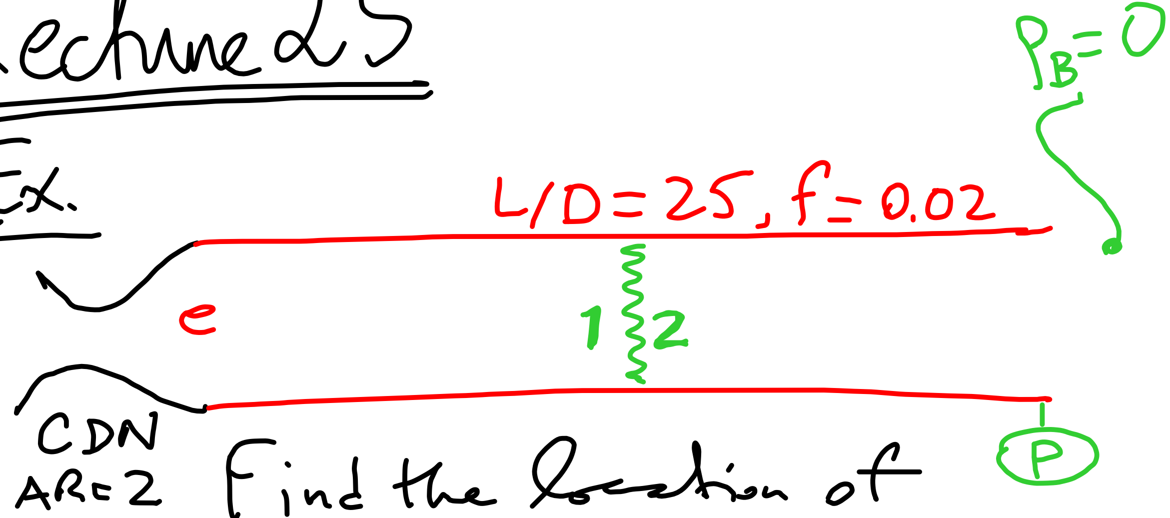
Lecture 25

Ex.



Find the location of the NSW & P_p when $P_B = 0$!

$$M_e = 2.2 \quad \text{for} \quad AR = 2$$

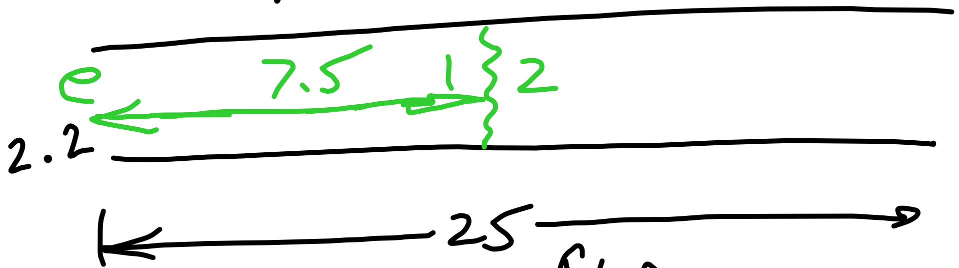
$$M_p = 1.0 \quad (\text{since } P_B = 0 \text{ \& } M_e \text{ can not be greater than } 1)$$

$$\left.\frac{fL}{D}\right|_{actual} = (0.02)(25) = 0.5 > \left.\frac{fL_{max}}{D}\right|_{M=2.2} = 0.36$$

Solution involves Trial & error:

$\boxed{M_p = 1}$ ⇒ ⇓ ⇐ Tables

1. Guess a shock wave location
2. Check $M_p \overset{?}{=} 1$
3. If not satisfied, guess again & repeat the steps.



$$L/D|_1 = 7.5 \Rightarrow \frac{fL}{D}\Big|_1 = (0.02)(7.5) = 0.15$$

$$\left.\frac{fL_{max}}{D}\right|_1 = 0.36 - 0.15 = 0.21 \Rightarrow \underline{\underline{M_1 = 1.71}}$$

sup.

