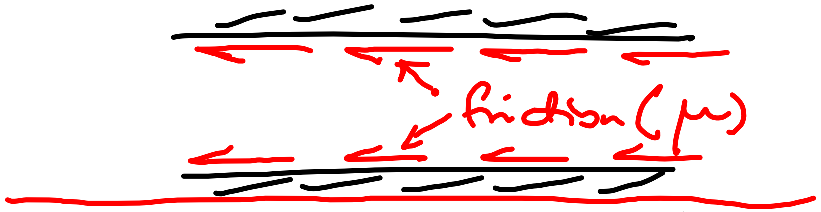
Lecture 22 (Fanno Flow - Cont'd)

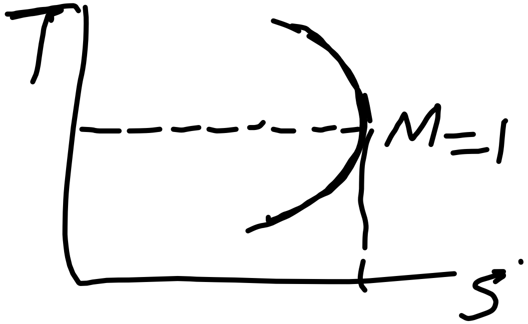
Last time:



* 1-D flow
* A = const & Adiabatic
* Perfect gas

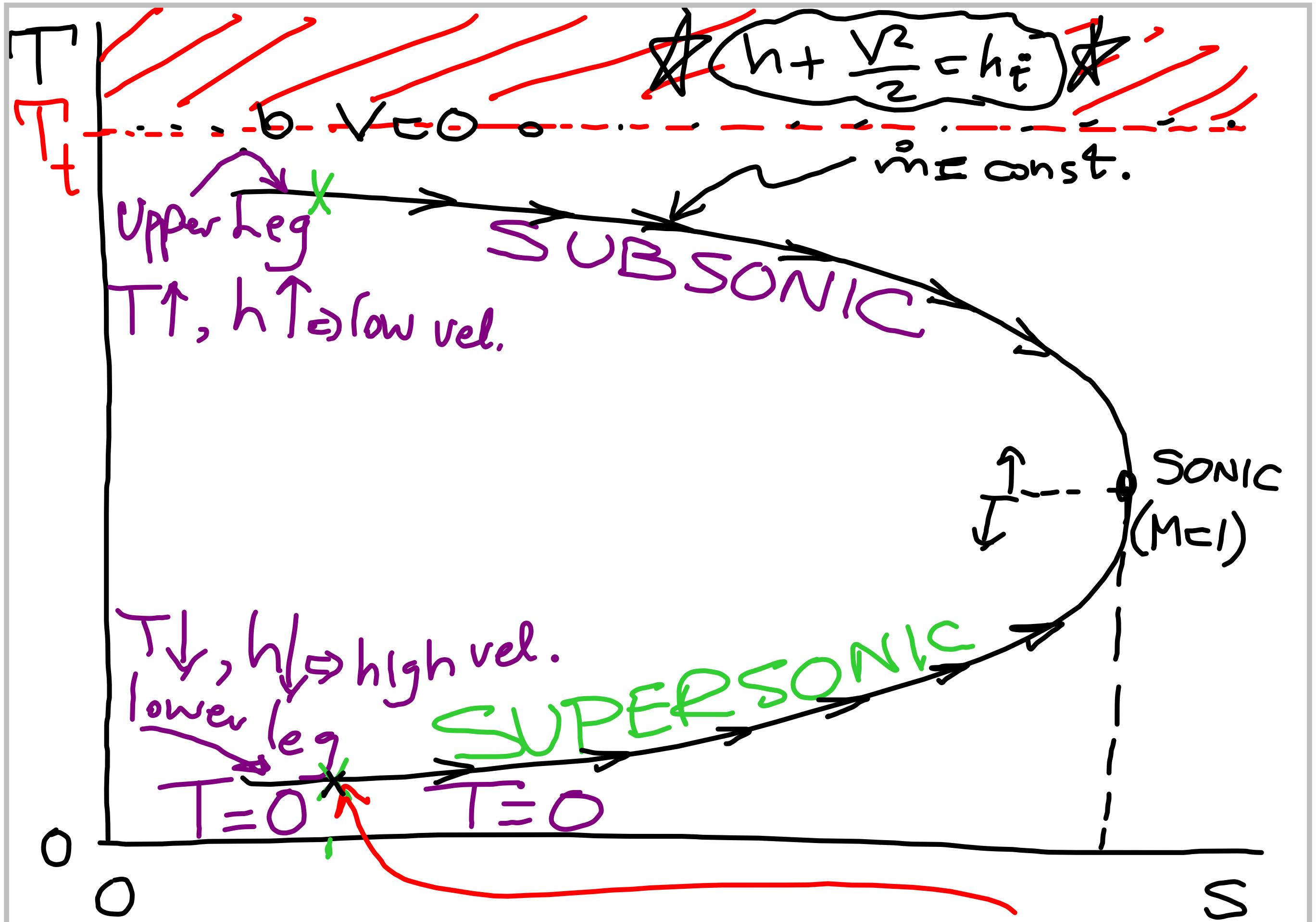
E.E. + Continuity + Perfect gas

