB} = \frac{6\mathbf{i} + 2\mathbf{j}}{\sqrt{6^2 + 2^2}} = 0.9487\mathbf{i} + 0.3162\mathbf{j}.$$

Choose a coordinate system with origin at A rotating with arm AB. The velocity of point B is

$$\mathbf{v}_B = v_{B\text{rel}} \mathbf{e}_{AB} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= v_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 6 & 2 & 0 \end{bmatrix}.$$

$$\mathbf{v}_B = v_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + \omega_{AB}(-2\mathbf{i} + 6\mathbf{j}).$$

Equate the expressions and separate components:

$$1.364 = 0.9487 v_{B\text{rel}} - 2\omega_{AB},$$

$$-0.6818 = 0.3162 v_{B\text{rel}} + 6\omega_{AB}.$$

Solve: $\omega_{AB} = -0.1704$ rad/s, $\boxed{v_{B\text{rel}} = 1.078 \text{ ft/s}}$ which is the rate of extension of the actuator.

Problem 17.135 The angular acceleration of the scoop in Problem 17.134 is zero. Determine the rate of change of the rate at which the hydraulic actuator AB is extending.

Solution: Choose a coordinate system with the origin at D and the x axis parallel to ADE. The vector locations of points A, B, C, and E are $\mathbf{r}_A = -5\mathbf{i}$ ft, $\mathbf{r}_B = 1\mathbf{i} + 2\mathbf{j}$ ft, $\mathbf{r}_C = 3.5\mathbf{i} + 1.5\mathbf{j}$ ft, $\mathbf{r}_E = 3.5\mathbf{i}$ ft. The vector AB is

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 6\mathbf{i} + 2\mathbf{j} \text{ (ft)},$$

$$\mathbf{r}_{B/D} = \mathbf{r}_B - \mathbf{r}_D = 1\mathbf{i} + 2\mathbf{j} \text{ (ft)}.$$

Assume that the scoop rotates at 1 rad/s about point E. The acceleration of point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{\text{Scoop}} \times 15\mathbf{j} - \omega_{\text{scoop}}^2(1.5\mathbf{j}) = -1.5\mathbf{j} \text{ (ft/s}^2),$$

since $\alpha_{\text{scoop}} = 0$. The vector from C to B is $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -2.5\mathbf{i} + 0.5\mathbf{j}$ (ft). The acceleration of point B in terms of point C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$$

$$= 1.5\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -2.5 & +0.5 & 0 \end{bmatrix} - \omega_{BC}^2(-2.5\mathbf{i} + 0.5\mathbf{j}),$$

from which

$$\mathbf{a}_B = -(0.5\alpha_{BC} - 2.5\omega_{BC}^2)\mathbf{i} - (1.5 + 2.5\alpha_{BC} + 0.5\omega_{BC}^2)\mathbf{j}.$$

The acceleration of B in terms of D is

$$\mathbf{a}_B = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2 \mathbf{r}_{B/D}$$

$$= \mathbf{a}_D + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 1 & 2 & 0 \end{bmatrix} - \omega_{BD}^2(\mathbf{i} + 2\mathbf{j}).$$

The acceleration of point D is zero, from which $\mathbf{a}_B = -(2\alpha_{BD} + \omega_{BD}^2)\mathbf{i} + (\alpha_{BD} - 2\omega_{BD}^2)\mathbf{j}$. Equate like terms in the two expressions for $\mathbf{a}_B$, $-(0.5\alpha_{BC} - 2.5\omega_{BC}^2) = -(2\alpha_{BD} + \omega_{BD}^2)$, $-(1.5 + 2.5\alpha_{BC} + 0.5\omega_{BC}^2) = (\alpha_{BD} - 2\omega_{BD}^2)$. From the solution to Problem 17.134, $\omega_{BC} = 0.2727$ rad/s, and $v_B = 1.525$ ft/s. The velocity of point B is normal to the link BD, from which

$$\omega_{BD} = \frac{v_B}{\sqrt{1^2 + 2^2}} = 0.6818 \text{ rad/s}.$$

Substitute and solve for the angular accelerations: $\alpha_{BC} = -0.1026$ rad/s^2, $\alpha_{BD} = -0.3511$ rad/s^2. From which the acceleration of point B is

$$\mathbf{a}_B = -(2\alpha_{BD} + \omega_{BD}^2)\mathbf{i} + (\alpha_{BD} - 2\omega_{BD}^2)\mathbf{j}$$

$$= 0.2372\mathbf{i} - 1.281\mathbf{j} \text{ (ft/s}^2).$$

The acceleration of point B in terms of the arm AB is

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{e}_{B/A} + 2\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{B/A} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}.$$

From the solution to Problem 17.134: $\mathbf{e}_{B/A} = 0.9487\mathbf{i} + 0.3162\mathbf{j}$, $v_{B\text{rel}}] = 1.078$ ft/s, $\omega_{AB} = -0.1705$ rad/s. From which

$$\mathbf{a}_B = a_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + 0.1162\mathbf{i} - 0.3487\mathbf{j}$$

$$+ \alpha_{AB}(-2\mathbf{i} + 6\mathbf{j}) - 0.1743\mathbf{i} - 0.0581\mathbf{j}.$$

Equate the accelerations of point B and separate components:

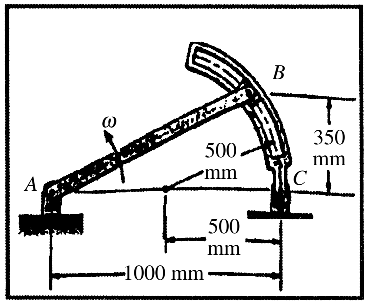
$$0.2371 = 0.9487a_{B\text{rel}} - 2\alpha_{AB} - 0.0581,$$

$$-1.281 = 0.3162a_{B\text{rel}} + 6\alpha_{AB} - 0.4068.$$

Solve: $\boxed{a_{B\text{rel}} = 0.0038 \text{ ft/s}^2}$, which is the rate of change of the rate at which the actuator is extending.

Problem 17.136 Suppose that the curved bar in Example 17.9 rotates with a counterclockwise angular velocity of 2 rad/s.

(a) What is the angular velocity of bar AB?
(b) What is the velocity of block B relative to the slot?



Solution: The angle defining the position of B in the circular slot is

$$\beta = \sin^{-1}\left(\frac{350}{500}\right) = 44.4^\circ.$$

The vectors are

$$\mathbf{r}_{B/A} = (500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} = 857\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{B/C} = (-500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

The unit vector tangent to the slot at B is given by

$$\mathbf{e}_B = -\sin\beta\mathbf{i} + \cos\beta\mathbf{j} = -0.7\mathbf{i} + 0.714\mathbf{j}.$$

The velocity of B in terms of AB is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 857 & 350 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-350\mathbf{i} + 857\mathbf{j}) \text{ (mm/s)}.$$

The velocity of B in terms of BC is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -142.9 & 350 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + (-700\mathbf{i} - 285.8\mathbf{j}) \text{ (mm/s)}$$

Equate the expressions for the velocity of B and separate components:

$$-350\omega_{AB} = -0.7v_{B\text{rel}} - 700,$$

$$-857\omega_{AB} = 0.714v_{B\text{rel}} - 285.8.$$

Solve:

(a) $\boxed{\omega_{AB} = -2 \text{ rad/s}}$ (clockwise).

(b) $\boxed{v_{B\text{rel}} = -2000 \text{ mm/s}}$ (toward C).

Problem 17.137 Suppose that the curved bar in Example 17.9 has a clockwise angular velocity of 4 rad/s and a counterclockwise angular acceleration of 10 rad/s^2. What is the angular acceleration of bar AB?

Solution: Use the solution to Problem 17.118 with new data.

Get the velocities: The angle defining the position of B in the circular slot is

$$\beta = \sin^{-1}\left(\frac{350}{500}\right) = 44.4^\circ.$$

The vectors

$$\mathbf{r}_{B/A} = (500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} = 857\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{B/C} = (-500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

The unit vector tangent to the slot at B

$$\mathbf{e}_B = -\sin\beta\mathbf{i} + \cos\beta\mathbf{j} = -0.7\mathbf{i} + 0.714\mathbf{j}.$$

The component normal to the slot at B is

$$\mathbf{e}_{NB} = \cos\beta\mathbf{i} + \sin\beta\mathbf{j} = 0.7141\mathbf{i} + 0.7\mathbf{j}.$$

The velocity of B in terms of AB

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 857 & 350 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-350\mathbf{i} + 857\mathbf{j}) \text{ (mm/s)}.$$

The velocity of B in terms of BC is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ -142.9 & 350 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + (1400\mathbf{i} + 571.6\mathbf{j}) \text{ (mm/s)}.$$

Equate the expressions for the velocity of B and separate components: $-350\omega_{AB} = -0.7v_{B\text{rel}} + 1400$, $857\omega_{AB} = 0.714v_{B\text{rel}} + 571.6$. Solve: $\omega_{AB} = 4$ rad/s (counterclockwise). $v_{B\text{rel}} = 4000$ mm/s (away from C).

Get the accelerations: The acceleration of point B in terms of the AB is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 857 & 350 & 0 \end{bmatrix} - \omega_{AB}^2(857\mathbf{i} + 350\mathbf{j}),$$

$$\mathbf{a}_B = \alpha_{AB}(-350\mathbf{i} + 857\mathbf{j}) - 138713\mathbf{i} - 5665\mathbf{j} (\text{mm/s}^2).$$

The acceleration in terms of the arm BC is

$$\mathbf{a}_B = \mathbf{a}_{B\text{rel}} + 2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}.$$

Expanding term by term:

$$\mathbf{a}_{B\text{rel}} = a_{B\text{rel}}\mathbf{e}_B - \left(\frac{v_{B\text{rel}}^2}{500}\right)\mathbf{e}_{NB}$$

$$= a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 22{,}852.6\mathbf{i} - 22{,}400\mathbf{j}.$$

Other terms:

$$2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}}\mathbf{e}_B = 22852\mathbf{i} + 22{,}400\mathbf{j},$$

$$\boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} = -3500\mathbf{i} - 1429.3\mathbf{j},$$

$$-\omega_{BC}^2\mathbf{r}_{B/C} = 2286.8\mathbf{i} - 5600\mathbf{j}.$$

Collect terms:

$$\mathbf{a}_B = a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 22852.6\mathbf{i} - 22400\mathbf{j} + 22852.6\mathbf{i}$$

$$+ 22400\mathbf{j} - 3500\mathbf{i} - 1429.3\mathbf{j} + 2286.9\mathbf{i} - 5600\mathbf{j}$$

$$\mathbf{a}_B = a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 1213\mathbf{i} - 7029.3\mathbf{j}.$$

Equate the two expressions for the acceleration of B to obtain the two equations:

$$-350\alpha_{AB} - 13{,}871 = -0.7a_{B\text{rel}} - 1213.1,$$

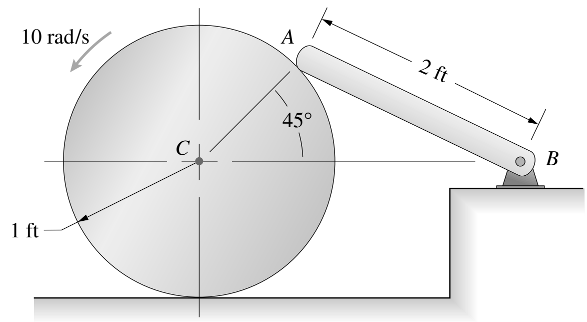
$$857\alpha_{AB} - 5665 = 0.7141a_{B\text{rel}} - 7029.3.$$

Solve:

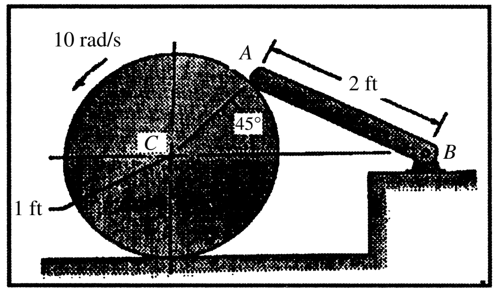
$$a_{B\text{rel}} = 29180 \text{ (mm/s}^2),$$

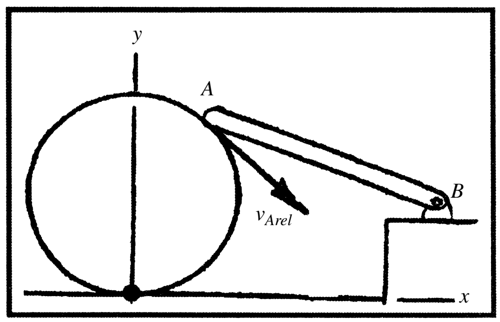
$\boxed{\alpha_{AB} = 22.65 \text{ rad/s}^2}$ (counterclockwise).

Problem 17.138 The disk rolls on the plane surface with a counterclockwise angular velocity of 10 rad/s. Bar AB slides on the surface of the disk at A. Determine the angular velocity of bar AB.



Solution: Choose a coordinate system with the origin at the point of contact between the disk and the plane surface, with the x axis parallel to the plane surface. Let A be the point of the bar in contact with the disk. The vector location of point A on the disk is





$$\mathbf{r}_A = \mathbf{i}\cos 45^\circ + \mathbf{j}(1 + \sin 45^\circ) = 0.707\mathbf{i} + 1.707\mathbf{j} \text{ (ft)}.$$

The unit vector parallel to the radius of the disk is

$$\mathbf{e}_A = \cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j} = 0.707\mathbf{i} + 0.707\mathbf{j}.$$

The unit vector tangent to the surface of the disk at A is

$$\mathbf{e}_{NA} = \mathbf{i}\sin 45^\circ - \mathbf{j}\cos 45^\circ = 0.707\mathbf{i} - 0.707\mathbf{j}.$$

The angle formed by the bar AB with the horizontal is

$$\beta = \sin^{-1}\left(\frac{\sin 45^\circ}{2}\right) = 20.7^\circ.$$

The velocity of point A in terms of the motion of bar AB is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2\cos\beta & 2\sin\beta & 0 \end{bmatrix}.$$

$$\mathbf{v}_A = \omega_{AB}(-0.707\mathbf{i} - 1.871\mathbf{j}) \text{ (ft/s)}.$$

The velocity of point A in terms of the point of the disk in contact with the plane surface is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e}_{NA} + \boldsymbol{\omega}_{\text{disk}} \times \mathbf{r}_A$$

$$= v_{A\text{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{disk}} \\ 0.707 & 1.707 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = v_{A\text{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + (-17.07\mathbf{i} + 7.07\mathbf{j}).$$

Equate the expressions and separate components:

$$-0.707\omega_{AB} = 0.707v_{A\text{rel}} - 17.07, \; -1.871\omega_{AB}$$

$$= -0.707v_{A\text{rel}} + 7.07.$$

Solve:

$v_{A\text{rel}} = 20.3$ ft/s,

$\boxed{\omega_{AB} = 3.88 \text{ rad/s}}$ (counterclockwise).

Problem 17.139 In Problem 17.138, the disk rolls on the plane surface with a constant counterclockwise angular velocity of 10 rad/s. Determine the angular acceleration of bar AB.

Solution: Use the results of the solution to Problem 17.138. Choose a coordinate system with the origin at the point of contact between the disk and the plane surface, with the x axis parallel to the plane surface. The vector location of point A on the disk is

$$\mathbf{r}_A = \mathbf{i}\cos 45^\circ + \mathbf{j}(1 + \sin 45^\circ) = 0.707\mathbf{i} + 1.707\mathbf{j} \text{ (ft)}.$$

The unit vector tangent to the surface of the disk at A is

$$\mathbf{e}_{NA} = \mathbf{i}\sin 45^\circ - \mathbf{j}\cos 45^\circ = 0.707\mathbf{i} - 0.707\mathbf{j}.$$

The angle formed by the bar AB with the horizontal is

$$\beta = \sin^{-1}(\sin 45^\circ/2) = 20.7^\circ.$$

Get the velocities: The velocity of point A in terms of the motion of bar AB is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2\cos\beta & 2\sin\beta & 0 \end{bmatrix}$$

$$= \omega_{AB}(-0.707\mathbf{i} - 1.871\mathbf{j}) \text{ (ft/s)}.$$

The acceleration of the center of the disk is zero. The velocity of point A in terms of the center of the disk is

$$\mathbf{v}_A = v_{A\mathrm{rel}}\mathbf{e}_{NA} + \boldsymbol{\omega}_{\mathrm{disk}} \times \mathbf{r}_A$$

$$= v_{A\mathrm{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\mathrm{disk}} \\ 0.707 & 1.707 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = v_{A\mathrm{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + (-17.07\mathbf{i} + 7.07\mathbf{j}).$$

Equate the expressions and separate components:

$$-0.707\omega_{AB} = 0.707v_{A\mathrm{rel}} - 17.07, \quad -1.871\omega_{AB} = -0.707v_{A\mathrm{rel}} + 7.07.$$

Solve:

$$v_{A\mathrm{rel}} = 20.3 \text{ ft/s},$$

$$\omega_{AB} = 3.88 \text{ rad/s} \quad \text{(counterclockwise)}.$$

Get the accelerations: The acceleration of point A in terms of the arm AB is

$$\mathbf{a}_A = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -1.87 & 0.707 & 0 \end{bmatrix} + 28.15\mathbf{i} - 10.64\mathbf{j} \text{ (ft/s}^2\text{)},$$

$$\mathbf{a}_A = \alpha_{AB}(-0.707\mathbf{i} - 1.87\mathbf{j}) + 28.15\mathbf{i} - 10.64\mathbf{j} \text{ (ft/s}^2\text{)}.$$

The acceleration of point A in terms of the disk is

$$\mathbf{a}_A = \mathbf{a}_{A\mathrm{rel}} + 2\boldsymbol{\omega}_{\mathrm{disk}} \times v_{A\mathrm{rel}}\mathbf{e}_{NA} + \boldsymbol{\alpha}_{\mathrm{disk}} \times \mathbf{r}_{A/C} - \omega_{\mathrm{disk}}^2\mathbf{r}_{A/C}.$$

Expanding term by term: The acceleration $\mathbf{a}_{A\mathrm{rel}}$ is composed of a tangential component and a radial component:

$$\mathbf{a}_{A\mathrm{rel}} = a_{A\mathrm{rel}}\mathbf{e}_{NA} - \left(\frac{v_{A\mathrm{rel}}^2}{1}\right)\mathbf{e}_A$$

$$= a_{A\mathrm{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) - 290.3\mathbf{i} - 290.3\mathbf{j}.$$

$$2\boldsymbol{\omega}_{\mathrm{disk}} \times v_{A\mathrm{rel}}\mathbf{e}_{NB} = 286.6\mathbf{i} + 286.6\mathbf{j}, \; \boldsymbol{\alpha}_{\mathrm{disk}} \times \mathbf{r}_A = 0,$$

since the acceleration of the disk is zero.