$M_2 \simeq 0.639$ sub. (NS table)

$$\left.\frac{fL_{max}}{D}\right|_2 = \underline{\underline{0.3564}}$$

$$\Rightarrow \left.\frac{fL}{D}\right|_{2-p} = 0.02\,(25-7.5) = \underline{\underline{0.35}}$$

$\Rightarrow$ Best Guess $\Rightarrow$ $\left.L/D\right|_1 = 7.7 \Rightarrow M_1 = 1.695$

$M_2 = 0.642$

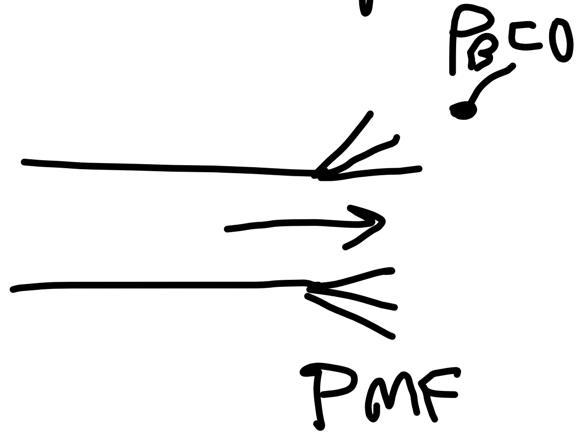
$$P_p = P^* = \frac{P^*}{P_2}\,\frac{P_2}{P_1}\,\frac{P_1}{P^{*+}}\,\frac{P^{*+}}{P_e}\,\frac{P_e}{P_0}\,P_0$$

NSW

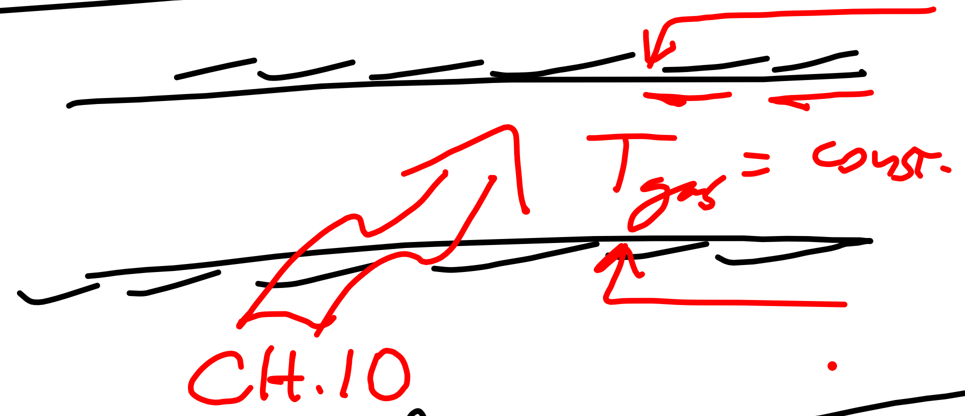
T.F.

isentropic

$$\Rightarrow P_p = 132.2\ \text{kPa}$$



9.4 Isothermal Flow



⇒ Derivation ⇒ Fanno flow

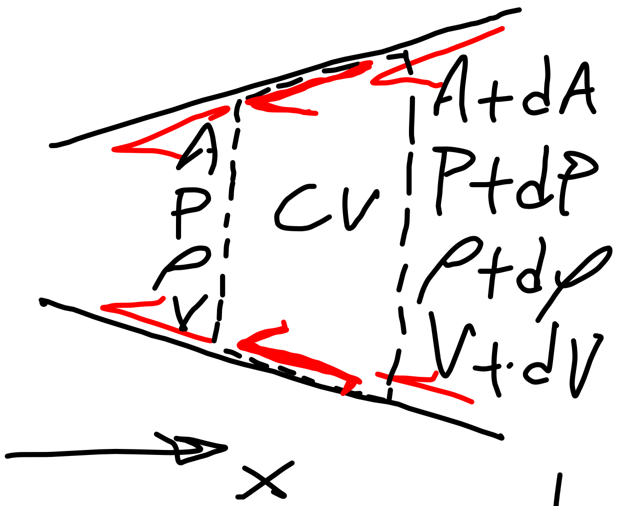
$$\frac{dM}{M} = \frac{f\,dx}{D}\,\frac{\gamma M^2/2}{(1-\gamma M^2)}$$

Critical M#

$M < 1/\sqrt{\gamma} \rightarrow M\uparrow \longrightarrow M = \frac{1}{\sqrt{\gamma}}$

$M > \text{''} \rightarrow M\downarrow$

9.5 Adiabatic Flow with Friction & Area Change



- steady
- 1-D (f(x))
- perfect gas

Continuity: $$\frac{d\rho}{\rho}+\frac{dV}{V}+\frac{dA}{A}=0$$

x-mom:

$$PA-(P+dP)(A+dA)+\underbrace{\left(P+\frac{dP}{2}\right)dA}_{\text{slanted wall}}-\tau_w A_s=\rho AV\,dV$$

