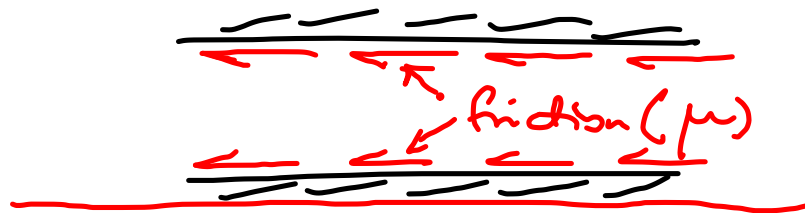


Lecture 22 (Fanno Flow - Cont'd)

lost time:

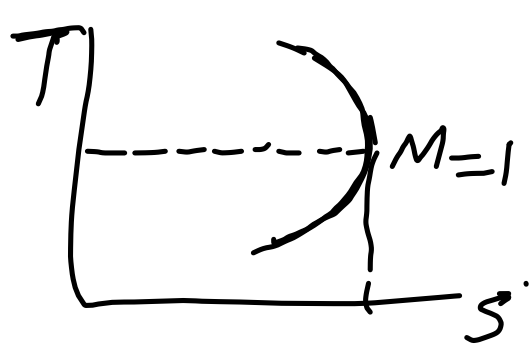


* 1-D flow * $A = \text{const}$ & Adiabatic

* Perfect gas

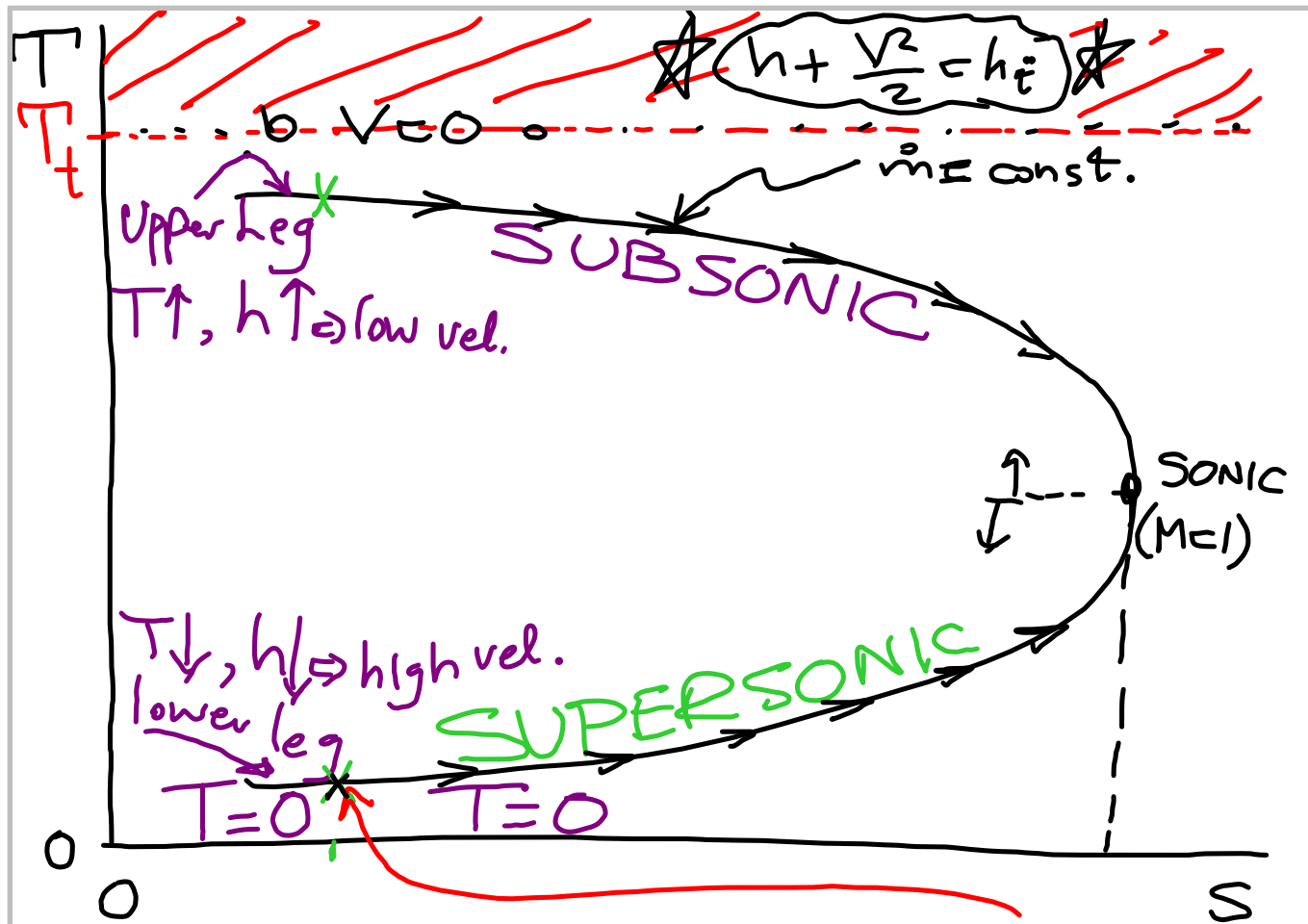
E.E. + Continuity + Perfect gas