$$-\omega_{\mathrm{disk}}^2\mathbf{r}_{A/C} = -70.7\mathbf{i} - 70.7\mathbf{j}.$$

Collect terms and separate components to obtain:

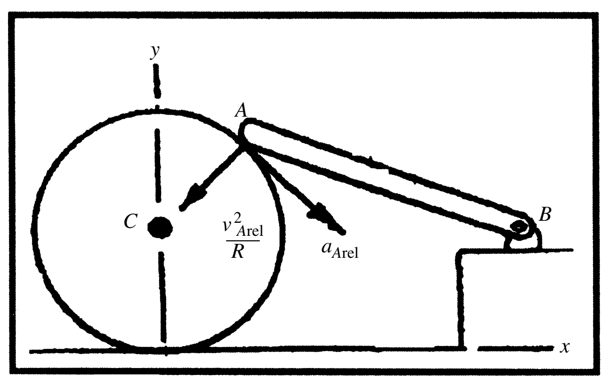
$$-0.707\alpha_{AB} + 28.15 = 0.707a_{A\mathrm{rel}} - 290.3 + 286.6 - 70.7,$$

$$-1.87\alpha_{AB} - 10.64 = -0.707a_{A\mathrm{rel}} - 290.3 + 286.6 - 70.7.$$

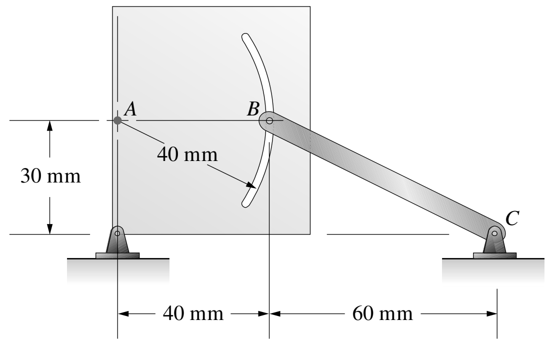
Solve:

$$a_{A\mathrm{rel}} = 80.6 \text{ ft/s}^2,$$

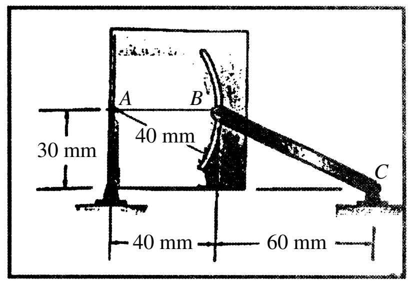
$$\boxed{\alpha_{AB} = 64.6 \text{ rad/s}^2} \quad \text{(counterclockwise)}.$$



Problem 17.140 Bar BC rotates with a counterclockwise angular velocity of 2 rad/s. A pin at B slides in a circular slot in the rectangular plate. Determine the angular velocity of the plate and the velocity at which the pin slides relative to the circular slot.



Solution: Choose a coordinate system with the origin O at the lower left pin and the x axis parallel to the plane surface. The unit vector parallel to AB is $\mathbf{e}_{AB} = \mathbf{i}$. The unit vector tangent to the slot at B is $\mathbf{e}_{NAB} = \mathbf{j}$. The velocity of the pin in terms of the motion of BC is $\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$.



$$\mathbf{v}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -60 & 30 & 0 \end{bmatrix} = 2(-30\mathbf{i} - 60\mathbf{j}) = -60\mathbf{i} - 120\mathbf{j} \text{ (mm/s)}.$$

The velocity of the pin in terms of the plate is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{j} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= v_{B\text{rel}}\mathbf{j} + \omega_{AB}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s)}.$$

Equate the expressions and separate components to obtain

$$-60 = -30\omega_{AB},$$

$$-120 = v_{B\text{rel}} + 40\omega_{AB}.$$

Solve:

$$\mathbf{v}_{B\text{rel}} = -200\mathbf{j} \text{ mm/s},$$

$\boxed{\omega_{AB} = 2 \text{ rad/s}}$ (counterclockwise).

Problem 17.141 Bar BC in Problem 17.140 rotates with a constant counterclockwise angular velocity of 2 rad/s. Determine the angular acceleration of the plate.

Solution: Choose the same coordinate system as in Problem 17.140. *Get the velocities*: The unit vector parallel to AB is $\mathbf{e}_{AB} = \mathbf{i}$. The unit vector tangent to the slot at B is $\mathbf{e}_{NAB} = \mathbf{j}$. The velocity of the pin in terms of the motion of BC is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -60 & 30 & 0 \end{bmatrix}$$

$$= (-60\mathbf{i} - 120\mathbf{j}) \text{ (mm/s)}.$$

The velocity of the pin in terms of the plate is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= v_{B\text{rel}}\mathbf{e}_{NAB} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/O}$$

$$= v_{B\text{rel}}\mathbf{j} + \omega_{AB}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s)}.$$

Equate the expressions and separate components to obtain

$$-60 = -30\omega_{AB},$$

$$-120 = v_{B\text{rel}} + 40\omega_{AB}.$$

Solve:

$$\mathbf{v}_{B\text{rel}} = -200\mathbf{j} \text{ mm/s},$$

$$\omega_{AB} = 2 \text{ rad/s} \quad \text{(counterclockwise)}.$$

Get the accelerations: The acceleration of the pin in terms of the arm BC is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}$$

$$= 0 - 4(-60\mathbf{i} + 30\mathbf{j})$$

$$= 240\mathbf{i} - 120\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of the pin in terms of the plate AB is

$$\mathbf{a}_B = \mathbf{a}_{B\text{rel}} + 2\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{NAB} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/O} - \omega_{AB}^2\mathbf{r}_{B/O}.$$

Expand term by term:

$$\mathbf{a}_{B\text{rel}} = a_{B\text{rel}}\mathbf{e}_{NAB} - \left(\frac{v_{B\text{rel}}^2}{40}\right)\mathbf{e}_{AB}$$

$$= a_{B\text{rel}}\mathbf{j} - 1000\mathbf{i} \text{ (mm/s}^2\text{)},$$

$$2\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{NAB} = 800\mathbf{i} \text{ (mm/s}^2\text{)}.$$

$$\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= \alpha_{BA}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s}^2\text{)},$$

$$-\omega_{AB}^2(40\mathbf{i} + 30\mathbf{j}) = -160\mathbf{i} - 120\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Collect terms and separate components to obtain:

$$240 = -1000 + 800 - 30\alpha_{BA} - 160,$$

$$-120 = a_{B\text{rel}} + 40\alpha_{BA} - 120.$$

Solve:

$$a_{B\text{rel}} = 800 \text{ mm/s}^2 \quad \text{(upward)},$$

$$\boxed{\alpha_{AB} = -20 \text{ rad/s}^2}, \text{ (clockwise)}.$$

Problem 17.142 By taking the derivative of Eq. (17.11) with respect to time and using Eq. (17.12), derive Eq. (17.13).

Solution: Eq (17.11) is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}.$$

Eq (17.12) is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \mathbf{k}\left(\frac{dz}{dt}\right)\mathbf{k}.$$

Assume that the coordinate system is body fixed and that B is a point on the rigid body, (A is not necessarily a point on the rigid body), such that $\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$, where $\mathbf{r}_{A/B} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and x, y, z are the coordinates of A in body fixed coordinates. Take the derivative of both sides of Eq (17.11):

$$\frac{d\mathbf{v}_A}{dt} = \frac{d\mathbf{v}_B}{dt} + \frac{d\mathbf{v}_{A\text{rel}}}{dt} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times \frac{d\mathbf{r}_{A/B}}{dt}.$$

By definition,

$$\frac{d\mathbf{v}_A}{dt} = \mathbf{a}_A,\ \frac{d\mathbf{v}_B}{dt} = \mathbf{a}_B, \quad \text{and}\ \frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{\alpha}.$$

The derivative:

$$\frac{d\mathbf{v}_{A\text{rel}}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k} + \frac{dx}{dt}\frac{d\mathbf{i}}{dt} + \frac{dy}{dt}\frac{d\mathbf{j}}{dt} + \frac{dz}{dt}\frac{d\mathbf{k}}{dt}.$$

Using the fact that the derivative of a unit vector represents a rotation of the unit vector,

$$\frac{d\mathbf{i}}{dt} = \boldsymbol{\omega} \times \mathbf{i},\ \frac{d\mathbf{j}}{dt} = \boldsymbol{\omega} \times \mathbf{j},\ \frac{d\mathbf{k}}{dt} = \boldsymbol{\omega} \times \mathbf{k}.$$

Substitute into the derivative:

$$\frac{d\mathbf{v}_{A\text{rel}}}{dt} = \mathbf{a}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}}.$$

Noting

$$\boldsymbol{\omega} \times \frac{d\mathbf{r}_{A/B}}{dt} = \boldsymbol{\omega} \times \left(\frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}\right)$$

$$+ \boldsymbol{\omega} \times \left(x\frac{d\mathbf{i}}{dt} + y\frac{d\mathbf{j}}{dt} + z\frac{d\mathbf{k}}{dt}\right)$$

$$= \boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

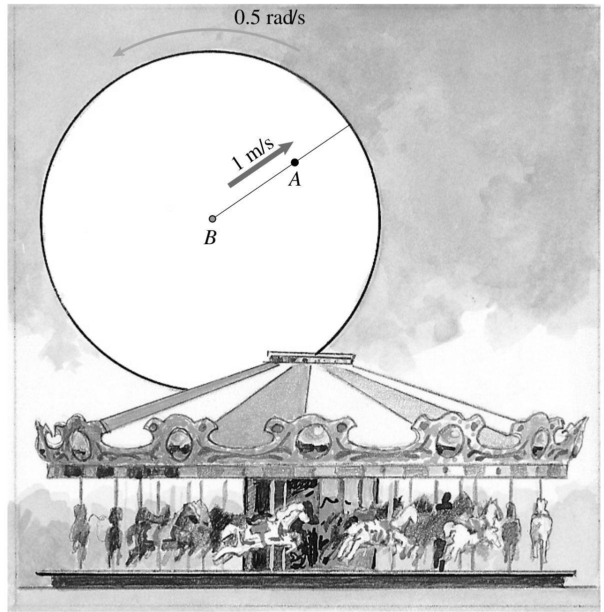
Collect and combine terms: the derivative of Eq (17.11) is

$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})},$$

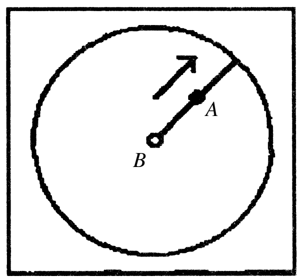
which is Eq (17.13).

Problem 17.143 A merry-go-round rotates at a constant angular velocity of 0.5 rad/s. Person A walks at a constant speed of 1 m/s along a radial line. Determine A's velocity and acceleration *relative to the earth* when she is 2 m from the center of the merry-go-round, using two methods:

(a) Express the velocity and acceleration in terms of polar coordinates.
(b) Use Eqs. (17.21) and (17.22) to express the velocity and acceleration in terms of a body-fixed coordinate system with its x axis aligned with the line along which A walks and its z axis perpendicular to the merry-go-round.



Solution:



(a) The velocity in polar coordinates is

$$\boxed{\mathbf{v}_A = 1\mathbf{e}_r + 0.5(2)\mathbf{e}_\theta = \mathbf{e}_r + \mathbf{e}_\theta \text{ (m/s)}}$$

The acceleration is

$$\mathbf{a}_A = \left(\left(\frac{d^2 v_A}{dt^2}\right) - r\left(\frac{d\theta}{dt}\right)^2\right)\mathbf{e}_r$$

$$+ \left(r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\right)\mathbf{e}_\theta.$$

Substitute noting

$$\frac{d\theta}{dt} = \omega = 0.5 \text{ rad/s},$$

$$\mathbf{a}_A = -r\omega^2\mathbf{e}_r + 2\omega v_A\mathbf{e}_\theta = -0.5\mathbf{e}_r + \mathbf{e}_\theta \text{ (m/s}^2).$$

(b) Eq. (17.19) is $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$. The center, point B, is stationary relative to the earth, and the relative velocity $v_{A\text{rel}} = 1\mathbf{i}$ (m/s). The vector $\mathbf{r}_{A/B} = 2\mathbf{i}$ (m). Substitute:

$$\boxed{\mathbf{v}_A = \mathbf{i} + \omega(\mathbf{k} \times 2\mathbf{i}) = \mathbf{i} + \mathbf{j} \text{ (m/s)}}$$

Eq (17.20) is

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

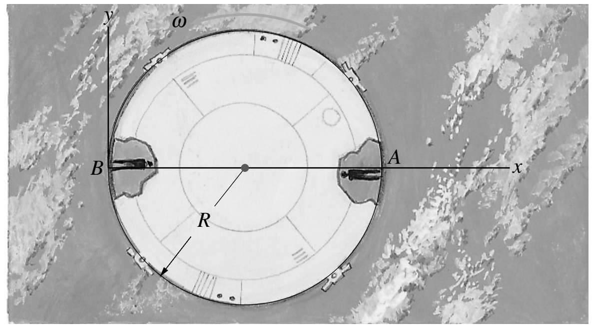
The point B is stationary, $\mathbf{a}_{A\text{rel}} = 0$, and $\boldsymbol{\alpha} = 0$. Substitute:

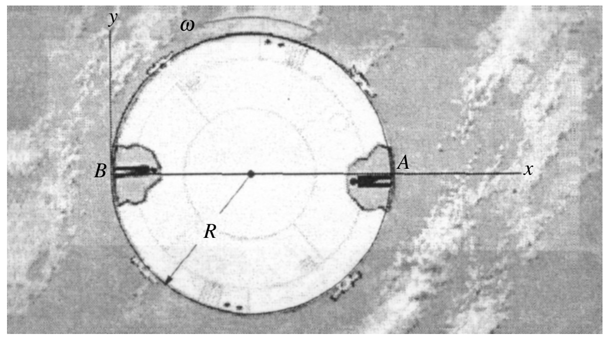
$$\mathbf{a}_A = 0 + 0 + 2\omega(\mathbf{k} \times \mathbf{i}) + 0 + \omega^2(\mathbf{k} \times (\mathbf{k} \times 2\mathbf{i}))$$

$$= 2\omega\mathbf{j} - 2\omega^2\mathbf{i} \quad \boxed{= -0.5\mathbf{i} + \mathbf{j} \text{ (m/s}^2)}$$

Problem 17.144 A disk-shaped space station of radius R rotates with constant angular velocity ω about the axis perpendicular to the page. Two persons A and B are stationary relative to the station. The coordinate system shown has its origin at B's location and is fixed with respect to the station.

(a) What are A's velocity and acceleration relative to the station-fixed coordinate system?
(b) What are A's velocity and acceleration relative to a nonrotating reference frame with its origin fixed at B's location?





Solution: $\mathbf{r}_{A/B} = ZR\mathbf{i}$

(a) A is not moving in station fixed coordinates.

$\therefore \mathbf{v}_{A\text{rel}} = 0$, $\mathbf{a}_{A\text{rel}} = 0$.

(b) $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \omega \times \mathbf{r}_{A/B}$ (Non-Rotating coordinates)

$\mathbf{v}_B = 0, \mathbf{v}_{A\text{rel}} = 0$

$\mathbf{v}_A = \omega\mathbf{k} \times (2R\mathbf{i})$

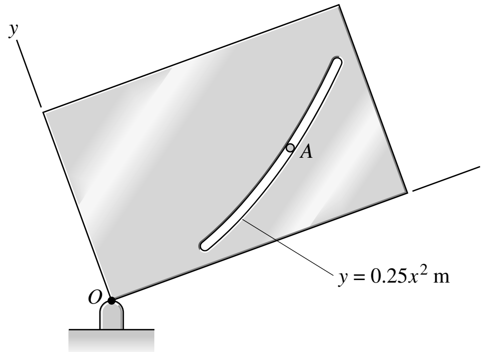
$\underline{\mathbf{v}_A = 2R\omega\mathbf{j}}$

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B\text{rel}} + \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$

$\mathbf{a}_B = 0, \mathbf{a}_{AB\text{rel}} = 0, \alpha = 0$

$\underline{\mathbf{a}_A = -2R\omega^2\mathbf{i}}$

Problem 17.145 The metal plate is attached to a fixed ball-and-socket support at O. The pin A slides in a slot in the plate. At the instant shown, $x_A = 1$ m, $dx_A/dt = 2$ m/s, and $d^2x_A/dt^2 = 0$, and the plate's angular velocity and angular acceleration are $\boldsymbol{\omega} = 2\mathbf{k}$ (rad/s) and $\boldsymbol{\alpha} = 0$. What are the x, y, and z components of the velocity and acceleration of A relative to a nonrotating reference frame with its origin at O?