$$\Rightarrow \quad \frac{s - s_1}{C_v} = \ln T + \frac{(\gamma - 1)}{2} \ln (T_t - T) + \text{Const.}$$



$$\frac{d}{dT}\left(\frac{s}{c_v}\right) = 0$$

$$V^2 = \gamma R T \Rightarrow \begin{array}{l} V = a \\ M = 1 \end{array}$$

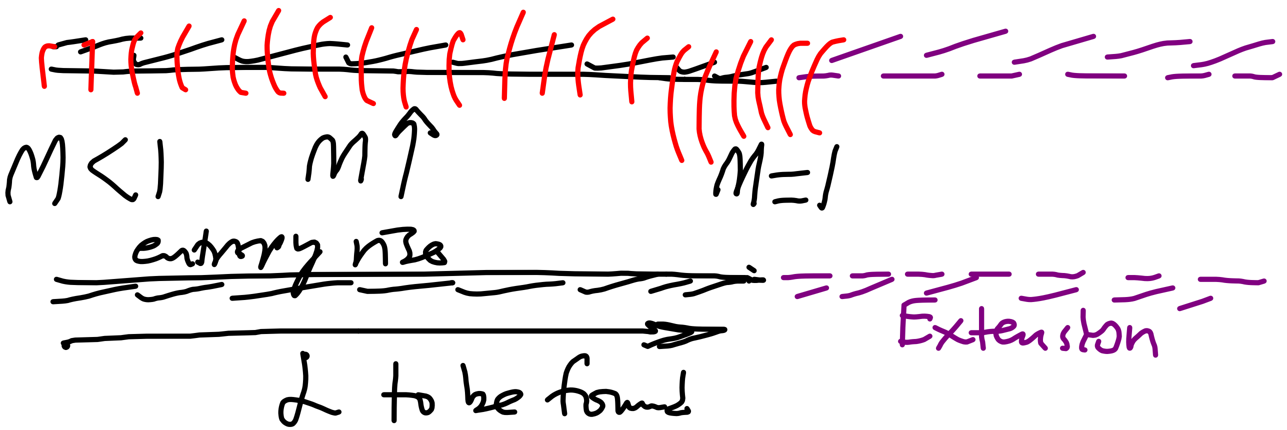


Ch. 3 (Isentropic flow)