$$\Rightarrow \frac{s - s_1}{C_v} = \ln T + \frac{(\gamma - 1)}{2} \ln(T_t - T) + \text{const.}$$



$$\frac{d}{dT} \left(\frac{s}{C_v} \right) = 0$$

$$V^2 = \gamma R T \Rightarrow \begin{matrix} V = a \\ M = 1 \end{matrix}$$

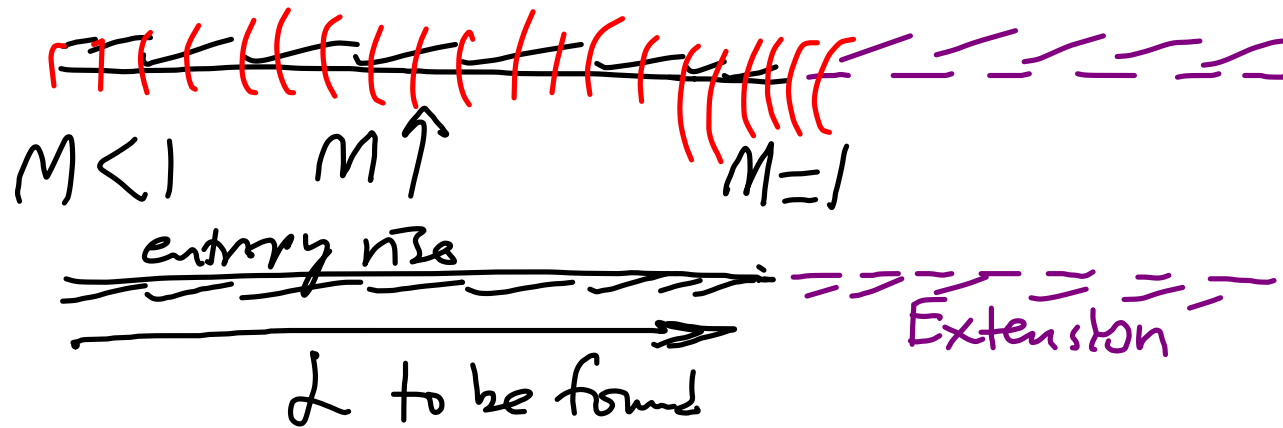


Ch. 3 (Isentropic flow)