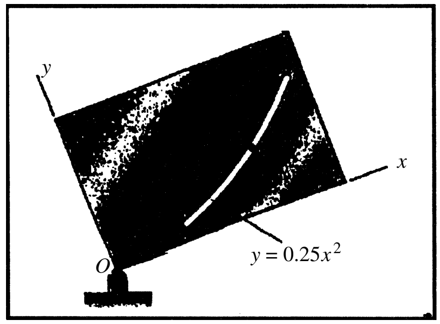


Solution: The velocity is $\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$. The relative velocity is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \left(\frac{dz}{dt}\right)\mathbf{k},$$

where $\dfrac{dx}{dt} = 2$ m/s, $\dfrac{dy}{dt} = \dfrac{d}{dt}0.25x^2 = 0.5x\dfrac{dx}{dt} = 1$ m/s, $\dfrac{dz}{dt} = 0$, and $\mathbf{r}_{A/O} = x\mathbf{i} + y\mathbf{j} + z\mathbf{j} = \mathbf{i} + 0.25\mathbf{j} + 0$, from which

$$\boxed{\begin{aligned}\mathbf{v}_A &= 2\mathbf{i} + \mathbf{j} + \omega(\mathbf{k} \times (\mathbf{i} + 0.25\mathbf{j})) = 2\mathbf{i} + \mathbf{j} + 2(-0.25\mathbf{i} + \mathbf{j})\\ &= 1.5\mathbf{i} + 3\mathbf{j}\ \text{(m/s)}.\end{aligned}}$$



The acceleration is

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/O}).$$

Noting

$$\mathbf{a}_{A\text{rel}} = \left(\frac{d^2x}{dt^2}\right)\mathbf{i} + \left(\frac{d^2y}{dt^2}\right)\mathbf{j} + \left(\frac{d^2z}{dt^2}\right)\mathbf{k},$$

where

$$\left(\frac{d^2x}{dt^2}\right) = 0,\ \frac{d^2y}{dt^2} = \frac{d^2}{dt^2}0.25x^2 = 0.5\left(\frac{dx}{dt}\right)^2 = 2,\ \left(\frac{d^2z}{dt^2}\right) = 0.$$

Substitute:

$$\mathbf{a}_A = 2\mathbf{j} + 2\omega(\mathbf{k} \times (2\mathbf{i} + \mathbf{j})) + \omega^2(\mathbf{k} \times (\mathbf{k} \times (\mathbf{i} + 0.25\mathbf{j})))$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 2 & 1 & 0 \end{bmatrix} + 4\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0.25 & 0 \end{bmatrix}$$

$$\mathbf{a}_A = 2\mathbf{j} - 4\mathbf{i} + 4\omega\mathbf{j} + 4\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -0.25 & 1 & 0 \end{bmatrix} \quad \boxed{= -8\mathbf{i} + 9\mathbf{j}\ (\text{m/s}^2)}$$

Problem 17.146 Suppose that at the instant shown in Problem 17.145, $x_A = 1$ m, $dx_A/dt = -3$ m/s, $d^2x_A/dt^2 = 4$ m/s^2, and the plate's angular velocity and angular acceleration are $\boldsymbol{\omega} = -4\mathbf{j} + 2\mathbf{k}$ (rad/s), and $\boldsymbol{\alpha} = 3\mathbf{i} - 6\mathbf{j}$ (rad/s^2). What are the x, y, z components of the velocity and acceleration of A relative to a non rotating reference frame that is stationary with respect to O?

Solution: The velocity is $\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$. The relative velocity is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \left(\frac{dz}{dt}\right)\mathbf{k},$$

where $\dfrac{dx}{dt} = -3$ m/s, $\dfrac{dy}{dt} = \dfrac{d}{dt}0.25x^2 = 0.5x\dfrac{dx}{dt} = -15$ m/s, $\dfrac{dz}{dt} = 0$, and $\mathbf{r}_{A/O} = x\mathbf{i} + y\mathbf{j} + z\mathbf{j} = \mathbf{i} + 0.25\mathbf{j} + 0$, from which $\mathbf{v}_A = -3\mathbf{i} - 1.5\mathbf{j} + \boldsymbol{\omega} \times (\mathbf{i} + 0.25\mathbf{j})$.

$$\mathbf{v}_A = -3\mathbf{i} - 1.5\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ 1 & 0.25 & 0 \end{bmatrix} = -3\mathbf{i} - 1.5\mathbf{j} - 0.5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$$

$$\boxed{= -3.5\mathbf{i} + 0.5\mathbf{j} + 4\mathbf{k} \text{ (m/s)}}$$

The acceleration is $\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/O})$. Noting

$$\mathbf{a}_{A\text{rel}} = \left(\frac{d^2x}{dt^2}\right)\mathbf{i} + \left(\frac{d^2y}{dt^2}\right)\mathbf{j} + \left(\frac{d^2z}{dt^2}\right)\mathbf{k},$$

where

$$\left(\frac{d^2x}{dt^2}\right) = 4 \text{ m/s}^2,$$

$$\frac{d^2y}{dt^2} = \frac{d^2}{dt^2}0.25x^2 = 0.5\left(\frac{dx}{dt}\right)^2 + 0.5x\left(\frac{d^2x}{dt^2}\right) = 6.5 \text{ (m/s}^2\text{)},$$

$$\left(\frac{d^2z}{dt^2}\right) = 0, \boldsymbol{\alpha} = 3\mathbf{i} - 6\mathbf{j} \text{ (rad/s}^2\text{)}, \mathbf{v}_{A\text{rel}} = -3\mathbf{i} - 1.5\mathbf{j},$$

and from above: $\boldsymbol{\omega} \times \mathbf{r}_{A/O} = -0.5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. Substitute:

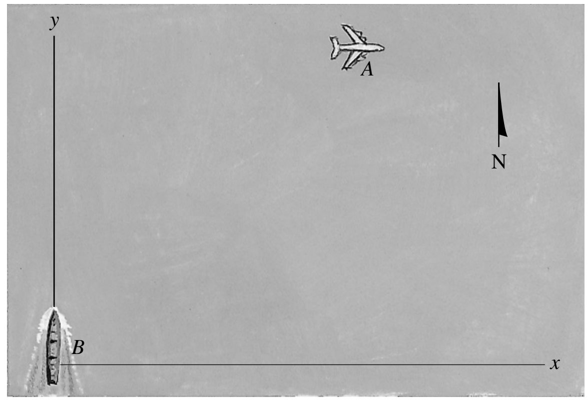
$$\mathbf{a}_A = 4\mathbf{i} + 6.5\mathbf{j} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ -3 & -1.5 & 0 \end{bmatrix} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 0 \\ 1 & 0.25 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ -0.5 & 2 & 4 \end{bmatrix}.$$

$$\mathbf{a}_A = 4\mathbf{i} + 6.5\mathbf{j} + 2(3\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}) + (6.75\mathbf{k}) + (-20\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\boxed{\mathbf{a}_A = -10\mathbf{i} - 6.5\mathbf{j} - 19.25\mathbf{k} \text{ (m/s}^2\text{)}}$$

Problem 17.147 The coordinate system shown is fixed relative to the ship B. At the given instant, the ship is sailing north at 10 ft/s relative to the earth and its angular velocity is 0.02 rad/s clockwise. The airplane is flying east at 400 ft/s relative to the earth, and its position relative to the ship is $\mathbf{r}_{A/B} = 2000\mathbf{i} + 2000\mathbf{j} + 1000\mathbf{k}$ (ft). If the ship uses its radar to measure the plane's velocity relative to the ship's body-fixed coordinate system, what is the result?

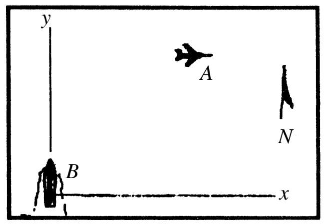


Solution: The relative velocity is

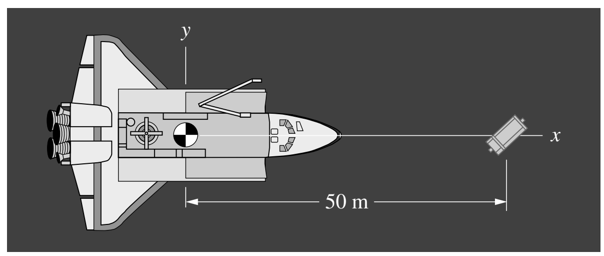
$$\mathbf{v}_{A\text{rel}} = \mathbf{v}_A - \mathbf{v}_B - \boldsymbol{\omega} \times \mathbf{r}_{A/B},$$

$$= 400\mathbf{i} - 10\mathbf{j} + 0.02(\mathbf{k} \times (2000\mathbf{i} + 2000\mathbf{j} + 100\mathbf{k}))\ \text{(ft/s)}$$

$$\boxed{\mathbf{v}_{A\text{rel}} = 360\mathbf{i} + 30\mathbf{j}\ \text{(ft/s)}}.$$



Problem 17.148 The space shuttle is attempting to recover a satellite for repair. At the current time, the satellite's position relative to a coordinate system fixed to the shuttle is $50\mathbf{i}$ (m). The rate gyros on the shuttle indicate that its current angular velocity is $0.05\mathbf{j} + 0.03\mathbf{k}$ (rad/s). The Shuttle pilot measures the velocity of the satellite relative to the body-fixed coordinate system and determines it to be $-2\mathbf{i} - 1.5\mathbf{j} + 2.5\mathbf{k}$ (rad/s). What are the x, y, and z components of the satellite's velocity relative to a non-rotating coordinate system with its origin fixed to the shuttle's center of mass?



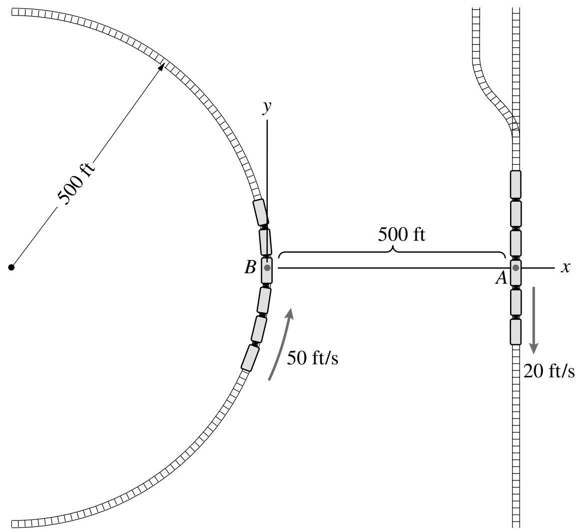
Solution: The velocity of the satellite is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 0 - 2\mathbf{i} - 1.5\mathbf{j} + 2.5\mathbf{k} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.05 & 0.03 \\ 50 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -2\mathbf{i} + 1.5\mathbf{j} + 2.5\mathbf{k} - 1.5\mathbf{j} - 2.5\mathbf{k} = -2\mathbf{i}\ \text{(m/s)}}$$

Problem 17.149 The train on the circular track is traveling at a constant speed of 50 ft/s in the direction shown. The train on the straight track is traveling at 20 ft/s in the direction shown and is increasing its speed at 2 ft/s^2. Determine the velocity of passenger A that passenger B observes relative to the given coordinate system, which is fixed to the car in which B is riding.



Solution:

The angular velocity of B is $\omega = \dfrac{50}{500} = 0.1$ rad/s.

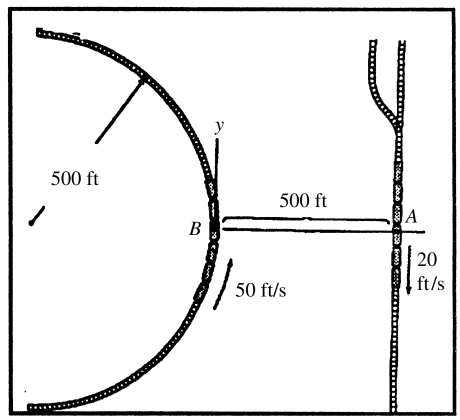
The velocity of A is $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$.

At the instant shown, $\mathbf{v}_A = -20\mathbf{j}$ (ft/s), $\mathbf{v}_B = +50\mathbf{j}$ (ft/s),

and $\mathbf{r}_{A/B} = 500\mathbf{i}$ (ft), from which

$$\mathbf{v}_{A\text{rel}} = -20\mathbf{j} - 50\mathbf{j} - \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ 500 & 0 & 0 \end{bmatrix} = -20\mathbf{j} - 50\mathbf{j} - 50\mathbf{j},$$

$\boxed{\mathbf{v}_{A\text{rel}} = -120\mathbf{j} \text{ (ft/s)}}$



Problem 17.150 In Problem 17.149, determine the acceleration of passenger A that passenger B observes relative to the coordinate system fixed to the car in which B is riding.

Solution:

Use the solution to Problem 17.149:

$\mathbf{v}_{A\text{rel}} = -120\mathbf{j}$ (ft/s),

$$\omega = \frac{50}{500} = 0.1 \text{ rad/s},$$

$\mathbf{r}_{A/B} = 500\mathbf{i}$ (ft).

The acceleration of A is $\mathbf{a}_A = -2\mathbf{j}$ (ft/s),

The acceleration of B is

$\mathbf{a}_B = -500(\omega^2)\mathbf{i} = -5\mathbf{i}$ ft/s^2,

and $\boldsymbol{\alpha} = 0$, from which

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

Rearrange:

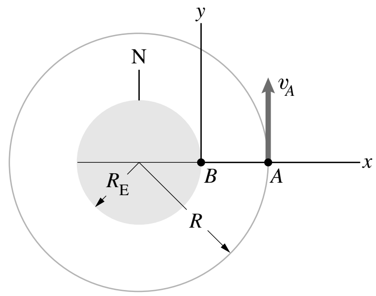
$$\mathbf{a}_{A\text{rel}} = \mathbf{a}_A - \mathbf{a}_B - 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

$$\mathbf{a}_{\text{Re}l} = -2\mathbf{j} + 5\mathbf{i} - 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0 & -120 & 0 \end{bmatrix} - \omega^2 \left(\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 500 & 0 & 0 \end{bmatrix} \right).$$

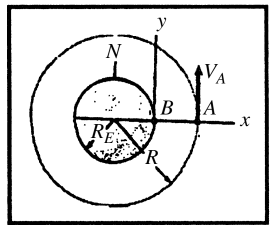
$$\mathbf{a}_{\text{Re}l} = -2\mathbf{j} + 5\mathbf{i} - 2(120\omega)\mathbf{i} - \omega^2 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 500 & 0 \end{bmatrix},$$

$\boxed{\mathbf{a}_{A\text{rel}} = -14\mathbf{i} - 2\mathbf{j} \text{ (ft/s}^2\text{)}}$.

Problem 17.151 The satellite A is in a circular polar orbit (a circular orbit that intersects the earth's axis of rotation). The radius of the orbit is R, and the magnitude of the satellite's velocity relative to a non-rotating reference frame with its origin at the center of the earth is v_A. At the instant shown, the satellite is above the equator. An observer B on the earth directly below the satellite measures its motion using the earth-fixed coordinate system shown. What are the velocity and acceleration of the satellite relative to B's earth-fixed coordinate system? The radius of the earth is R_E and the angular velocity of the earth is ω_E.



Solution: From the sketch, in the coordinate system shown, the location of the satellite in this system is $\mathbf{r}_A = (R - R_E)\mathbf{i}$, from which $\mathbf{r}_{A/B} = \mathbf{r}_A - 0 = (R - R_E)\mathbf{i}$. The angular velocity of the observer is $\boldsymbol{\omega}_E = -\omega_E \mathbf{k}$. The velocity of the observer is $\mathbf{v}_B = -\omega_E R_E \mathbf{k}$. The velocity of the satellite is $\mathbf{v}_A = v_A \mathbf{j}$.
The relative velocity is



$$\mathbf{v}_{A\text{rel}} = \mathbf{v}_A - \mathbf{v}_B - \boldsymbol{\omega}_E \times \mathbf{r}_{A/B},$$

$$\mathbf{v}_{A\text{rel}} = v_A \mathbf{j} + \omega_E R_E \mathbf{k} - (\omega_E) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ R - R_E & 0 & 0 \end{bmatrix}$$

$$\boxed{= v_A \mathbf{j} + R\omega_E \mathbf{k}}.$$

From Eqs (17.26) and (17.27)

$$\mathbf{a}_{A\text{rel}} = \mathbf{a}_A - \mathbf{a}_B - 2\boldsymbol{\omega}_E \times \mathbf{v}_{A\text{rel}} - \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times \mathbf{r}_{A/B}).$$

The accelerations:

$$\mathbf{a}_A = -\omega_A^2 R \mathbf{i} = -\left(\frac{v_A^2}{R}\right)\mathbf{i},\ \mathbf{a}_B = -\omega_E^2 R_E \mathbf{i},\ \boldsymbol{\alpha} = 0, \text{ from which}$$

$$\mathbf{a}_{A\text{rel}} = -\left(\frac{v_A^2}{R}\right) + \mathbf{i}\omega_E^2 R_E - 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ 0 & v_A & R\omega_E \end{bmatrix}$$

$$-\boldsymbol{\omega}_E \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ R - R_E & 0 & 0 \end{bmatrix}.$$

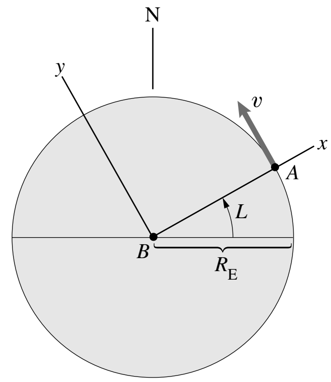
$$\mathbf{a}_{A\text{rel}} = -\omega_A^2 R \mathbf{i} + \omega_E^2 R_E \mathbf{i}$$

$$-2\omega_E^2 R \mathbf{i} - \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ 0 & 0 & -\omega_E(R - R_E) \end{bmatrix}$$

$$= -\left(\frac{v_A^2}{R}\right)\mathbf{i} + \omega_E^2 R_E \mathbf{i} - 2\omega_E^2 R \mathbf{i} + \omega_E^2 (R - R_E)\mathbf{i}$$

$$\boxed{\mathbf{a}_{A\text{rel}} = -\left(\left(\frac{v_A^2}{R}\right) + \omega_E^2 R\right)\mathbf{i}}$$

Problem 17.152 A car A at north latitude L drives north on a north–south highway with constant speed v. The earth's radius is R_E, and the earth's angular velocity is ω_E. (The earth's angular velocity vector points north.) The coordinate system is earth fixed, and the x axis passes through the car's position at the instant shown. Determine the car's velocity and acceleration (a) relative to the earth-fixed coordinate system and (b) relative to a nonrotating reference frame with its origin at the center of the earth.