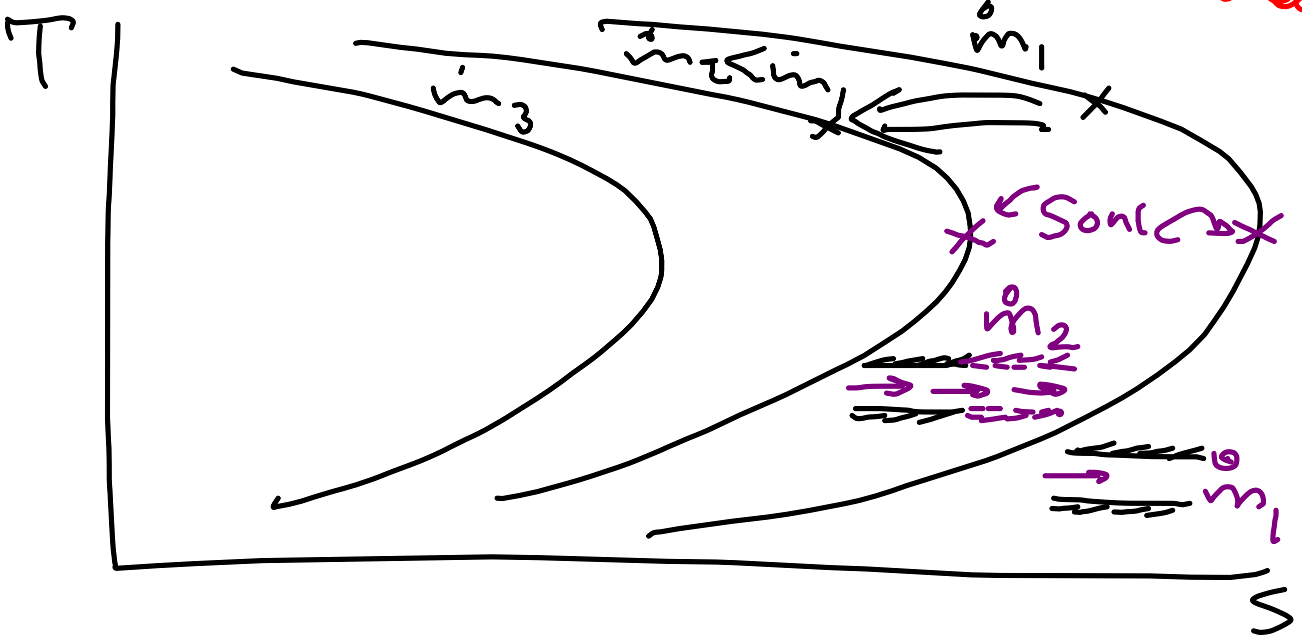
(ds=0) ⇒

2nd Law of Thermodynamics

$dS > 0$

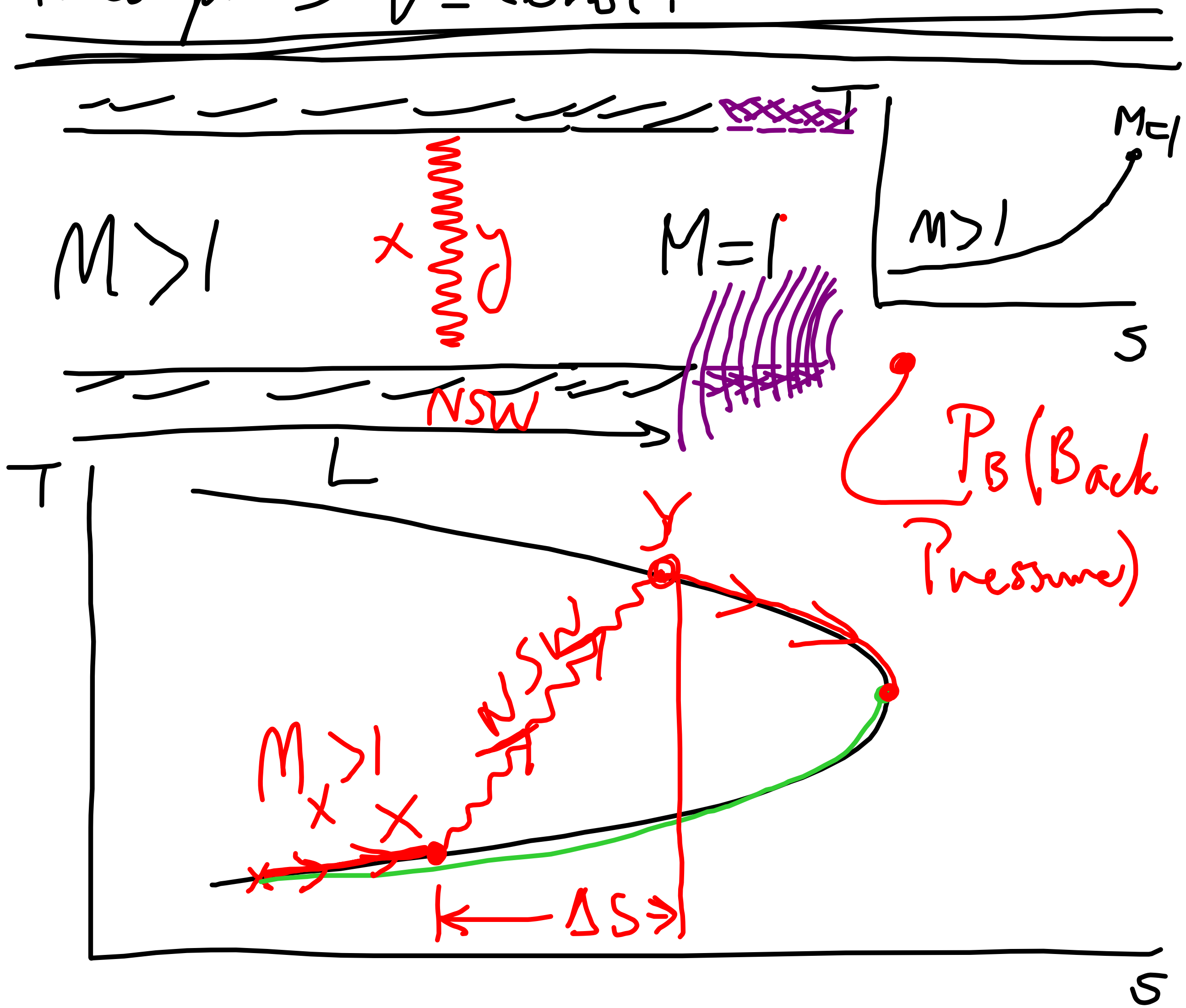


- System will supply less fluid ($\dot{m}\downarrow$) and Fanno line will be switched,



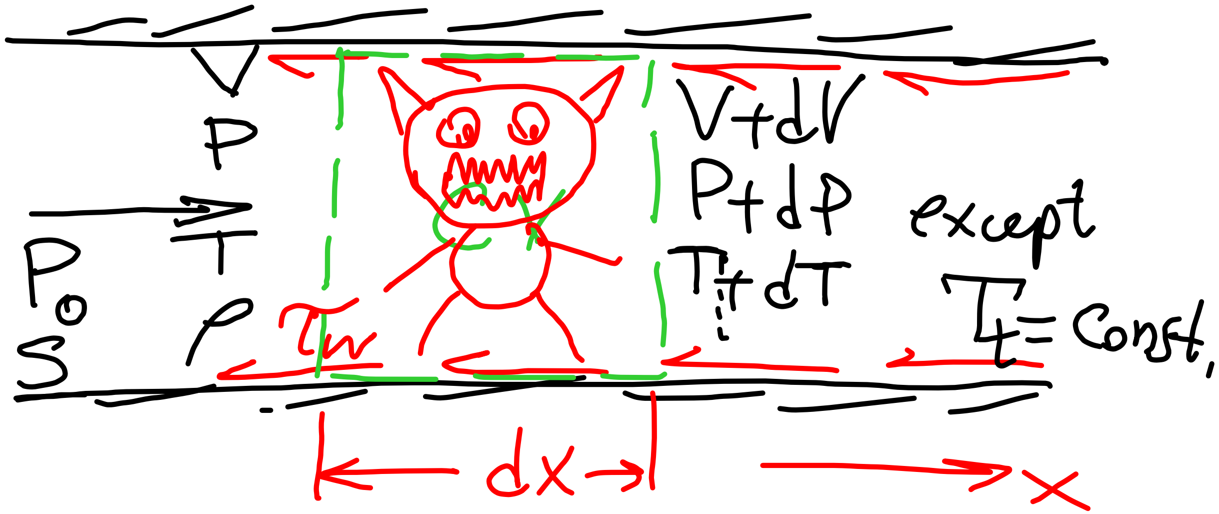
$\rho A V = \text{Const.} \Rightarrow A = C_0 \Rightarrow \rho V = \text{Const}$

incomp. $\Rightarrow V = \text{const.}$



9.2 Fanno Flow

- 1-D Steady flow
- Perfect gas (Const. C_p & C_v)
- $Q = 0$ with no external work



x-direction momentum:

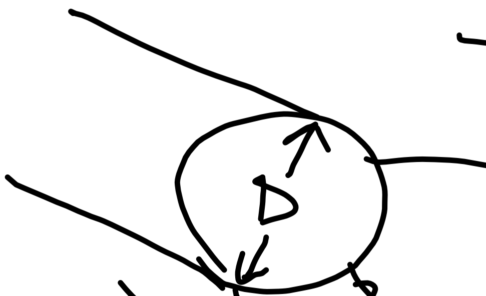
$$+PA - (P+dP)A - \tau_w A_\tau = \rho A V (V+dV-V)$$