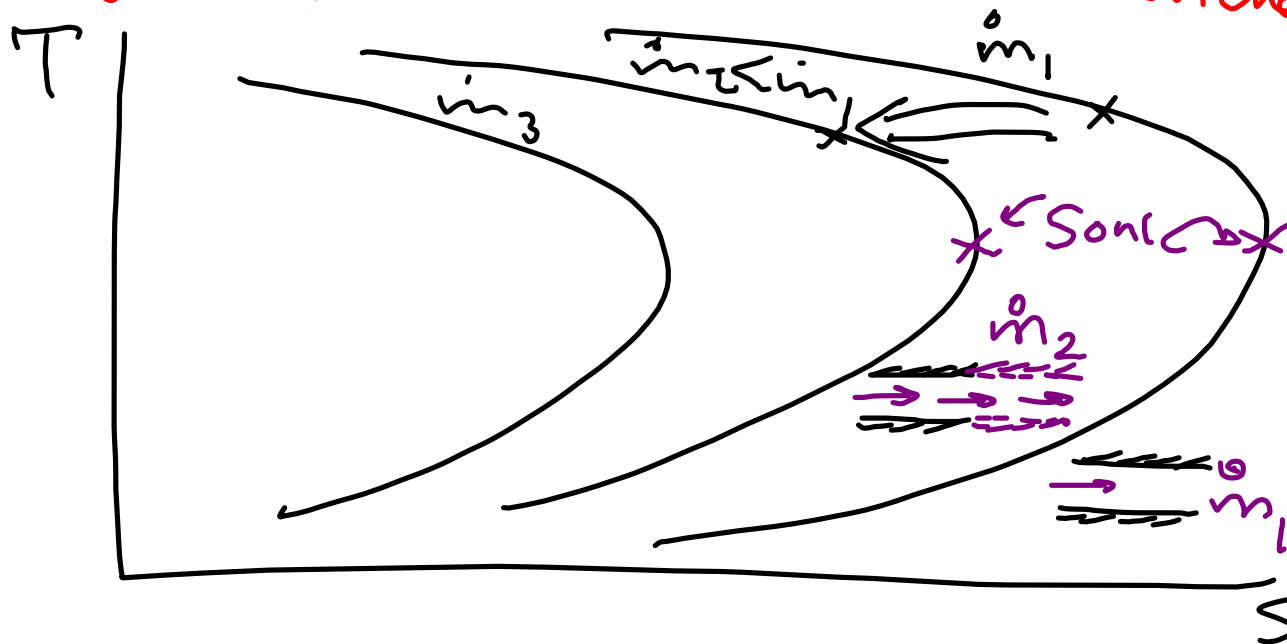
$(ds=0) \Rightarrow$

2nd Law of Thermodynamics
 $dS > 0$

$M > 1$
 isentropic

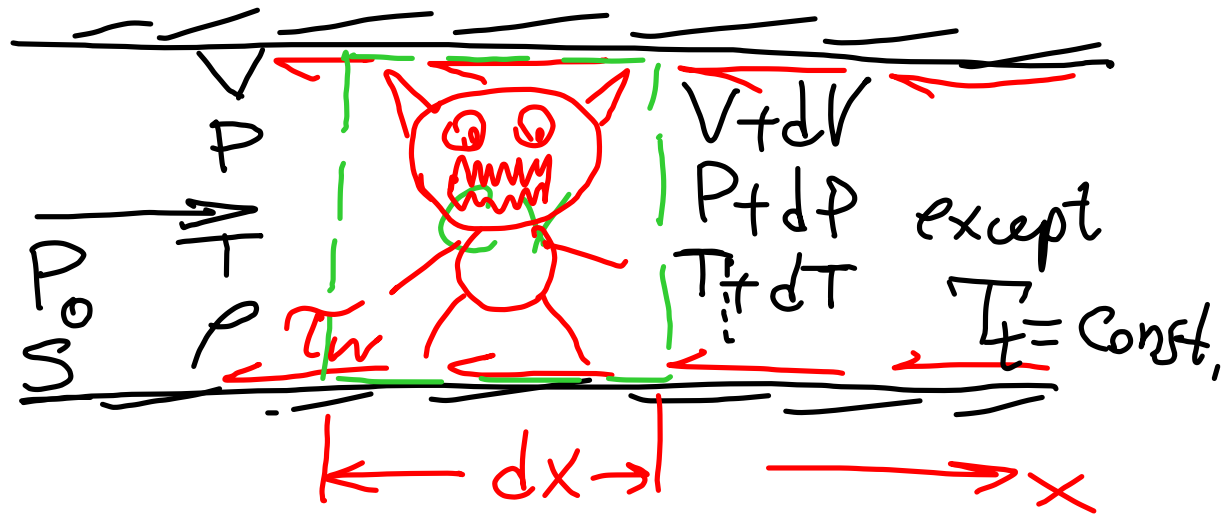


- System will supply less fluid ($\dot{m} \downarrow$) and Fanno line will be switched.