Solution:

(a) In earth fixed coords,

$$\mathbf{v}_{rel} = v\mathbf{j},$$

$$\mathbf{a}_{rel} = -v^2/R_E\mathbf{i}. \text{ (motion in a circle)}$$

(b) $\mathbf{v}_A = \mathbf{v}_{Arel} + \boldsymbol{\omega}_E \times \mathbf{r}_{A/B} + \mathbf{v}_B (\mathbf{v}_B = 0)$

$$= v\mathbf{j} + (\omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}) \times R_E\mathbf{i}$$

$$\underline{\mathbf{v}_A = v\mathbf{j} - \omega_E R_E \cos L\mathbf{k}}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{Arel} + 2\boldsymbol{\omega}_E \times \mathbf{v}_{Arel} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B}$$

$$+ \boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times \mathbf{r}_{A/B})$$

where $\boldsymbol{\omega}_E = \omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}$

and $\mathbf{r}_{A/B} = R_E\mathbf{i}$

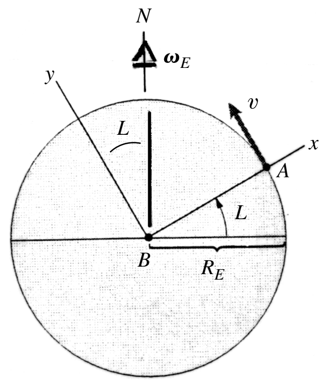
$$\mathbf{a}_A = 0 - \frac{v^2}{R_E}\mathbf{i} + 2v\omega_E \sin L\mathbf{k}$$

$$+ (\omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}) \times (-\omega_E R_E \cos L\mathbf{k})$$

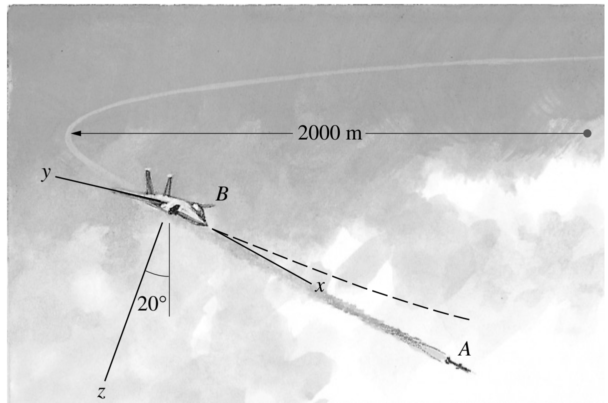
$$\mathbf{a}_A = -\left(\frac{v^2}{R_E} + \omega_E^2 R_E \cos^2 L\right)\mathbf{i}$$

$$+ (\omega_E^2 R_E \sin L \cos L)\mathbf{j}$$

$$+ 2v\omega_E \sin L\mathbf{k}$$

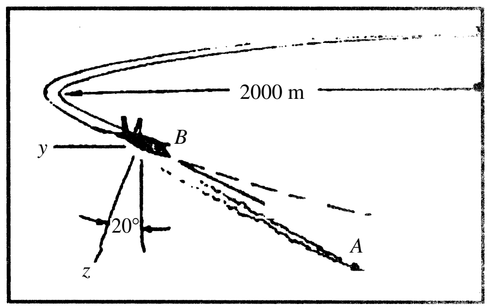


Problem 17.153 The airplane B conducts flight tests of a missile. At the instant shown, the airplane is traveling at 200 m/s relative to the earth in a circular path of 2000-m radius *in the horizontal plane*. The coordinate system is fixed relative to the airplane. The x axis is tangent to the plane's path and points forward. The y axis points out the plane's right side, and the z axis points out the bottom of the plane. The plane's bank angle (the inclination of the z axis from the vertical) is constant and equal to 20°. *Relative to the airplane's coordinate system*, the pilot measures the missile's position and velocity and determines them to be $\mathbf{r}_{A/B} = 1000\mathbf{i}$ (m) and $\mathbf{v}_{A/B} = 100.0\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k}$ (m/s).



(a) What are the x, y, and z components of the airplane's angular velocity vector?
(b) What are the x, y, and z components of the missile's velocity relative to the earth?

Solution:



(a) The bank angle is a rotation about the x axis; assume that the rotation is counterclockwise, so that the z axis is rotated toward the positive y axis. The magnitude of the angular velocity is

$$\omega = \frac{200}{2000} = 0.1 \text{ rad/s}.$$

In terms of airplane fixed coordinates,

$$\boldsymbol{\omega} = 0.1(\mathbf{i}\sin 20^\circ - \mathbf{j}\cos 20^\circ) \text{ (rad/s)}.$$

$$\boxed{\boldsymbol{\omega} = 0.03242\mathbf{j} - 0.0940\mathbf{k} \text{ rad/s}}$$

(b) The velocity of the airplane in earth fixed coordinates is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

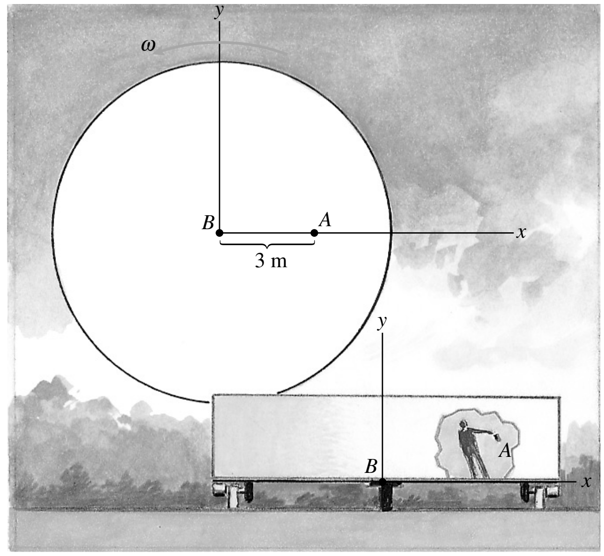
$$= 200\mathbf{i} + 100\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.0342 & -0.940 \\ 1000 & 0 & 0 \end{bmatrix}$$

$$\boxed{\mathbf{v}_A = 300\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k} - 94.0\mathbf{j} - 34.2\mathbf{k} = 300\mathbf{i} \text{ (m/s)}}$$

Problem 17.154 To conduct experiments related to long-term spaceflight, engineers construct a laboratory on earth that rotates about the vertical axis at B with a constant angular velocity ω of one revolution every 6 s. They establish a laboratory-fixed coordinate system with its origin at B and the z axis pointing upward. An engineer holds an object stationary relative to the laboratory at point A, 3 m from the axis of rotation, and releases it. At the instant he drops the object, determine its acceleration relative to the laboratory-fixed coordinate system,

(a) assuming that the laboratory-fixed coordinate system is inertial and
(b) not assuming that the laboratory-fixed coordinate system is inertial, but assuming that an earth-fixed coordinate system with its origin at B is inertial.



Solution: (a) If the laboratory system is inertial, Newton's second law is $\mathbf{F} = m\mathbf{a}$. The only force is the force of gravity; so that as the object free falls the acceleration is

$$\boxed{-g\mathbf{k} = -9.81\mathbf{k}(\text{m/s}^2)}.$$

If the earth fixed system is inertial, the acceleration observed is the centripetal acceleration and the acceleration of gravity:

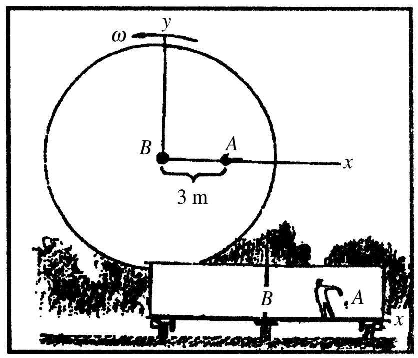
$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_B) - \mathbf{g},$$

where the angular velocity is the angular velocity of the coordinate system relative to the inertial frame.

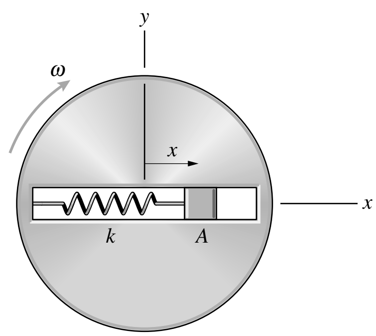
$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) - \mathbf{g} = -\left(\frac{2\pi}{6}\right)^2 \left(\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix}\right) - 9.81\mathbf{k}$$

$$= \left(\frac{\pi^2}{3}\right)\mathbf{i} - 9.81\mathbf{k}$$

$$\boxed{= 3.29\mathbf{i} - 9.81\mathbf{k}\ (\text{m/s}^2)}$$

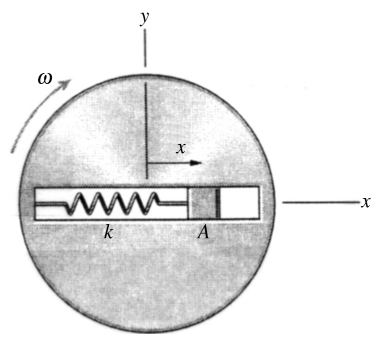


Problem 17.155 The disk rotates *in the horizontal plane* about a fixed shaft at the origin with constant angular velocity $w = 10$ rad/s. The 2-kg slider A moves in a smooth slot in the disk. The spring is unstretched when $x = 0$ and its constant is $k = 400$ N/m. Determine the acceleration of A relative to the body-fixed coordinate system when $x = 0.4$ m.



Strategy: Use Eq. (17.30) to express Newton's second law for the slider in terms of the body-fixed coordinate system.

Solution:



$$\mathbf{T} = -kx = (-400)(0.4)$$

$$\mathbf{T} = -160\mathbf{i} \text{ Newtons}$$

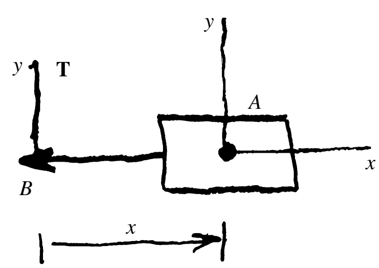
$$\sum \mathbf{F} = -160\mathbf{i} = m\mathbf{a}_{A\text{rel}} + m[\cancel{\mathbf{a}}_B^0 + 2\cancel{\boldsymbol{\omega}}^0 \times \mathbf{v}_{\text{rel}}$$

$$+ \cancel{\boldsymbol{\alpha}}^0 \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

where $\boldsymbol{\omega} = -10\mathbf{k}$, $\mathbf{r}_{A/B} = 0.4\mathbf{i}$, $m = 2$

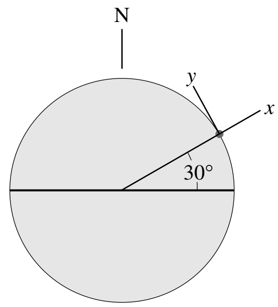
$$-160\mathbf{i} = 2\mathbf{a}_{A\text{rel}} - 80\mathbf{i}$$

$$\underline{\mathbf{a}_{A\text{rel}} = -40\mathbf{i} \text{ (m/s}^2\text{)}}.$$



Problem 17.156 Engineers conduct flight test of a rocket at 30° north latitude. They measure the rocket's motion using an earth-fixed coordinate system with the x axis pointing upward and the y axis directed northward. At a particular instant, the mass of the rocket is 4000 kg, the velocity of the rocket relative to the engineers' coordinate system is $2000\mathbf{i} + 2000\mathbf{j}$ (m/s), and the sum of the forces exerted on the rocket by its thrust, weight, and aerodynamic forces is $400\mathbf{i} + 400\mathbf{j}$ (N). Determine the rocket's acceleration relative to the engineers' coordinate system,

(a) assuming that their earth-fixed coordinate system is inertial and
(b) not assuming that their earth-fixed coordinate system is inertial.



Solution: Use Eq. (17.22):

$$\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

$$= m\mathbf{a}_{A\text{rel}}.$$

(a) If the earth fixed coordinate system is assumed to be inertial, this reduces to $\sum \mathbf{F} = m\mathbf{a}_{A\text{rel}}$, from which

$$\mathbf{a}_{A\text{rel}} = \frac{1}{m}\sum \mathbf{F} = \frac{1}{4000}(400\mathbf{i} + 400\mathbf{j})$$

$$\boxed{= 0.1\mathbf{i} + 0.1\mathbf{j} \ (\text{m/s}^2)}$$

(b) If the earth fixed system is not assumed to be inertial, $\mathbf{a}_B = -R_E\omega_E^2 \cos^2\lambda\mathbf{i} + R_E\omega_E^2 \cos\lambda \sin\lambda\mathbf{j}$, the angular velocity of the rotating coordinate system is $\boldsymbol{\omega} = \omega_E \sin\lambda\mathbf{i} + \omega_E \cos\lambda\mathbf{j}$ (rad/s). The relative velocity in the earth fixed system is $\mathbf{v}_{A\text{rel}} = 2000\mathbf{i} + 2000\mathbf{j}$ (m/s), and $\mathbf{r}_{A/B} = R_E\mathbf{i}$ (m).

$$2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} = 2\omega_E \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin\lambda & \cos\lambda & 0 \\ 2000 & 2000 & 0 \end{bmatrix}$$

$$= 4000\omega_E(\sin\lambda - \cos\lambda)\mathbf{k}$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) = \boldsymbol{\omega} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_E \sin\lambda & \omega_E \cos\lambda & 0 \\ R_E & 0 & 0 \end{bmatrix}$$

$$= \boldsymbol{\omega} \times (-R_E\omega_E \cos\lambda)\mathbf{k}$$

$$\boldsymbol{\omega} \times (-R_E\omega_E \cos\lambda)\mathbf{k} = R_E\omega_E^2 \cos\lambda \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= (-R_E\omega_E^2 \cos^2\lambda)\mathbf{i} + (R_E\omega_E^2 \cos\lambda \sin\lambda)\mathbf{j}.$$

Collect terms,

$$\mathbf{a}_{A\text{rel}} = +(R_E\omega_E^2 \cos^2\lambda)\mathbf{i} - (R_E\omega_E^2 \cos\lambda \sin\lambda)\mathbf{j}$$

$$- 4000\omega_E(\sin\lambda - \cos\lambda)\mathbf{k} + 0.1\mathbf{i} + 0.1\mathbf{j}.$$

Substitute values:

$R_E = 6336 \times 10^3$ m, $\omega_E = 0.73 \times 10^{-4}$ rad/s, $\lambda = 30°$,

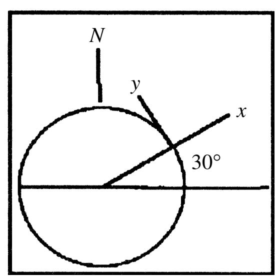
$$\boxed{\mathbf{a}_{A\text{rel}} = 0.125\mathbf{i} + 0.0854\mathbf{j} + 0.1069\mathbf{k} \ (\text{m/s}^2)}$$

Note: The last two terms in the parenthetic expression for $\mathbf{a}_A$ in

$$\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

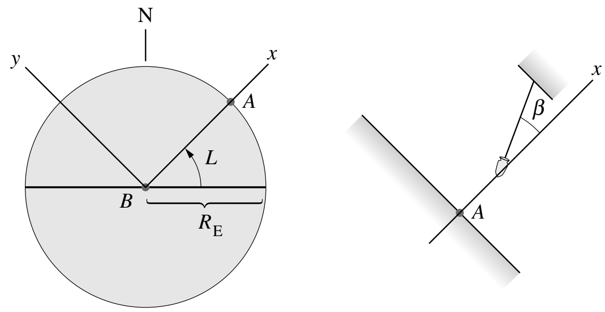
$$= m\mathbf{a}_{A\text{rel}}$$

can be neglected without significant change in the answers.



Problem 17.157 Consider a point A on the surface of the earth at north latitude L. The radius of the earth is R_E and its angular velocity is ω_E. A plumb bob suspended just above the ground at point A will hang at a small angle β relative to the vertical because of the earth's rotation. Show that β is related to the latitude by

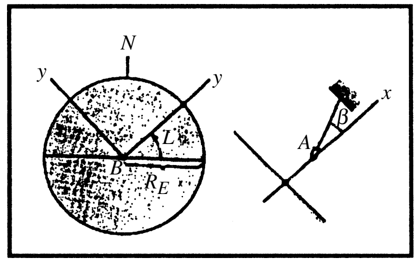
$$\tan\beta = \frac{\omega_E^2 R_E \sin L \cos L}{g - \omega_E^2 R_E \cos^2 L}.$$

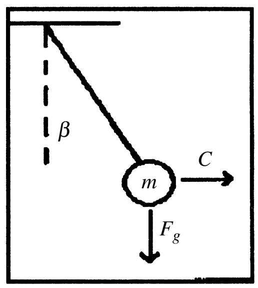


Strategy: Use the earth-fixed coordinate system shown, express Newton's second law in the form given by Eq. (6.27).

Solution:

Use Eq. (17.27). $\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})] = m\mathbf{a}_{A\text{rel}}$. The bob is stationary, so that $\mathbf{v}_{A\text{rel}} = 0$. The origin of the coordinate system is stationary, so that $\mathbf{a}_B = 0$. The external force is the weight of the bob $\sum \mathbf{F} = m\mathbf{g}$. The relative acceleration is the apparent acceleration due to gravity, $m\mathbf{a}_{A\text{rel}} = m\mathbf{g}_{\text{Apparent}}$. Substitute:





$$\mathbf{g}_{\text{Apparent}} = \mathbf{g} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$$

$$= g\mathbf{i} - \boldsymbol{\omega} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_E \sin L & \omega_E \cos L & 0 \\ R_E & 0 & 0 \end{bmatrix}$$

$$\mathbf{g}_{\text{Apparent}} = g\mathbf{i} - \boldsymbol{\omega} \times (-R_E\omega_E \cos L)\mathbf{k}$$

$$= g\mathbf{i} + R_E\omega_E^2 \cos L \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin L & \cos L & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{g}_{\text{Apparent}} = g\mathbf{i} - (R_E\omega_E^2 \cos^2 L)\mathbf{i} + (R_E\omega_E^2 \cos L \sin L)\mathbf{j}.$$

The vertical component of the apparent acceleration due to gravity is $g_{\text{vertical}} = g - R_E\omega_E^2 \cos^2 L$. The horizontal component of the apparent acceleration due to gravity is $g_{\text{horizontal}} = R_E\omega_E^2 \cos L \sin L$. From equation of angular motion, the moments about the bob suspension are $M_{\text{vertical}} = (\lambda \sin\beta) m g_{\text{vertical}}$ and $M_{\text{horizontal}} = (\lambda \cos\beta) m g_{\text{horizontal}}$, where λ is the length of the bob, and m is the mass of the bob. In equilibrium, $M_{\text{vertical}} = M_{\text{horizontal}}$, from which $g_{\text{vertical}} \sin\beta = g_{\text{horizontal}} \cos\beta$. Substitute and rearrange:

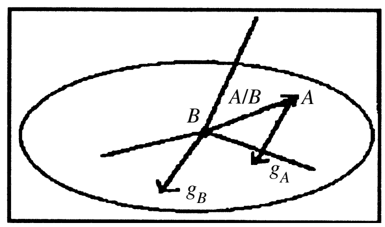
$$\boxed{\tan\beta = \frac{g_{\text{horizontal}}}{g_{\text{vertical}}} = \frac{R_E\omega_E^2 \cos L \sin L}{g - R_E\omega_E^2 \cos^2 L}}.$$

Problem 17.158 Suppose that a space station is in orbit around the earth and two astronauts on the station toss a ball back and forth. They observe that the ball appears to travel between them in a straight line at constant velocity.

(a) Write Newton's second law for the ball as it travels between the astronauts in terms of a nonrotating coordinate system with its origin fixed to the station. What is the term $\sum F$? Use the equation you wrote to explain the behavior of the ball observed by the astronauts.