(same as Ch. 4)

Substitute for τ_w in terms of "f":

$$dp + \frac{1}{2}\rho V^2 \frac{f\,dx}{D} + \rho V\,dV = 0$$

continuity

$M = V/a$

Perfect gas

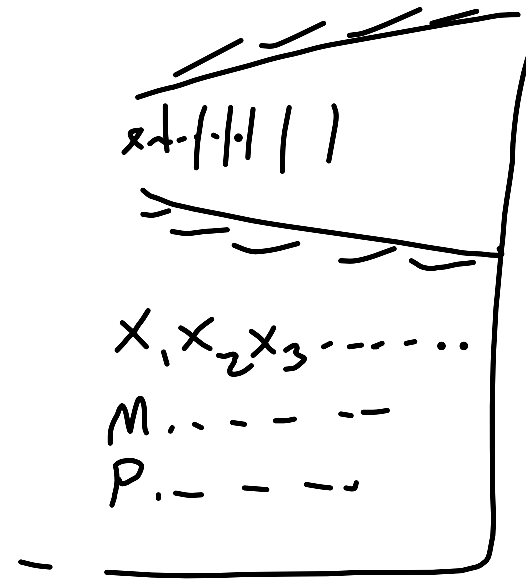
$$\boxed{\frac{1}{2}\gamma M^2 \frac{f}{D} = -\frac{1}{A}\frac{dA}{dx} + \frac{dM}{dx}\,\frac{(1-M^2)/M}{1+\frac{\gamma-1}{2}M^2}}$$

Direct integration is not possible!

$\Rightarrow$ Numerical integration is required!

MECH 2XYZ ; MATH 66XY

Fall 2002 → MATHLAB



$$\Rightarrow \frac{dA}{dx} = 0$$

⇒ "Fanno Flow"

$$\Rightarrow \frac{dM}{dx} = 0$$

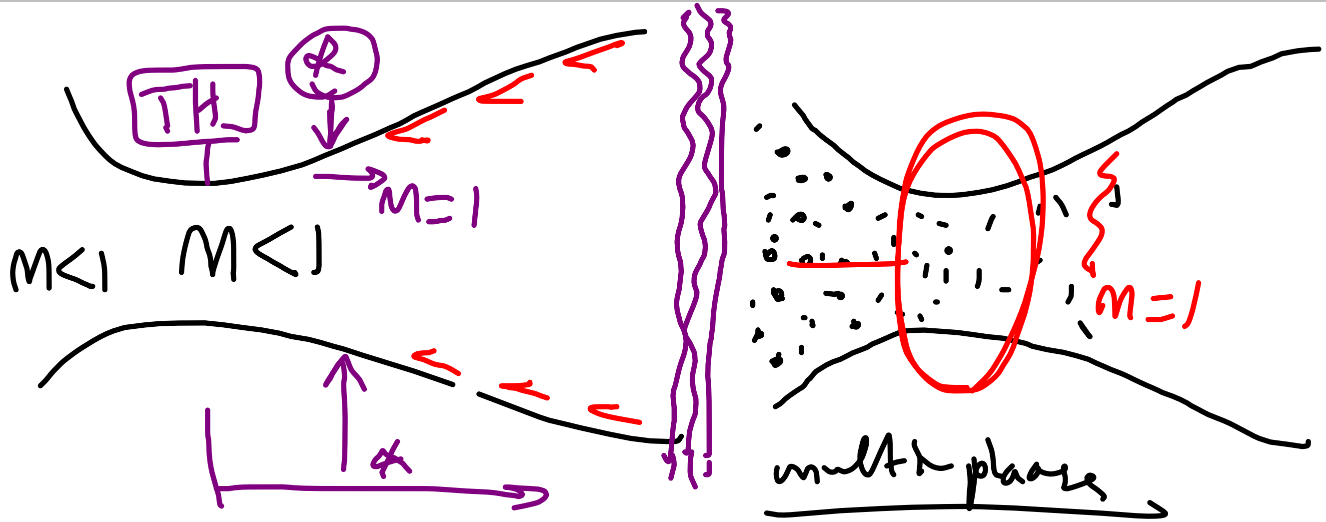
$$\frac{1}{2}\gamma M^2 \frac{f}{D} = \frac{1}{A}\frac{dA}{dx} \xrightarrow[\text{circular cross section}]{} = \frac{2}{D}\frac{dD}{dx}$$

$$\boxed{\frac{dD}{dx} = \frac{\gamma M^2}{4} f} > 0$$

$\Rightarrow$ Last 2 cases $\Rightarrow$ <u>Limiting Cases</u>

Significan of the above relation when applied to nozzles (CDN)