9.2 Fanno Flow

- 1-D Steady flow
- Perfect gas (const. C_p & C_v)
- $Q = 0$ with no external work



x -direction momentum:

$$+PA - (P+dP)A - \tau_w A = \rho A V \uparrow$$

$\underbrace{\hspace{10em}}_{(V+dV-V)}$

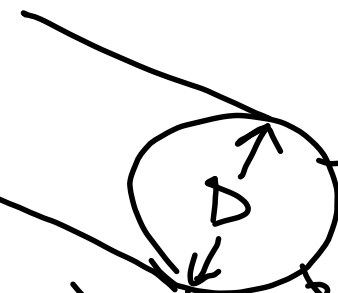
$$-AdP - \tau_w A_\tau = \rho A V dV$$

$\uparrow \quad \uparrow$
 $\tau_w \quad A_\tau$

Hydraulic diameter (D_h) is defined:

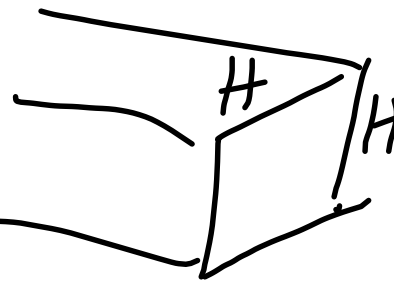
$$D_h = \frac{4A}{\text{Perimeter}}$$

Pipe



$$D_h = \frac{4 \pi D^2}{\pi D} = D$$

Square duct



$$D_h = H$$

2-D Channel



$$D_h = 2H.$$

$$-A dp - \tau_w \frac{4A}{D_h} (dx) = \rho A V dV$$

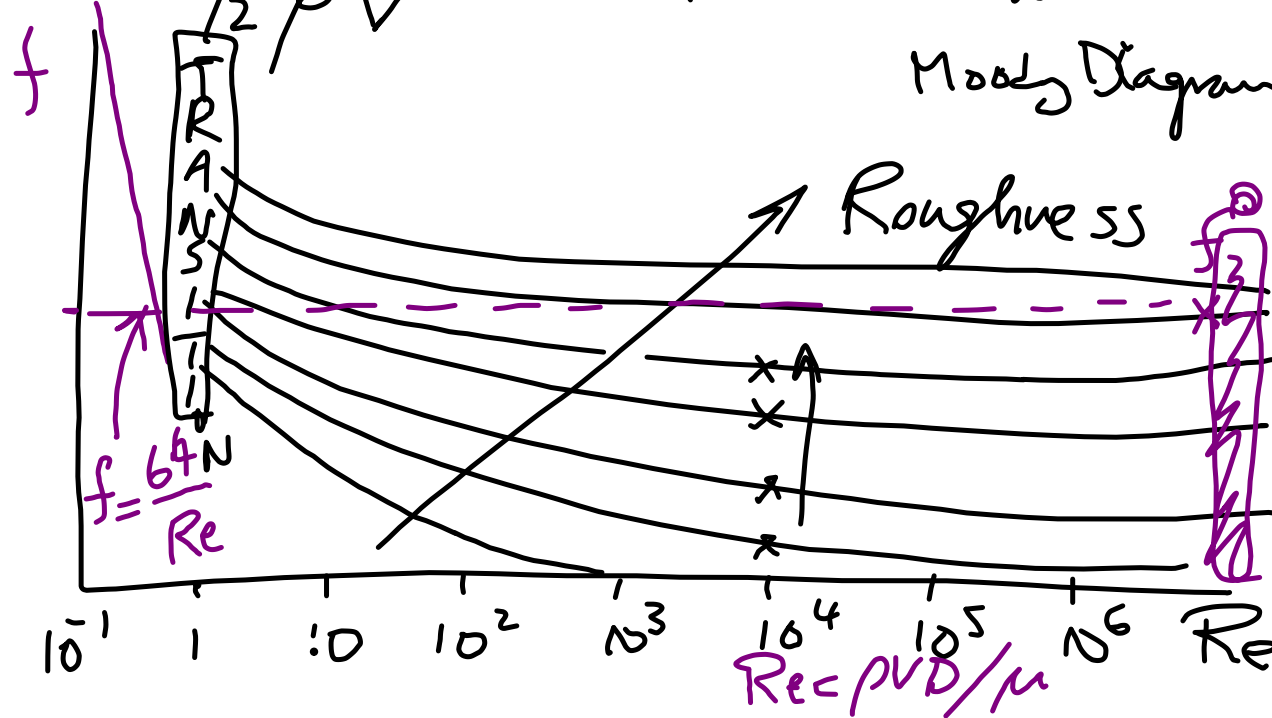
← wall shear stress

⇒ Define a friction coefficient

$$f = \frac{4 \tau_w}{\frac{1}{2} \rho V^2} = f(Re, \frac{\epsilon}{D_h})$$

roughness

Moozy Diagram



$$-dP - \frac{1}{2} \rho V^2 f \frac{dx}{D_h} = \rho V dV$$

Divide by P:

$$-\frac{dP}{P} - \frac{1}{2} \left[\frac{\rho}{P} \right] V^2 f \frac{dx}{D_h} = \left[\frac{\rho}{P} \right] V dV$$

$$\frac{dP}{P} + \frac{1}{2} \gamma M^2 f \frac{dx}{D_h} + \gamma M^2 \frac{dV}{V} = 0$$

Continuity: $\rho V = C \Rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} = 0$

Perfected Gas: $P = \rho R T$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad \left[\begin{array}{l} \text{Differential} \\ \text{eq.} \end{array} \right]$$

Similarly:

$$V = M \sqrt{\gamma R T} \Rightarrow M = \frac{V}{\sqrt{\gamma R T}}$$

$$\Rightarrow \frac{dM}{M} = \frac{dV}{V} - \frac{1}{2} \frac{dT}{T}$$

$\Rightarrow$ All these go into the momentum equation:

$$\frac{dT}{T} - \frac{1}{2} \frac{dT}{T} - \frac{dM}{M} \cdot \frac{dx}{D_h} f \dots = 0$$

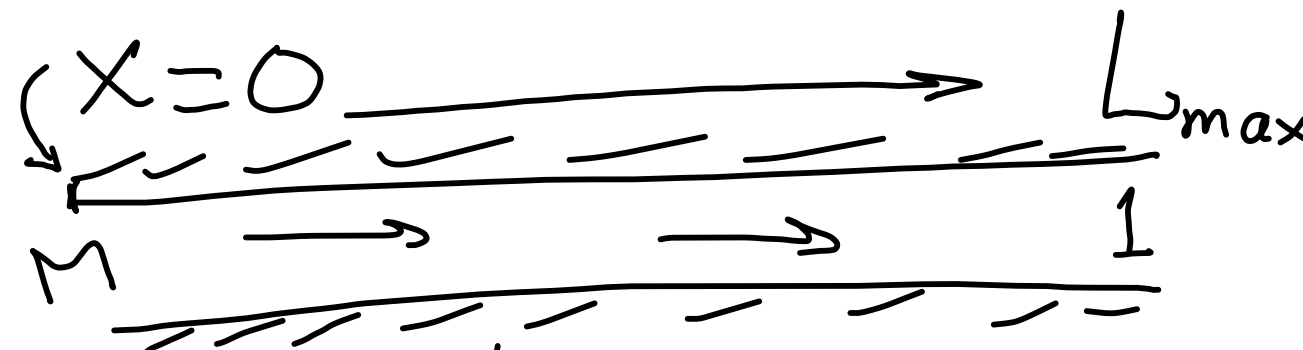
Since flow is adiabatic:

$$T_0 = T \left(1 + \frac{\gamma-1}{2} M^2 \right)$$

$$\frac{dT}{T} + \frac{(\gamma-1) M^2 \frac{dM}{M}}{1 + \frac{\gamma-1}{2} M^2} = 0$$

$$f \frac{dx}{D_h} = 2 \frac{dM}{M} \left[\frac{(1-M^2)}{\left(1 + \frac{\gamma-1}{2} M^2 \right) \gamma M^2} \right]$$

Integrate with limits;



The diagram shows a horizontal duct of length L_{max} . At the inlet (left), the mass flow rate is M . A velocity profile is shown as a dashed line with a parabolic shape, indicating a fully developed flow. The duct is labeled with $X=0$ at the inlet and L_{max} at the outlet. Below the duct, the integral equation is written:

$$f \frac{L_{max}}{D_h} = \int_M^1 \frac{dM}{M}$$

[Analytic solution; Saad's text]

Fanno line Table (App. F)
($\gamma = 1.4$)

$\frac{M}{0.0}$

M	T/T^*	P/P^*	P_t/P_t^*	ρ/ρ^*	f	L_{max}/D
0.0	1.2	∞	∞	$\left(\frac{\infty}{1}\right)$		∞
SUB						

1.0	1.0	1.0	1.0	1.0	0
SUP					