(b) Write Newton's second law for the ball as it travels between the astronauts in terms of a nonrotating coordinate system with its origin fixed to the center of the earth. What is the term $\sum \mathbf{F}$? Explain the difference between this equation and the one you obtained in (a).

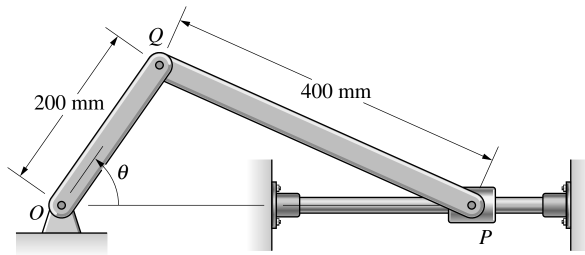
Solution: An earth centered, non rotating coordinate system can be treated as inertial for analyzing the motions of objects near the earth (See Section 7.2.) Let O be the reference point of this reference frame, and let B be the origin of the non rotating reference frame fixed to the space station, and let A denote the ball. The orbiting station and its contents and the station-fixed non rotating frame are in free fall about the earth (they accelerate relative to the earth due to the earth's gravitational attraction), so that the forces on the ball in the fixed reference frame *exclude the earth's gravitational attraction*. Let $\mathbf{g}_B$ be the station's acceleration, and let $\mathbf{g}_A$ be the ball's acceleration relative to the earth due to the earth's gravitational attraction. Let $\sum F$ be the sum of all forces on the ball, *not including the earth's gravitational attraction*. Newton's second law for the ball of mass m is $\sum \mathbf{F} + m\mathbf{g}_A = m\mathbf{a}_A = m(\mathbf{a}_B + \mathbf{a}_{A/B}) = m\mathbf{g}_B + m\mathbf{a}_{A/B}$. Since the ball is within a space station whose dimensions are small compared to the distance from the earth, $\mathbf{g}_A$ is equal to $\mathbf{g}_B$ within a close approximation, from which $\sum \mathbf{F} = m\mathbf{a}_{A/B}$. The sum of the forces on the ball *not including the force exerted by the earth's gravitational attraction* equals the mass times the ball's acceleration relative to a reference frame fixed with respect to the station. As the astronauts toss the ball back and forth, the only other force on it is aerodynamic drag. Neglecting aerodynamic drag, $\mathbf{a}_{A/B} = 0$, from which the ball will travel in a straight line with constant velocity.



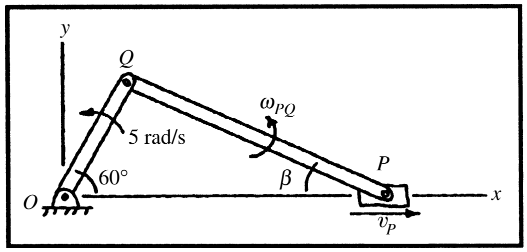
(b) Relative to the earth centered non rotating reference frame, Newton's second law for the ball is $\sum \mathbf{F} = m\mathbf{a}_A$ where $\sum \mathbf{F}$ is the sum of all forces on the ball, including aerodynamic drag *and the force due to the earth's gravitational attraction*. Neglect drag, from which $\mathbf{a}_A = \mathbf{g}_A$; the ball's acceleration is its acceleration due to the earth's gravitational attraction, because in this case we are determining the ball's motion relative to the earth.

Note: An obvious unstated assumption is that the time of flight of the ball as it is tossed between the astronauts is much less than the period of an orbit. Thus the very small acceleration differences $\mathbf{g}_A - \mathbf{g}_B$ will have a negligible effect on the path of the ball over the short time interval.

Problem 17.159 If $\theta = 60^\circ$ and bar OQ is rotating in the counterclockwise direction at 5 rad/s, what is the angular velocity of bar PQ?



Solution: By applying the law of sines, $\beta = 25.7^\circ$. The velocity of Q is



$\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{OQ} \times \mathbf{r}_{Q/O}$ or

$$\mathbf{v}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 0.2\cos 60^\circ & 0.2\sin 60^\circ & 0 \end{vmatrix} = -\sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}.$$

The velocity of P is

$$v_p \mathbf{i} = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$$

$$= -\sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components $v_P = -\sin 60^\circ + 0.4\omega_{PQ}\sin\beta$, and $0 = \cos 60^\circ + 0.4\omega_{PQ}\cos\beta$. Solving, we obtain $v_P = -1.11$ m/s and $\omega_{PQ} = -1.39$ rad/s.

Problem 17.160 Consider the system shown in Problem 17.159. If $\theta = 55^\circ$ and the sleeve P is moving to the left at 2 m/s, what is the angular velocity of the bar PQ?

Solution: By applying the law of sines, $\beta = 24.2^\circ$ The velocity of Q is

$$\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{OQ} \times \mathbf{r}_{Q/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{OQ} \\ 0.2\cos 55^\circ & 0.2\sin 55^\circ & 0 \end{vmatrix}$$

$$= -0.2\omega_{OQ}\sin 55^\circ \mathbf{i} + 0.2\omega_{OQ}\cos 55^\circ \mathbf{j} \qquad \textbf{(1)}$$

We can also express $\mathbf{v}_Q$ as

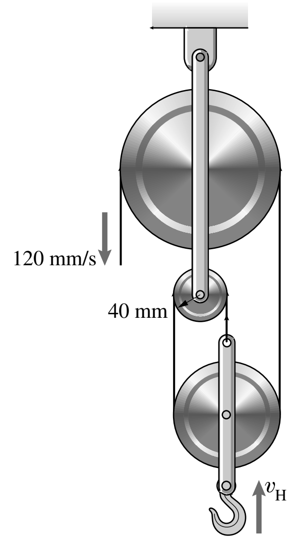
$$\mathbf{v}_Q = \mathbf{v}_P + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{Q/P}$$

$$= -2\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ -0.4\cos\beta & 0.4\sin\beta & 0 \end{vmatrix}.$$

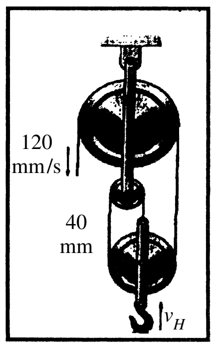
Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (1) and (2), we get $-0.2\omega_{OQ}\sin 55^\circ = -2 - 0.4\omega_{PQ}\sin\beta$, and $0.2\omega_{OQ}\cos 55^\circ = -0.4\omega_{PQ}\cos\beta$. Solving, we obtain

$\omega_{OQ} = 9.29$ rad/s$\omega_{PQ} = -2.92$ rad/s.

Problem 17.161 Determine the vertical velocity v_H of the hook and the angular velocity of the small pulley.



Solution: The upper pulley is fixed so that it cannot move, from which the upward velocity of the rope on the right is equal to the downward velocity on the left, and the upward velocity of the rope on the right of the lower pulley is 120 mm/s. The small pulley is fixed so that it does not move. The upward velocity on the right of the small pulley is v_H mm/s, from which the downward velocity on the left is v_H mm/s. The upward velocity of the center of the bottom pulley is the mean of the difference of the velocities on the right and left, from which



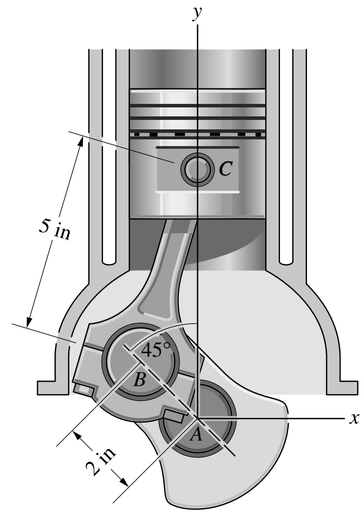
$$v_H = \frac{120 - v_H}{2}.$$

Solve, $\boxed{v_H = 40 \text{ mm/s}}$.

The angular velocity of the small pulley is

$$\boxed{\omega = \frac{v_H}{R} = \frac{40}{40} = 1 \text{ rad/s}}.$$

Problem 17.162 If the crankshaft AB is turning in the counterclockwise direction at 2000 rpm (revolutions per minute), what is the velocity of the piston?



Solution: The angle of the crank with the vertical is 45°. The angular velocity of the crankshaft is

$$\omega = 2000\left(\frac{2\pi}{60}\right) = 209.44 \text{ rad/s.}$$

The vector location of point B (the main rod bearing) $\mathbf{r}_B = 2(-\mathbf{i}\sin 45^\circ + \mathbf{j}\cos 45^\circ) = 1.414(-\mathbf{i}+\mathbf{j})$ in. The velocity of point B (the main rod bearing) is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 1.414\omega \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -296.2(\mathbf{i}+\mathbf{j}) \text{ (in/s).}$$

From the law if sines the interior angle between the connecting rod and the vertical at the piston is obtained from $\dfrac{2}{\sin\theta} = \dfrac{5}{\sin 45^\circ}$, from which

$$\theta = \sin^{-1}\left(\frac{2\sin 45^\circ}{5}\right) = 16.43^\circ.$$

The location of the piston is $\mathbf{r}_C = (2\sin 45^\circ + 5\cos\theta)\mathbf{j} = 6.21\mathbf{j}$ (in.). The vector $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -1.414\mathbf{i} - 4.796\mathbf{j}$ (in.). The piston is constrained to move along the y axis. In terms of the connecting rod the velocity of the point B is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = v_C\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 1.414 & -4.796 & 0 \end{bmatrix}$$

$$= v_C\mathbf{j} + 4.796\omega_{BC}\mathbf{i} - 1.414\omega_{BC}\mathbf{j} \text{ (in/s).}$$

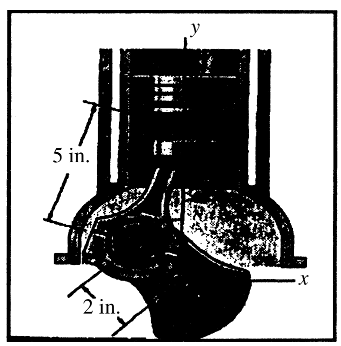
Equate expressions for $\mathbf{v}_B$ and separate components:

$$-296.2 = 4.796\omega_{BC},$$

$$-296.2 = v_C - 1.414\omega_{BC}.$$

Solve: $\boxed{\mathbf{v}_C = -383.5\mathbf{j} \text{ (in/s)} = -32\mathbf{j} \text{ (ft/s)}}$,

$$\omega_{BC} = -61.8 \text{ rad/s.}$$



Problem 17.163 In Problem 17.162, if the piston is moving with velocity $\mathbf{v}_C = 20\mathbf{j}$ (ft/s), what are the angular velocities of the crankshaft AB and the connecting rod BC?

Solution: Use the solution to Problem 17.162. The vector location of point B (the main rod bearing) $\mathbf{r}_B = 1.414(-\mathbf{i} + \mathbf{j})$ in. From the law if sines the interior angle between the connecting rod and the vertical at the piston is

$$\theta = \sin^{-1}\left(\frac{2\sin 45^\circ}{5}\right) = 16.43^\circ.$$

The location of the piston is $\mathbf{r}_C = (2\sin 45^\circ + 5\cos\theta)\mathbf{j} = 6.21\mathbf{j}$ (in.). The piston is constrained to move along the y axis. In terms of the connecting rod the velocity of the point B is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 240\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -1.414 & -4.796 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = 240\mathbf{j} + 4.796\omega_{BC}\mathbf{i} - 1.414\omega_{BC}\mathbf{j} \text{ (in/s)}.$$

In terms of the crank angular velocity, the velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 1.414\omega_{AB} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -1.414\omega_{AB}(\mathbf{i} + \mathbf{j}) \text{ (in/s)}.$$

Equate expressions and separate components:

$$4.796\omega_{BC} = -1.414\omega_{BC} = -1.414\omega_{AB}.$$

Solve:

$\boxed{\omega_{BC} = 38.65 \text{ rad/s}}$ (counterclockwise).

$\boxed{\omega_{AB} = 131.1 \text{ rad/s} = -12515 \text{ rpm}}$ (clockwise).

Problem 17.164 In Problem 17.162, if the piston is moving with velocity $\mathbf{v}_C = 20\mathbf{j}$ (ft/s), and its acceleration is zero, what are the angular accelerations of crankshaft AB and the connecting rod BC?

Solution: Use the solution to Problem 17.163. $\mathbf{r}_{B/A} = 1.414(-\mathbf{i} + \mathbf{j}$ in., $\omega_{AB} = -131.1$ rad/s, $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -1.414\mathbf{i} - 4.796\mathbf{j}$ (in.), $\omega_{BC} = 38.65$ rad/s. For point B,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= 1.414\alpha_{AB} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} - 1.414\omega_{AB}^2(-\mathbf{i} + \mathbf{j}),$$

$$\mathbf{a}_B = -1.414\alpha_{AB}(\mathbf{i} + \mathbf{j}) + 24291(\mathbf{i} - \mathbf{j}) \text{ (in/s}^2).$$

In terms of the angular velocity of the connecting rod,

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C},$$

$$\mathbf{a}_B = \alpha_{BC} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1.414 & -4.796 & 0 \end{bmatrix} - \omega_{BC}^2(-1.414\mathbf{i} - 4.795\mathbf{j}) \text{ (in/s}^2),$$

$$\mathbf{a}_B = 4.796\alpha_{BC}\mathbf{i} - 1.414\alpha_{BC}\mathbf{j} + 2112.3\mathbf{i} + 71630\mathbf{j} \text{ (in/s}^2).$$

Equate expressions and separate components:

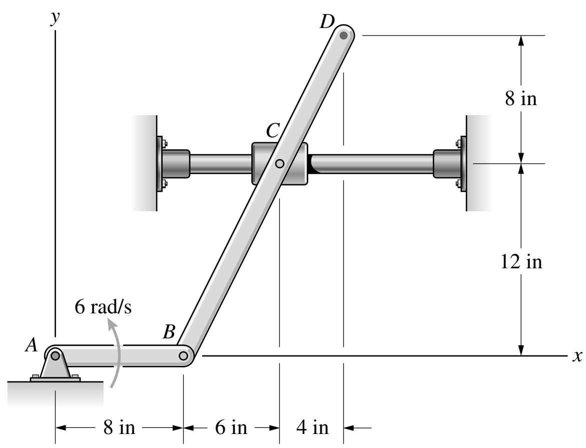
$$-1.414\alpha_{AB} + 24291 = 4.796\alpha_{BC} + 2112.3,$$

$$-1414\alpha_{AB} - 24291 = 1.414\alpha_{BC} + 7163.$$

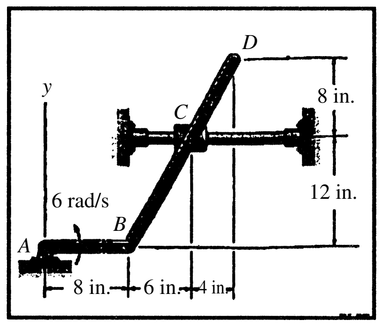
Solve: $\boxed{\alpha_{AB} = -13{,}605 \text{ rad/s}^2}$ (clockwise).

$\boxed{\alpha_{BC} = 8636.5 \text{ rad/s}^2}$ (counterclockwise).

Problem 17.165 Bar AB rotates at 6 rad/s in the counterclockwise direction. Use instantaneous centers to determine the angular velocity of bar BCD and the velocity of point D.



Solution: The strategy is to determine the angular velocity of bar BC from the instantaneous center; using the constraint on the motion of C. The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (8\mathbf{i} + 4\mathbf{j}) - 4\mathbf{j} = 8\mathbf{i}$ (in.). The velocity of point B is $\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \omega_{AB}(\mathbf{k} \times 8\mathbf{i}) = 48\mathbf{j}$ (in/s). The velocity of point B is normal to the x axis, and the velocity of C is parallel to the x axis. The instantaneous center of bar BC has the coordinates (14, 0). The vector



$$\mathbf{r}_{B/IC} = \mathbf{r}_B - \mathbf{r}_{IC} = (8\mathbf{i} + 4\mathbf{j}) - (14\mathbf{i} - 4\mathbf{j}) = -6\mathbf{i} \text{ (in.)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -6 & 0 & 0 \end{bmatrix} = -6\omega_{BC}\mathbf{j} = 48\mathbf{j},$$

from which

$$\boxed{\omega_{BC} = -\frac{48}{6} = -8 \text{ rad/s}}.$$

The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & 12 & 0 \end{bmatrix} = 96\mathbf{i} \text{ (in/s)}.$$

The velocity of point D is normal to the unit vector parallel to BCD

$$\mathbf{e} = \frac{6\mathbf{i} + 12\mathbf{j}}{\sqrt{6^2 + 12^{12}}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The intersection of the projection of this unit vector with the projection of the unit vector normal to velocity of C is occurs at point C, from which the coordinates of the instantaneous center for the part of the bar CD are (14, 12). The instantaneous center is translating at velocity $\mathbf{v}_C$, from which the velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{D/ICD} = 96\mathbf{i} - 8(\mathbf{k} \times (4\mathbf{i} + 8\mathbf{j}))$$

$$= 160\mathbf{i} - 32\mathbf{j} \text{ (in/s)}.$$

Problem 17.166 In Problem 17.165, bar AB rotates with a constant angular acceleration of 6 rad/s in the counterclockwise direction. Determine the acceleration of point D.

Solution: Use the solution to Problem 17.165. The accelerations are determined from the angular velocity, the known accelerations of B, and the constraint on the motion of C. The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 8\mathbf{i}$ (in.). The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{A/B} = 0 - 36(8\mathbf{i}) = -288\mathbf{i} \text{ (in/s}^2\text{)}.$$

From the solution to Problem 17.165, $\omega_{BC} = -8$ rad/s. (clockwise).