- $M = 1$ happends for $\frac{dA}{dx} > 0$ (Diverging section)



Nozzle Vel. Coeff. $\equiv C_V = \dfrac{\text{actual exit vel.}}{\text{isentropic vel.}}$

Enthalpy Coeff. $\equiv C_h = \dfrac{\Delta h_{actual}}{\Delta h_{isentropic}}$

If $V_{inlet} \approx 0 \Rightarrow \boxed{C_h = C_V^2}$

Common practice: $\boxed{C_v \approx \underline{0.95}}$

Chapter 10: Flow with Heat Transfer

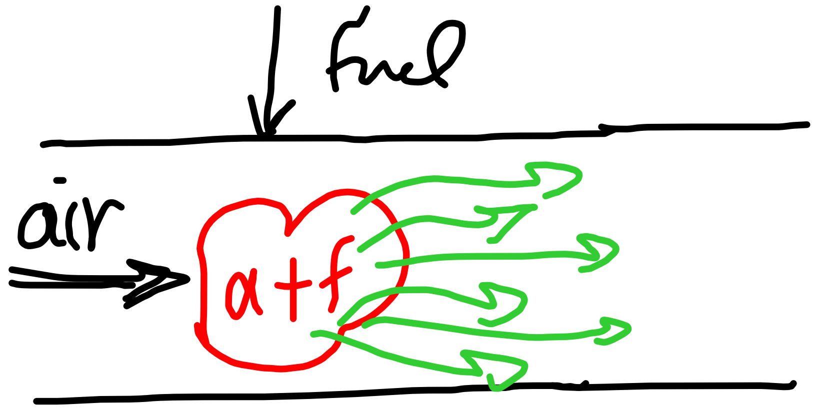
Ch. 9 Ch. 10

Stagnation Press. $\downarrow$

$\Delta S > 0$

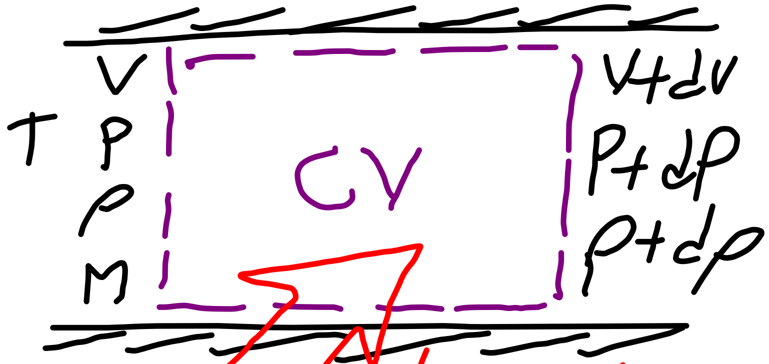
- Const. area duct.
- HT + area change
- friction + HT + area change

* 1-D
* steady
* perfect



Gas Turbines

10.2 Rayleigh Line Flows



dq (+ or −) [J/kg]

- steady flow
- A = const.

Continuity: $\rho V = (\rho + d\rho)(V + dV)$

$$\rho V = C \Rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} = 0$$

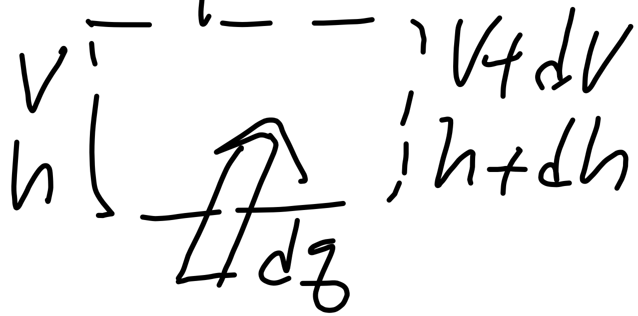
Momentum:

$$dp + \rho V dV = 0$$

since $\rho V = \text{const.}$

$$\Rightarrow P + \rho V^2 = \text{const.}$$

E.E.



$$dq = V dV + dh$$

$\Rightarrow$ Perfect Gas: $dh = c_p dT$.

$$C_p\,dT + V\,dV = dq$$

Stagnation temp (T_0):

$$C_p(T_0 - T) = \frac{V^2}{2}$$

$$\Rightarrow \boxed{C_p\,dT_0 = dq} \quad 1-9$$

$$\boxed{dq = 0}$$