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{v}_B = (14\mathbf{i} + 12\mathbf{j}) - (8\mathbf{i}) = 6\mathbf{i} + 12\mathbf{j} \text{ (in.)}.$$

The vector $\mathbf{r}_{B/C} = -\mathbf{r}_{C/B}$. The acceleration of point B in terms of the angular acceleration of point C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{BC} - \omega_{BC}^2 \mathbf{r}_{B/C}$$

$$= \mathbf{a}_C\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -6 & -12 & 0 \end{bmatrix} - 64(-6\mathbf{i} - 12\mathbf{j}).$$

$$\mathbf{a}_B = a_C\mathbf{i} + 12\alpha_{BC}\mathbf{i} - 6\alpha_{BC}\mathbf{j} + 384\mathbf{i} + 768\mathbf{j}.$$

Equate the expressions and separate components:

$$-288 = a_C + 12\alpha_{BC} + 384, \quad 0 = -6\alpha_{BC} + 768.$$

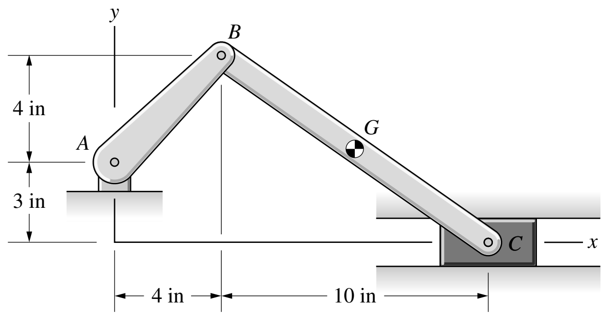
Solve $\alpha_{BC} = 128$ rad/s^2 (counterclockwise), $a_C = -2208$ in/s^2. The acceleration of point D is

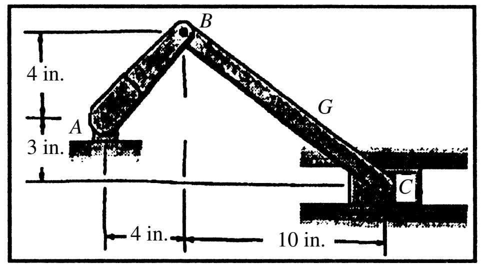
$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{D/C} - \omega_{BC}^2 \mathbf{r}_{D/C}$$

$$= -2208\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 4 & 8 & 0 \end{bmatrix} - \omega_{BC}^2(4\mathbf{i} + 8\mathbf{j}).$$

$$\boxed{\begin{aligned} \mathbf{a}_D &= -2208\mathbf{i} + (128)(-8\mathbf{i} + 4\mathbf{j}) - (64)(4\mathbf{i} + 8\mathbf{j}) \\ &= -3490\mathbf{i} \text{ (in/s}^2\text{)} \end{aligned}}.$$

Problem 17.167 Point C is moving to the right at 20 in./s. What is the velocity of the midpoint G of bar BC?





Solution:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{w}_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 4 & 4 & 0 \end{vmatrix}.$$

Also,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{w}_{BC} \times \mathbf{r}_{B/C} = 20\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & 7 & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in these two expressions, $-4\omega_{AB} = 20 - 7\omega_{BC}$, $4\omega_{AB} = -10\omega_{BC}$, and solving, we obtain $\omega_{AB} = -2.94$ rad/s, $\omega_{BC} = 1.18$ rad/s. Then the velocity of G is

$$\mathbf{v}_G = \mathbf{v}_C + \mathbf{w}_{BC} \times \mathbf{r}_{G/C} = 20\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -5 & 3.5 & 0 \end{vmatrix}$$

$$= \underline{15.88\,\mathbf{i} - 5.88\,\mathbf{j}\ \text{(in/s)}}.$$

Problem 17.168 In Problem 17.167, point C is moving to the right with a constant velocity of 20 in./s. What is the acceleration of the midpoint G of bar BC?

Solution: See the solution of Problem 17.167.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 4 & 4 & 0 \end{vmatrix} - \omega_{AB}^2(4\mathbf{i} + 4\mathbf{j}).$$

Also, $\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -10 & 7 & 0 \end{vmatrix} - \omega_{BC}^2(-10\mathbf{i} + 7\mathbf{j}).$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in these two expressions,

$$-4\alpha_{AB} - 4\omega_{AB}^2 = -7\alpha_{BC} + 10\omega_{BC}^2,$$

$$4\alpha_{AB} - 4\omega_{AB}^2 = -10\alpha_{BC} - 7\omega_{BC}^2,$$

and solving yields $\alpha_{AB} = -4.56$ rad/s^2, $\alpha_{BC} = 4.32$ rad/s^2.

Then $\mathbf{a}_G = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/C} - \omega_{BC}^2 \mathbf{r}_{G/C}$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -5 & 3.5 & 0 \end{vmatrix} - \omega_{BC}^2(-5\mathbf{i} + 3.5\mathbf{j})$$

$$= \underline{-8.18\mathbf{i} - 26.4\mathbf{j}\ \text{(in/s}^2\text{)}}.$$

Problem 17.169 In Problem 17.167, if the velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (in./s), what are the angular velocity vectors of arms AB and BC?

Solution: Use the solution to Problem 17.167: The velocity of the point B is determined from the known velocity of point C and the known velocity of C:

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 1.0\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \boldsymbol{\omega}_{BC} \\ -10 & 7 & 0 \end{bmatrix}$$

$$= 1.0\mathbf{i} - 7\omega_{BC}\mathbf{i} - 10\boldsymbol{\omega}_{BC}\mathbf{j}.$$

The angular velocity of bar AB is determined from the velocity of B.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \boldsymbol{\omega}_{AB} \\ 4 & 4 & 0 \end{bmatrix} = -4\boldsymbol{\omega}_{\mathbf{AB}}(\mathbf{i} - \mathbf{j}) \text{ (in/s)}$$

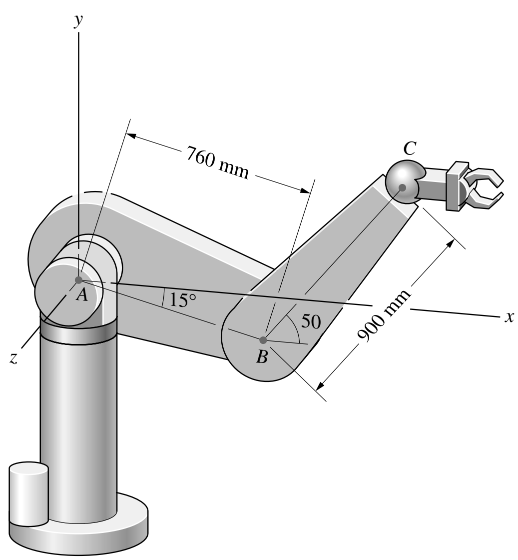
Equate expressions, separate components,

$$1.0 - 7\omega_{BC} = -4\omega_{AB}, -10\omega_{BC} = 4\omega_{AB}.$$

Solve: $\omega_{AB} = -0.147$ rad/s, $\omega_{BC} = 0.0588$ rad/s, from which

$\boxed{\boldsymbol{\omega}_{AB} = -0.147\mathbf{k} \text{ (rad/s)}}$, $\boxed{\boldsymbol{\omega}_{BC} = 0.0588\mathbf{k} \text{ (rad/s)}}$

Problem 17.170 Points B and C are in the x-y plane. The angular velocity vectors of arms AB and BC are $\boldsymbol{\omega}_{AB} = -0.5\mathbf{k}$ (rad/s) and $\boldsymbol{\omega}_{BC} = 2.0\mathbf{k}$ (rad/s). Determine the velocity of point C.



Solution: The vector

$$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j} \text{ (mm)}.$$

The vector

$$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

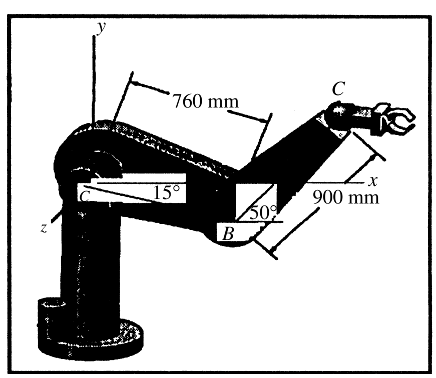
$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -0.5 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = -98.35\mathbf{i} - 367.1\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= -98.35\mathbf{i} - 367.1\mathbf{j} + (2)(\mathbf{k} \times (578.5\mathbf{i} + 689.4\mathbf{j})),$$



$$\boxed{\begin{aligned} \mathbf{v}_C &= -98.35\mathbf{i} - 367.1\mathbf{j} - 1378.9\mathbf{i} + 1157.0\mathbf{j} \\ &= -1477.2\mathbf{i} + 790\mathbf{j} \text{ (mm/s)} \end{aligned}}$$

Problem 17.171 In Problem 17.170, if the velocity vector of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s), what are the angular velocity vectors of arms AB and BC?

Solution: Use the solution to Problem 17.170. The vector

$$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j} \text{ (mm)}.$$

The vector

$$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$= 196.7\boldsymbol{\omega}_{AB}\mathbf{i} + 734.1\boldsymbol{\omega}_{AB}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 196.7\boldsymbol{\omega}_{AB}\mathbf{i} + 734.1\boldsymbol{\omega}_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 578.5 & 687.4 & 0 \end{bmatrix},$$

$$1000\mathbf{i} = 196.7\boldsymbol{\omega}_{AB}\mathbf{i} + 734.1\boldsymbol{\omega}_{AB}\mathbf{j} - 687.4\boldsymbol{\omega}_{BC}\mathbf{i} + 578.5\boldsymbol{\omega}_{BC}\mathbf{j} \text{ (mm/s)}.$$

Separate components:

$$1000 = 196.7\boldsymbol{\omega}_{AB} - 687.4\boldsymbol{\omega}_{BC},\ 0 = 734.1\boldsymbol{\omega}_{AB} + 578.5\boldsymbol{\omega}_{BC}.$$

Solve: $\boxed{\boldsymbol{\omega}_{AB} = 0.933\mathbf{k} \text{ (rad/s)}}$, $\boxed{\boldsymbol{\omega}_{BC} = -1.184\mathbf{k} \text{ (rad/s)}}$

Problem 17.172 In Problem 17.170, if the angular velocity vectors of arms AB and BC are $\boldsymbol{\omega}_{AB} = -0.5\mathbf{k}$ (rad/s) and $\boldsymbol{\omega}_{BC} = 2.0\mathbf{k}$ (rad/s), and their angular accelerations are $\boldsymbol{\alpha}_{AB} = 1.0\mathbf{k}$ (rad/s^2), and $\alpha_{BC} = 1.0\mathbf{k}$ (rad/s^2), what is the acceleration of point C?

Solution: Use the solution to Problem 17.170. The vector

$$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j} \text{ (mm)}.$$

The vector

$$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j} \text{ (mm)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$- (0.5^2)(734.1\mathbf{i} - 196.7\mathbf{j}) \text{ (mm/s}^2).$$

$$\mathbf{a}_B = 196.7\mathbf{i} + 734.1\mathbf{j} - 183.5\mathbf{i} + 49.7\mathbf{j} = 13.2\mathbf{i} + 783.28\mathbf{j} \text{ (mm/s}^2).$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 578.5 & 689.4 & 0 \end{bmatrix} - (2^2)(578.5\mathbf{i} + 689.4\mathbf{j})$$

$$\mathbf{a}_C = 13.2\mathbf{i} + 783.3\mathbf{j} - 689.4\mathbf{i} + 578.5\mathbf{j} - 2314\mathbf{i} - 2757.8\mathbf{j} \text{ (mm/s}^2).$$

$$\boxed{\mathbf{a}_C = -2990\mathbf{i} - 1396\mathbf{j} \text{ (mm/s}^2)}$$

Problem 17.173 In Problem 17.170 if the velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s) and $\mathbf{a}_C = 0$, what are the angular velocity and angular acceleration vectors of arm BC?

Solution: Use the solution to Problem 17.171. The vector $\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j}$ (mm). The vector $\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 578.5 & 689.4 & 0 \end{bmatrix}$$

$$1000\mathbf{i} = 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} - 689.4\omega_{BC}\mathbf{i}$$

$$+ 578.5\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

Separate components:

$$1000 = 196.7\omega_{AB} - 689.4\omega_{BC},$$

$$0 = 734.1\omega_{AB} + 578.5\omega_{BC}.$$

Solve: $\boldsymbol{\omega}_{AB} = 0.933\mathbf{k}$ (rad/s), $\boxed{\boldsymbol{\omega}_{BC} = -1.184\mathbf{k} \text{ (rad/s)}}$.

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$- (\omega_{AB}^2)(734.1\mathbf{i} - 196.7\mathbf{j}) \text{ (mm/s)}^2.$$

$$\mathbf{a}_B = 196.7\alpha_{AB}\mathbf{i} + 734.1\alpha_{AB}\mathbf{j} - 639.0\mathbf{i} + 171.3\mathbf{j} \text{ (mm/s}^2\text{)}$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 578.5 & 689.4 & 0 \end{bmatrix} - (\omega_{BC}^2)(578.5\mathbf{i} + 689.4)$$

$$\mathbf{a}_C = 0 = \alpha_{AB}(196.7\mathbf{i} + 734.1\mathbf{j}) + \alpha_{BC}(-689.4\mathbf{i} + 578.5\mathbf{j})$$

$$- 811.0\mathbf{i} - 966.5\mathbf{j} - 639.0\mathbf{i} + 171.3\mathbf{j} \text{ (mm/s}^2\text{)}$$

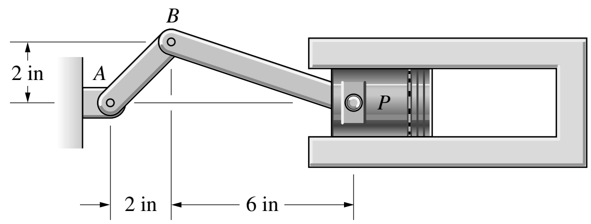
Separate components:

$$196.7\alpha_{AB} - 689.4\alpha_{BC} - 811.3 - 639.0 = 0,$$

$$734.1\alpha_{AB} + 578.5\alpha_{BC} - 966.5 + 171.3 = 0.$$

Solve: $\alpha_{AB} = 2.24$ rad/s^2, $\boxed{\alpha_{BC} = -1.465 \text{ (rad/s}^2\text{)}}$

Problem 17.174 The crank AB has a constant clockwise angular velocity of 200 rpm. What are the velocity and acceleration of the piston P?



Solution:

200 rpm $= 200(2\pi)/60 = 20.9$ rad/s.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -20.9 \\ 2 & 2 & 0 \end{vmatrix}.$$

Also, $\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega}_{BP} \times \mathbf{r}_{B/P} = v_P\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BP} \\ -6 & 2 & 0 \end{vmatrix}.$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in these two expressions,

$-(-20.9)(2) = v_P - 2\omega_{BP}$, $(-20.9)(2) = -6\omega_{BP}$,

we obtain $\omega_{BP} = 6.98$ rad/s and $\underline{v_P = 55.9 \text{ in/s}}$.

$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$

$= \mathbf{0} + \mathbf{0} - (-20.9)^2(2\mathbf{i} + 2\mathbf{j})$.

Also, $\mathbf{a}_B = \mathbf{a}_P + \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{B/P} - \omega_{BP}^2\mathbf{r}_{B/P}$

$$= a_P\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BP} \\ -6 & 2 & 0 \end{vmatrix} - \omega_{BP}^2(-6\mathbf{i} + 2\mathbf{j}).$$

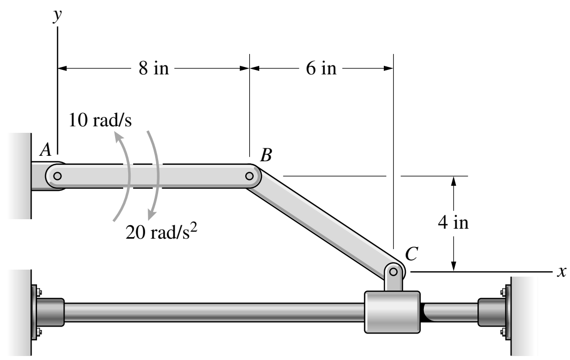
Equating $\mathbf{i}$ and $\mathbf{j}$ components,

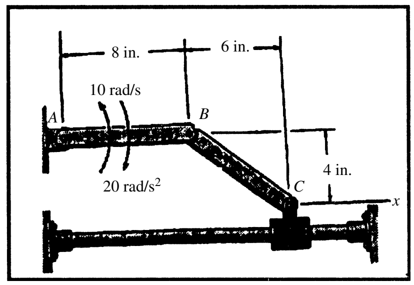
$-2(20.9)^2 = a_P - 2\alpha_{BP} + 6\omega_{BP}^2$,

$-2(20.9)^2 = -6\alpha_{BP} - 2\omega_{BP}^2$,

and solving, we obtain $\underline{a_P = -910 \text{ in/s}^2}$.

Problem 17.175 Bar AB has a counterclockwise angular velocity of 10 rad/s and a clockwise angular acceleration of 20 rad/s^2. Determine the angular acceleration of bar BC and the acceleration of point C.





Solution: Choose a coordinate system with the origin at the left end of the horizontal rod and the x axis parallel to the horizontal rod. The strategy is to determine the angular velocity of bar BC from the instantaneous center; using the angular velocity and the constraint on the motion of C, the accelerations are determined.

The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (8\mathbf{i} + 4\mathbf{j}) - 4\mathbf{j} = 8\mathbf{i}$ (in.).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix} = 80\mathbf{j} \text{ (in/s).}$$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -20 \\ 8 & 0 & 0 \end{bmatrix} - 100(8\mathbf{i})$$

$$= -800\mathbf{i} - 160\mathbf{j} \text{ (in/s}^2\text{).}$$

The velocity of point B is normal to the x axis, and the velocity of C is parallel to the x axis. The line perpendicular to the velocity at B is parallel to the x-axis, and the line perpendicular to the velocity at C is parallel to the y axis. The intercept is at (14, 4), which is the instantaneous center of bar BC. Denote the instantaneous center by C''.

The vector $\mathbf{r}_{B/C''} = \mathbf{r}_B - \mathbf{r}_{C''} = (8\mathbf{i} + 4\mathbf{j}) - (14\mathbf{i} - 4\mathbf{j}) = -6\mathbf{i}$ (in.).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -6 & 0 & 0 \end{bmatrix} = -6\omega_{BC}\mathbf{j} = 80\mathbf{j},$$

from which $\omega_{BC} = -\dfrac{80}{6} = -13.33$ rad/s.

The vector $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = (14\mathbf{i}) - (8\mathbf{i} + 4\mathbf{j}) = 6\mathbf{i} - 4\mathbf{j}$ (in.).

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -800\mathbf{i} - 160\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 6 & -4 & 0 \end{bmatrix} - 1066.7\mathbf{i} + 711.1\mathbf{j} \text{ (in/s}^2\text{).}$$

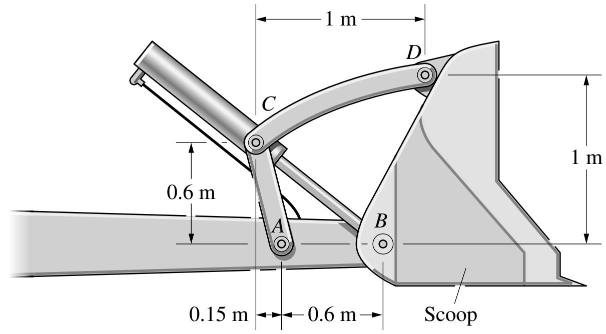
The acceleration of point C is constrained to be parallel to the x axis. Separate components:

$$a_C = -800 + 4\alpha_{BC} - 1066.7, \quad 0 = -160 + 6\alpha_{BC} + 711.1.$$

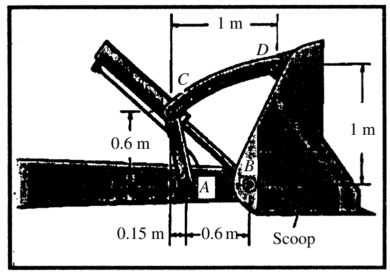
Solve:

$\boxed{\mathbf{a}_C = -2234\mathbf{i} \text{ (in/s}^2\text{)}}$, $\boxed{\alpha_{BC} = -91.9 \text{ rad/s}^2}$ (clockwise).

Problem 17.176 The angular velocity of arm AC is 1 rad/s counterclockwise. What is the angular velocity of the scoop?



Solution: Choose a coordinate system with the origin at A and the y axis vertical. The vector locations of B, C and D are $\mathbf{r}_B = 0.6\mathbf{i}$ (m), $\mathbf{r}_C = -0.15\mathbf{i} + 0.6\mathbf{j}$ (m), $\mathbf{r}_D = (1 - 0.15)\mathbf{i} + 1\mathbf{j} = 0.85\mathbf{i} + \mathbf{j}$ (m), from which $\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = 1\mathbf{i} + 0.4\mathbf{j}$ (m), and $\mathbf{r}_{D/B} = \mathbf{r}_D - \mathbf{r}_B = 0.25\mathbf{i} + \mathbf{j}$ (m). The velocity of point C is



$$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix}$$

$$= -0.6\mathbf{i} - 0.15\mathbf{j} \text{ (m/s)}.$$

The velocity of D in terms of the velocity of C is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -0.6\mathbf{i} - 0.15\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 1 & 0.4 & 0 \end{bmatrix}$$

$$= -0.6\mathbf{i} - 0.15\mathbf{j} + \omega_{CD}(-0.4\mathbf{i} + \mathbf{j}).$$

The velocity of point D in terms of the angular velocity of the scoop is

$$\mathbf{v}_D = \boldsymbol{\omega}_{DB} \times \mathbf{r}_{D/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DB} \\ 0.25 & 1 & 0 \end{bmatrix} = \omega_{DB}(-\mathbf{i} + 0.25\mathbf{j}).$$

Equate expressions and separate components:

$$-0.6 - 0.4\omega_{CD} = -\omega_{DB}, \; -0.15 + \omega_{CD} = 0.25\omega_{DB}.$$

Solve:

$\omega_{CD} = 0.333$ rad/s, $\boxed{\omega_{DB} = 0.733 \text{ rad/s}}$ (counterclockwise).

Problem 17.177 The angular velocity of arm AC in Problem 17.176 is 2 rad/s counterclockwise, and its angular acceleration is 4 rad/s^2 clockwise. What is the angular acceleration of the scoop?

Solution: Use the solution to Problem 17.176. Choose a coordinate system with the origin at A and the y axis vertical. The vector locations of B, C and D are $\mathbf{r}_B = 0.6\mathbf{i}$ (m), $\mathbf{r}_C = -0.15\mathbf{i} + 0.6\mathbf{j}$ (m), $\mathbf{r}_D = (1 - 0.15)\mathbf{i} + 1\mathbf{j} = 0.85\mathbf{i} + \mathbf{j}$ (m), from which $\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = 1\mathbf{i} + 0.4\mathbf{j}$ (m), and $\mathbf{r}_{D/B} = \mathbf{r}_D - \mathbf{r}_B = 0.25\mathbf{i} + \mathbf{j}$ (m). The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix} = -1.2\mathbf{i} - 0.3\mathbf{j} \text{ (m/s)}.$$

The velocity of D in terms of the velocity of C is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -1.2\mathbf{i} - 0.3\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 1 & 0.4 & 0 \end{bmatrix}$$

$$= -1.2\mathbf{i} - 0.3\mathbf{j} + \omega_{CD}(-0.4\mathbf{i} + \mathbf{j}).$$

The velocity of point D in terms of the angular velocity of the scoop is

$$\mathbf{v}_D = \boldsymbol{\omega}_{DB} \times \mathbf{r}_{D/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DB} \\ 0.25 & 1 & 0 \end{bmatrix} = \omega_{DB}(-\mathbf{i} + 0.25\mathbf{j}).$$

Equate expressions and separate components:

$$-1.2 - 0.4\omega_{CD} = -\omega_{DB}, -0.3 + \omega_{CD} = 0.25\omega_{DB}.$$

Solve: $\omega_{CD} = 0.667$ rad/s, $\omega_{DB} = 1.47$ rad/s. The angular acceleration of the point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} - \omega_{AC}^2 \mathbf{r}_{C/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \boldsymbol{\alpha}_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix} - \omega_{AC}^2(-0.15\mathbf{i} + 0.6\mathbf{j}),$$

$$\mathbf{a}_C = 2.4\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{i} - 2.4\mathbf{j} = 3\mathbf{i} - 1.8\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point D in terms of the acceleration of point C is

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2 \mathbf{r}_{D/C}$$

$$= 3\mathbf{i} - 1.8\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 1 & 0.4 & 0 \end{bmatrix} - \omega_{CD}^2(\mathbf{i} + 0.4\mathbf{j}).$$

$$\mathbf{a}_C = \alpha_{CD}(-0.4\mathbf{i} + \mathbf{j}) + 2.56\mathbf{i} - 1.98\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point D in terms of the angular acceleration of point B is

$$\mathbf{a}_D = \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 0.25 & 1 & 0 \end{bmatrix} - \omega_{BD}^2(0.25\mathbf{i} + \mathbf{j}).$$

$$\mathbf{a}_D = \alpha_{BD}(-\mathbf{i} + 0.25\mathbf{j}) - 0.538\mathbf{i} - 2.15\mathbf{j}.$$

Equate expressions for $\mathbf{a}_D$ and separate components:

$$-0.4\alpha_{CD} + 2.56 = -\alpha_{BD} - 0.538,$$

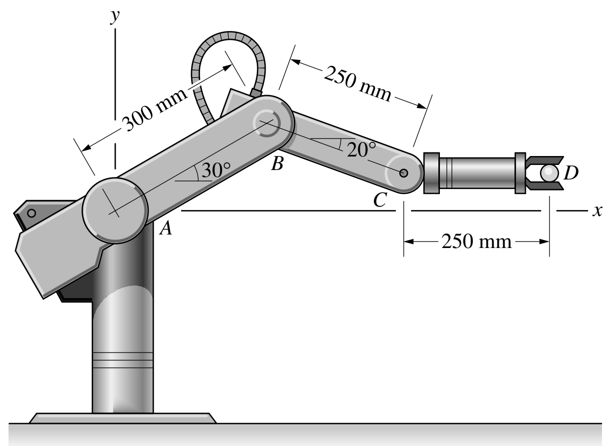
$$\alpha_{CD} - 1.98 = 0.25\alpha_{BD} - 2.15.$$

Solve:

$$\alpha_{CD} = -1.052 \text{ rad/s}^2, \boxed{\alpha_{BD} = -3.51 \text{ rad/s}^2},$$

where the negative sign means a clockwise acceleration.

Problem 17.178 If you want to program the robot so that, at the instant shown, the velocity of point D is $\mathbf{v}_D = 0.2\mathbf{i} + 0.8\mathbf{j}$ (m/s) and the angular velocity of arm CD is 0.3 rad/s counterclockwise, what are the necessary angular velocities of arms AB and BC?

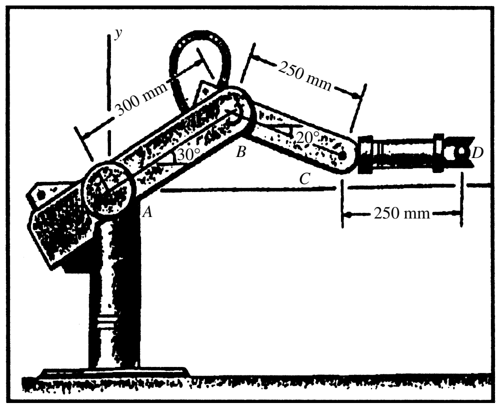


Solution: The position vectors are:

$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 259.8\mathbf{i} + 150\mathbf{j}$ (mm),

$\mathbf{r}_{C/B} = 250(\mathbf{i}\cos 20^\circ - \mathbf{j}\sin 20^\circ) = 234.9\mathbf{i} - 85.5\mathbf{j}$ (mm),

$\mathbf{r}_{C/D} = -250\mathbf{i}$ (mm).



The velocity of the point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 259.8 & 150 & 0 \end{bmatrix}$$

$\mathbf{v}_B = -150\omega_{AB}\mathbf{i} + 259.8\omega_{AB}\mathbf{j}$.

The velocity of point C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 234.9 & -85.5 & 0 \end{bmatrix}$$

$$= -150\omega_{AB}\mathbf{i} + 259.8\omega_{AB}\mathbf{j} + 85.5\omega_{BC}\mathbf{i} + 234.9\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C in terms of the velocity of point D is

$$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = 200\mathbf{i} + 800\mathbf{j} + 0.3 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -250 & 0 & 0 \end{bmatrix}$$

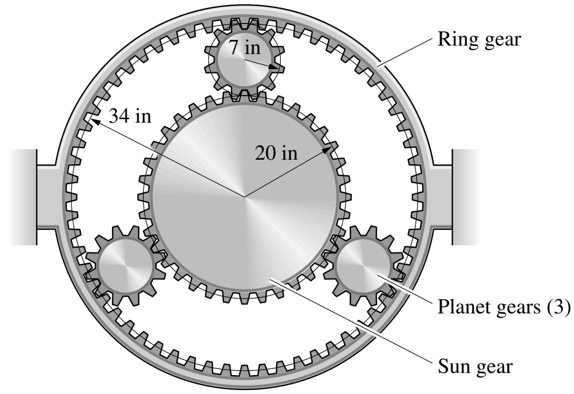
$$= 200\mathbf{i} + 725\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components:

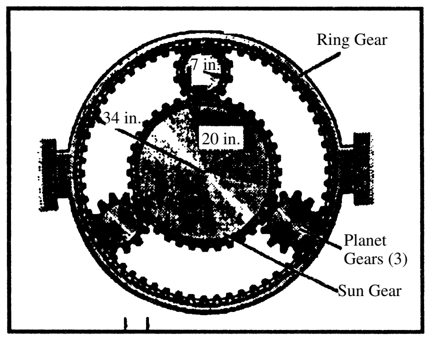
$-150\omega_{AB} + 85.5\omega_{BC} = 200$, and $259.8\omega_{AB} + 234.9\omega_{BC} = 725$.

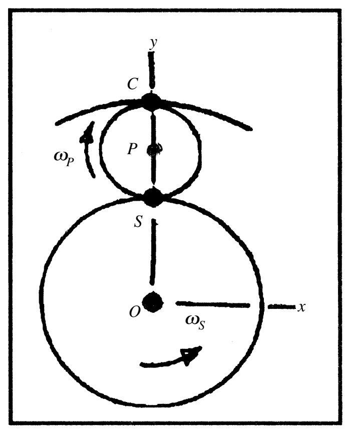
Solve: $\boxed{\omega_{AB} = 0.261 \text{ rad/s}}$, $\boxed{\omega_{BC} = 2.80 \text{ rad/s}}$.

Problem 17.179 The ring gear is stationary, and the sun gear rotates at 120 rpm (revolutions per minute) in the counterclockwise direction. Determine the angular velocity of the planet gears and the magnitude of the velocity of their centerpoints.



Solution: Denote the point O be the center of the sun gear, point S to be the point of contact between the upper planet gear and the sun gear, point P be the center of the upper planet gear, and point C be the point of contact between the upper planet gear and the ring gear. The angular velocity of the sun gear is





$$\omega_S = \frac{120(2\pi)}{60} = 4\pi \text{ rad/s},$$

from which $\boldsymbol{\omega}_S = 4\pi\mathbf{k}$ (rad/s). At the point of contact between the sun gear and the upper planet gear the velocities are equal. The vectors are: from center of sun gear to S is $\mathbf{r}_{P/S} = 20\mathbf{j}$ (in.), and from center of planet gear to S is $\mathbf{r}_{S/P} = -7\mathbf{j}$ (in.). The velocities are:

$$\mathbf{v}_{S/O} = \mathbf{v}_O + \boldsymbol{\omega}_S \times (20\mathbf{j}) = 0 + \omega_S(20)(\mathbf{k} \times \mathbf{j})$$

$$\mathbf{v}_{S/O} = 20\omega_S \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = -20(\omega_S)$$

$$\mathbf{i} = -251.3\mathbf{i} \text{ (in/s)}.$$

From equality of the velocities, $\mathbf{v}_{S/P} = \mathbf{v}_{S/O} = -251.3\mathbf{i}$ (in/s). The point of contact C between the upper planet gear and the ring gear is stationary, from which

$$\mathbf{v}_{S/P} = -251.3\mathbf{i} = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{C/S}$$

$$= 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_P \\ -14 & 0 & 0 \end{bmatrix} = 14\omega_P\mathbf{i} = -251.3\mathbf{i}$$

from which $\boxed{\omega_P = 17.95 \text{ rad/s.}}$

The velocity of the centerpoint of the top most planet gear is

$$\mathbf{v}_P = \mathbf{v}_{S/P} + \boldsymbol{\omega}_P \times \mathbf{r}_{P/S} = -251.3\mathbf{i} + (-17.95)(-7)(\mathbf{k} \times \mathbf{j})$$

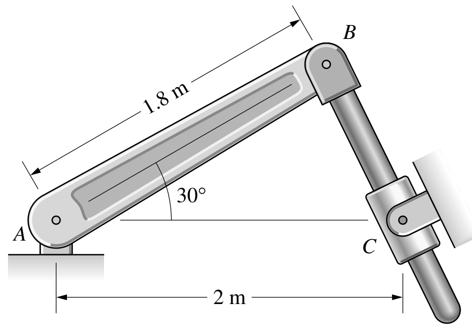
$$= -251.3\mathbf{i} + 125.65 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

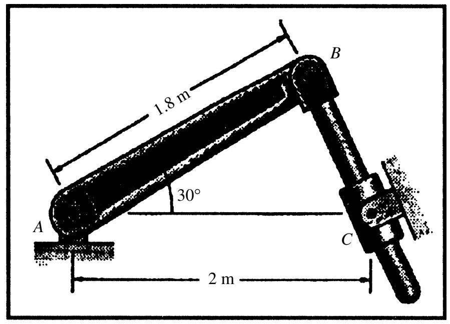
$$\mathbf{v}_P = -125.7\mathbf{i} \text{ (in./s)}$$

The magnitude is $\boxed{v_{PO} = 125.7 \text{ in./s}}$

By symmetry, the magnitudes of the velocities of the centerpoints of the other planetary gears is the same.

Problem 17.180 Arm AB is rotating at 10 rad/s in the clockwise direction. Determine the angular velocity of the arm BC and the velocity at which is slides relative to the sleeve at C.





Solution: The position vectors are

$$\mathbf{r}_{B/A} = 1.8(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.56\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

$$\mathbf{r}_{B/C} = \mathbf{r}_{B/A} - 2\mathbf{i} = -0.441\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ 1.56 & 0.9 & 0 \end{bmatrix} = 9\mathbf{i} - 15.6\mathbf{j} \text{ (m/s)}$$

The unit vector from B to C is

$$\mathbf{e}_{BC} = \frac{-\mathbf{r}_{B/C}}{|\mathbf{r}_{B/C}|} = 0.4401\mathbf{i} - 0.8979\mathbf{j}.$$

The relative velocity is parallel to this vector:

$$\mathbf{v}_{C\text{rel}} = v_{C\text{rel}}\mathbf{e}_{BC} = v_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j})\text{(m/s)}$$

The velocity of B in terms of the velocity of C is

$$\mathbf{v}_B = \mathbf{v}_{\text{rel}} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \mathbf{v}_{\text{rel}} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -0.441 & 0.9 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = 0.4401 v_{C\text{rel}}\mathbf{i} - 0.8979 v_{C\text{rel}}\mathbf{j} - 0.9\omega_{BC}\mathbf{i} - 0.441\omega_{BC}\mathbf{j} \text{ (m/s)}.$$

Equate the expressions for $\mathbf{v}_B$ and separate components:

$$9 = 0.4401 v_{C\text{rel}} - 0.9\omega_{BC}, \text{ and}$$

$$-15.6 = -0.8979 v_{C\text{rel}} - 0.441\omega_{BC}.$$

Solve:

$\boxed{v_{C\text{rel}} = 17.96 \text{ m/s}}$ (toward C).

$\boxed{\omega_{BC} = -1.22 \text{ rad/s}}$ (clockwise)

Problem 17.181 In Problem 17.180, arm AB is rotating with an angular velocity of 10 rad/s and an angular acceleration of 20 rad/s^2, both in the clockwise direction. Determine the angular acceleration of arm BC.

Solution: Use the solution to 17.180. The vector

$$\mathbf{r}_{B/A} = 1.8(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.56\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

$$\mathbf{r}_{B/C} = \mathbf{r}_{B/A} - 2\mathbf{i} = -0.441\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

The angular velocity:

$$\omega_{BC} = -1.22 \text{ rad/s},$$

and the relative velocity is $v_{C\text{rel}} = 17.96$ m/s.

The unit vector parallel to bar BC is $\mathbf{e} = 0.4401\mathbf{i} - 0.8979\mathbf{j}$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 1.56 & 0.9 & 0 \end{bmatrix} - \omega_{AB}^2(1.56\mathbf{i} + 0.9\mathbf{j}),$$

$$\mathbf{a}_B = 18\mathbf{i} - 31.2\mathbf{j} - 155.9\mathbf{i} - 90\mathbf{j} = -137.9\mathbf{i} - 121.2\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point B in terms of the acceleration of bar BC is

$$\mathbf{a}_B = \mathbf{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}.$$

Expanding term by term:

$$\mathbf{a}_{C\text{rel}} = a_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j}) \text{ (m/s}^2),$$

$$2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} = 2v_{C\text{rel}}\omega_{BC} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0.440 & -0.8979 & 0 \end{bmatrix}$$

$$= -39.26\mathbf{i} - 19.25\mathbf{j} \text{ (m/s}^2),$$

$$\boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -0.4411 & 0.9 & 0 \end{bmatrix}$$

$$= \alpha_{BC}(-0.9\mathbf{i} - 0.4411\mathbf{j}) \text{ (m/s}^2)$$

$$- \omega_{BC}^2(-0.4411\mathbf{i} + 0.9\mathbf{j})$$

$$= 0.6539\mathbf{i} - 1.334\mathbf{j} \text{ (m/s}^2).$$

Collecting terms,

$$\mathbf{a}_B = a_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j}) - \alpha_{BC}(0.9\mathbf{i} + 0.4411\mathbf{j})$$

$$- 38.6\mathbf{i} - 20.6\mathbf{j} \text{ (m/s}^2).$$

Equate the two expressions for $\mathbf{a}_B$ and separate components:

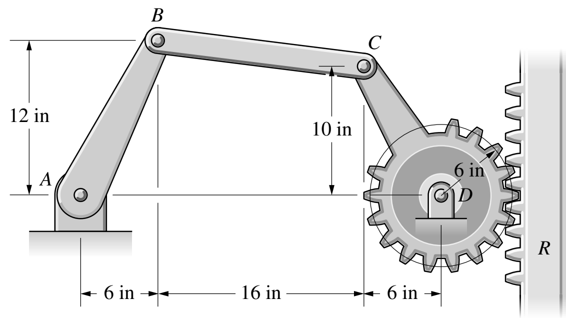
$$-137.9 = 0.4401a_{C\text{rel}} - 0.9\alpha_{BC} - 38.6,$$

and $-121.2 = -0.8979a_{C\text{rel}} - 0.4411\alpha_{BC} - 20.6.$

Solve: $a_{C\text{rel}} = 46.6$ m/s^2 (toward C)

$$\boxed{\alpha_{BC} = 133.1 \text{ rad/s}^2}$$

Problem 17.182 Arm AB is rotating with a constant counterclockwise angular velocity of 10 rad/s. Determine the vertical velocity and acceleration of the rack R of the rack and pinion gear.



Solution: The vectors:

$\mathbf{r}_{B/A} = 6\mathbf{i} + 12\mathbf{j}$ (in.). $\mathbf{r}_{C/B} = 16\mathbf{i} - 2\mathbf{j}$ (in.).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 6 & 12 & 0 \end{bmatrix} = -120\mathbf{i} + 60\mathbf{j} \text{ (in/s).}$$

The velocity of point C in terms of the velocity of point B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 16 & -2 & 0 \end{bmatrix}$$

$$= -120\mathbf{i} + 60\mathbf{j} + \omega_{BC}(2\mathbf{i} + 16\mathbf{j}) \text{ (in/s)}$$

The velocity of point C in terms of the velocity of the gear arm CD is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -6 & 10 & 0 \end{bmatrix}$$

$$= -10\omega_{CD}\mathbf{i} - 6\omega_{CD}\mathbf{j} \text{ (in/s).}$$

Equate the two expressions for $\mathbf{v}_C$ and separate components:

$$-120 + 2\omega_{BC} = -10\omega_{CD}, \quad 60 + 16\omega_{BC} = -6\omega_{CD}.$$

Solve: $\omega_{BC} = -8.92$ rad/s, $\omega_{CD} = 13.78$ rad/s,

where the negative sign means a clockwise rotation. The velocity of the rack is

$$\mathbf{v}_R = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{R/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 6 & 0 & 0 \end{bmatrix} = 6\omega_{CD}\mathbf{j},$$

$$\boxed{\mathbf{v}_R = 82.7\mathbf{j} \text{ (in/s)} = 6.89\mathbf{j} \text{ (ft/s)}}$$

The angular acceleration of point B is

$$\mathbf{a}_B = -\omega_{AB}^2\mathbf{r}_{B/A} = -100(6\mathbf{i} + 12\mathbf{j}) = -600\mathbf{i} - 1200\mathbf{j} \text{ (in/s}^2\text{).}$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B},$$

$$\mathbf{a}_C = \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 16 & -2 & 0 \end{bmatrix} - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j})$$

$$= \mathbf{a}_B + 2\alpha_{BC}\mathbf{i} + 16\alpha_{BC}\mathbf{j} - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j}).$$

Noting

$$\mathbf{a}_B - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j}) = -600\mathbf{i} - 1200\mathbf{j} - 1272.7\mathbf{i} + 159.1\mathbf{j}$$

$$= -1872.7\mathbf{i} - 1040.9\mathbf{j},$$

from which $\mathbf{a}_C = +\alpha_{BC}(2\mathbf{i} + 16\mathbf{j}) - 1873\mathbf{i} - 1041\mathbf{j}$ (in/s^2)

The acceleration of point C in terms of the gear arm is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -6 & 10 & 0 \end{bmatrix} - \omega_{CD}^2(-6\mathbf{i} + 10\mathbf{j}) \text{ (in/s}^2\text{),}$$

$$\mathbf{a}_C = -10\alpha_{CD}\mathbf{i} - 6\alpha_{CD}\mathbf{j} + 1140\mathbf{i} - 1900\mathbf{j} \text{ (in/s}^2\text{).}$$

Equate expressions for $\mathbf{a}_C$ and separate components:

$$2\alpha_{BC} - 1873 = -10\alpha_{CD} + 1140,$$

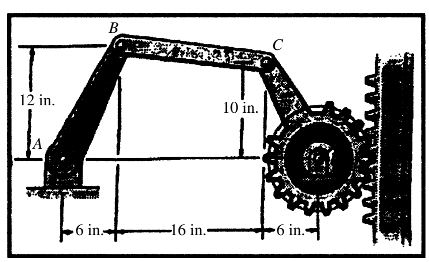
$$16\alpha_{BC} - 1041 = -6\alpha_{CD} - 1900.$$

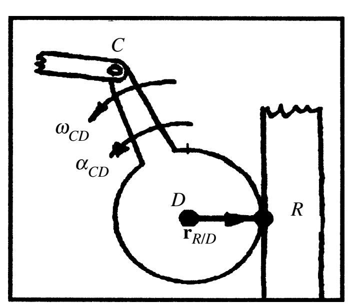
Solve: $\alpha_{CD} = 337.3$ rad/s^2, and $\alpha_{BC} = -180.2$ rad/s^2.

The acceleration of the rack R is the tangential component of the acceleration of the gear at the point of contact with the rack:

$$\mathbf{a}_R = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{R/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 6 & 0 & 0 \end{bmatrix} = 6\alpha_{CD}\mathbf{j} \text{ (in/s}^2\text{).}$$

$$\boxed{\mathbf{a}_R = 2024\mathbf{j} \text{ (in/s}^2\text{)} = 169\mathbf{j} \text{ (ft/s}^2\text{)}}$$





Problem 17.183 In Problem 17.182, if the rack R of the rack-and-pinion gear is moving upward with a constant velocity of 10 ft/s, what are the angular velocity and acceleration of bar BC?

Solution: The constant velocity of the rack R implies that the angular acceleration of the gear is zero, and the angular velocity of the gear is $\omega_{CD} = \dfrac{120}{6} = 20$ rad/s. The velocity of point C in terms of the gear angular velocity is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 20 \\ -6 & 10 & 0 \end{bmatrix} = -200\mathbf{i} - 120\mathbf{j} \text{ (in/s)}.$$

The velocity of point B in terms of the velocity of point C is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \mathbf{v}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -16 & 2 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = -200\mathbf{i} - 120\mathbf{j} - 2\omega_{BC}\mathbf{i} - 16\omega_{BC}\mathbf{j} \text{ (in/s)}.$$

The velocity of point B in terms of the angular velocity of the arm AB is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 6 & 12 & 0 \end{bmatrix}$$

$$= -12\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} \text{ (in/s)}.$$

Equate the expressions for $\mathbf{v}_B$ and separate components

$$-200 - 2\omega_{BC} = -12\omega_{AB}, -120 - 16\omega_{BC} = 6\omega_{AB}.$$

Solve: $\omega_{AB} = 14.5$ rad/s, $\boxed{\omega_{BC} = -12.94 \text{ rad/s}}$, where the negative sign means a clockwise rotation. The angular acceleration of the point C in terms of the angular velocity of the gear is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D} = 0 - \omega_{CD}^2(-6\mathbf{i} + 10\mathbf{j})$$

$$= 2400\mathbf{i} - 4000\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point B in terms of the acceleration of C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -16 & 2 & 0 \end{bmatrix} - \omega_{BC}^2(-16\mathbf{i} + 2\mathbf{j}).$$

$$\mathbf{a}_B = \alpha_{BC}(-2\mathbf{i} - 16\mathbf{j}) + 2400\mathbf{i} - 4000\mathbf{j} + 2680\mathbf{i} - 335\mathbf{j}.$$

$$\mathbf{a}_B = -2\alpha_{BC}\mathbf{i} - 16\alpha_{BC}\mathbf{j} + 5080\mathbf{i} - 433.5\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point B in terms of the angular acceleration of arm AB is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \alpha_{AB}(\mathbf{k} \times (6\mathbf{i} + 12\mathbf{j})) - \omega_{AB}^2(6\mathbf{i} + 12\mathbf{j}) \text{ (in/s}^2\text{)}$$

$$\mathbf{a}_B = \alpha_{AB}(-12\mathbf{i} + 6\mathbf{j}) - 1263.2\mathbf{i} - 2526.4\mathbf{j} \text{ (in/s}^2\text{)}.$$

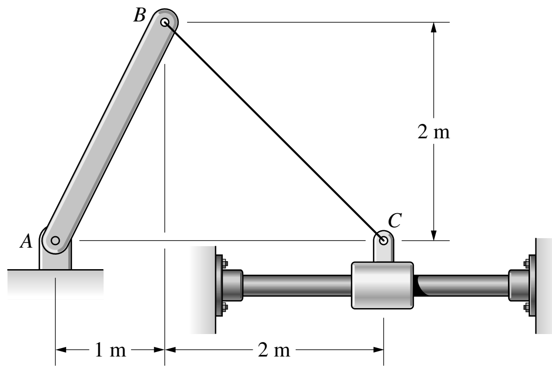
Equate the expressions for $\mathbf{a}_B$ and separate components:

$$-2\alpha_{BC} + 5080 = -12\alpha_{AB} - 1263.2,$$

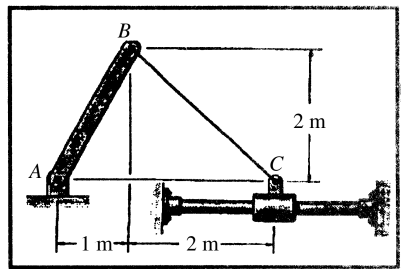
$$-16\alpha_{BC} - 4335 = 6\alpha_{AB} - 2526.4.$$

Solve: $\alpha_{AB} = -515.2$ rad/s^2, $\boxed{\alpha_{BC} = 80.17 \text{ rad/s}^2}$

Problem 17.184 Bar AB has a constant counterclockwise angular velocity of 2 rad/s. The 1-kg collar C slides on the smooth horizontal bar. At the instant shown, what is the tension in the cable BC?



Solution:



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{w}_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -4\mathbf{i} + 2\mathbf{j} \text{ (m/s)}.$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{w}_{BC} \times \mathbf{r}_{C/B} :$$

$$v_C\mathbf{i} = -4\mathbf{i} + 2\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2 & -2 & 0 \end{vmatrix}.$$

From the **i** and **j** components of this equation,

$$v_C = -4 + 2\omega_{BC},$$

$$0 = 2 + 2\omega_{BC},$$

we obtain $\omega_{BC} = -1$ rad/s.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \mathbf{0} + \mathbf{0} - (2)^2(\mathbf{i} + 2\mathbf{j})$$

$$= -4\mathbf{i} - 8\mathbf{j} \text{ (m/s}^2\text{)}.$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B} :$$

$$a_C\mathbf{i} = -4\mathbf{i} - 8\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 2 & -2 & 0 \end{vmatrix} - (-1)^2(2\mathbf{i} - 2\mathbf{j}).$$

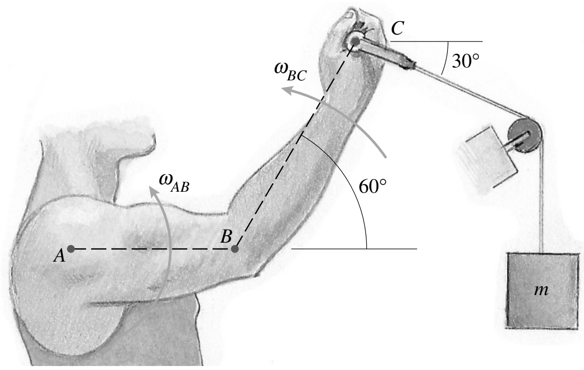
From the **i** and **j** components of this equation,

$$a_C = -4 + 2\alpha_{BC} - 2,$$

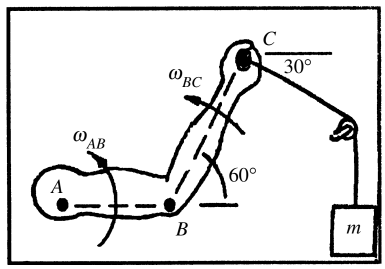
$$0 = -8 + 2\alpha_{BC} + 2,$$

we obtain $a_C = 0$. The force exerted on the collar at this instant is zero, so $\underline{T_{BC} = 0}$.

Problem 17.185 An athlete exercises his arm by raising the 8-kg mass m. The shoulder joint A is stationary. The distance AB is 300 mm, the distance BC is 400 mm, and the distance from C to the pulley is 340 mm. The angular velocities $\omega_{AB} = 1.5$ rad/s and $\omega_{BC} = 2$ rad/s are constant. What is the tension in the cable?



Solution:



$$\mathbf{a}_B = -\omega_{AB}^2 r_{B/A}\mathbf{i} = -(1.5)^2(0.3)\mathbf{i}$$

$$= -0.675\mathbf{i} \text{ (m/s}^2).$$

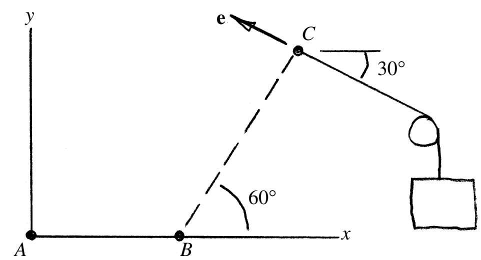
$$\mathbf{a}_C = \mathbf{a}_B - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= -0.675\mathbf{i} - (2)^2(0.4\cos 60^\circ\mathbf{i} + 0.4\sin 60^\circ\mathbf{j})$$

$$= -1.475\mathbf{i} - 1.386\mathbf{j} \text{(m/s}^2).$$

$$\mathbf{a}_C \cdot \mathbf{e} = (-1.475)(-\cos 30^\circ) + (-1.386)(\sin 30^\circ)$$

$$= 0.585 \text{ m/s}^2.$$



This is the upward acceleration of the mass, so

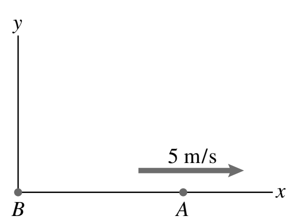
$$T - mg = m(0.585),$$

$$T = (8)(9.81 + 0.585)$$

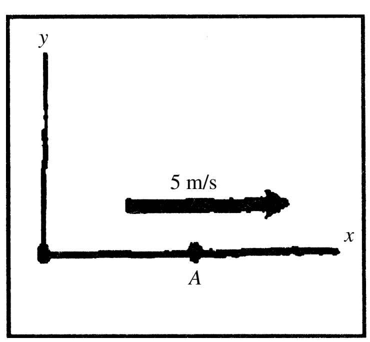
$$= 83.2 \text{ N}.$$

Problem 17.186 The secondary reference frame shown rotates with a constant angular velocity $\boldsymbol{\omega} = 2\mathbf{k}$ (rad/s) relative to a primary reference frame. The point A moves outward along the x axis at a constant rate of 5 m/s.

(a) What are the velocity and acceleration of A relative to the secondary reference frame?
(b) If the origin B is stationary relative to the primary reference frame, what are the velocity and acceleration of A relative to the primary reference frame when A is at the position $x = 1$ m?



Solution:



(a) $\mathbf{v}_{A\text{rel}} = 5\mathbf{i}$ (m/s), $\quad \mathbf{a}_{A\text{rel}} = 0;$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B} = 0 + 5\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{vmatrix}, \quad \text{or}$$

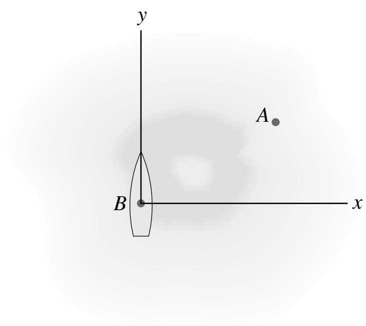
$$\mathbf{v}_A = 5\mathbf{i} + 2\mathbf{j} \text{ (m/s)}$$

(b) $\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$

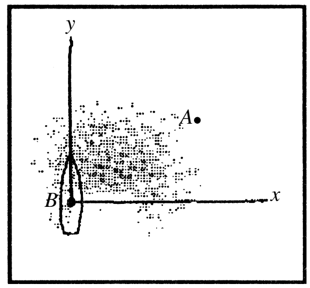
$$\mathbf{a}_A = 0 + 0 + 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 5 & 0 & 0 \end{vmatrix} + 0 - (2)^2(\mathbf{i})$$

$$= +20\mathbf{j} \text{ (m/s}^2) + 0 - (2)^2(\mathbf{i}) = -4\mathbf{i} + 20\mathbf{j} \text{ (m/s}^2)$$

Problem 17.187 The coordinate system shown is fixed relative to the ship B. The ship uses its radar to measure the position of a stationary buoy A and determines it to be $400\mathbf{i} + 200\mathbf{j}$ (m). The ship also measures the velocity of the buoy relative to its body-fixed coordinate system and determines it to be $2\mathbf{i} - 8\mathbf{j}$ (m/s). What are the ship's velocity and angular velocity relative to the earth? (Assume that the ship's velocity is in the direction of the y axis).



Solution:



$$\mathbf{v}_A = 0 = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= v_B\mathbf{j} + 2\mathbf{i} - 8\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 400 & 200 & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components to zero, $0 = 2 - 200\omega$ $0 = v_B - 8 + 400\omega$ we obtain $\omega = 0.01$ rad/s and $v_B = 4$ m/s.