$$-A\,dP - \tau_w A_\tau = \rho A V dV$$

<u>Hydraulic diameter (D_h) is defined:</u>

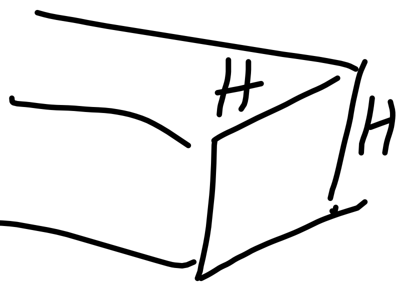
$$D_h = \frac{4A}{\text{Perimeter}}$$

<u>Pipe</u>



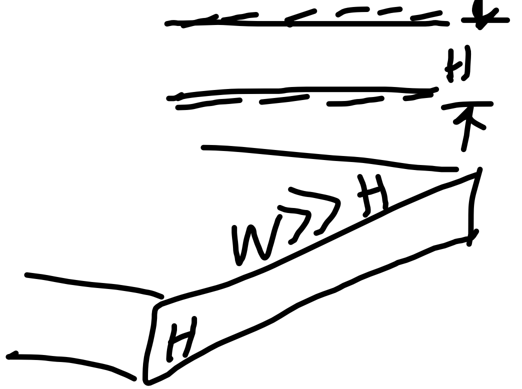
$$D_h = \frac{\cancel{4}\,\pi D^2}{\cancel{\pi} D \cdot \cancel{4}} = D$$

<u>Square duct</u>



$$D_h = H$$

<u>2-D Channel</u>



$$D_h = 2H.$$

$$-\cancel{A}dp - \tau_w \frac{4\cancel{A}}{D_h}(dx) = \rho \cancel{A} V dV$$

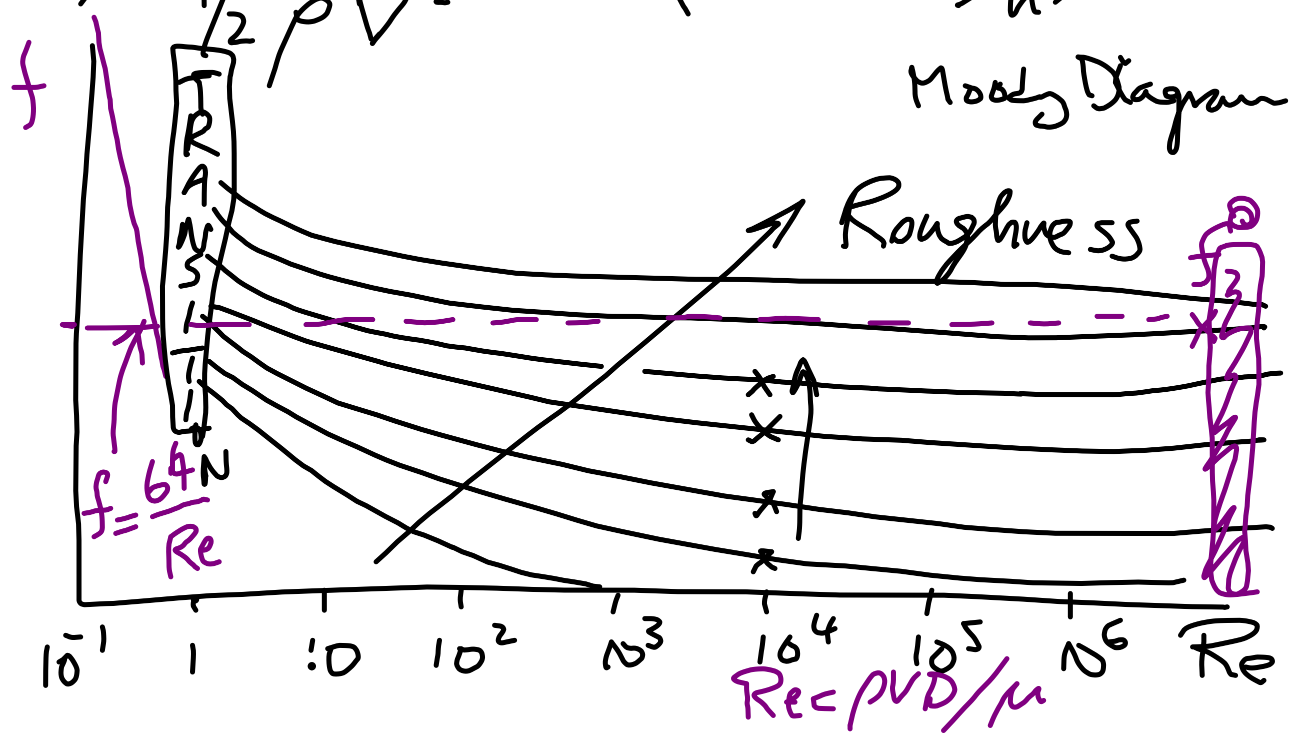
wall shear stress

$\Rightarrow$ Define a friction coefficient

$$f = \frac{4\tau_w}{\frac{1}{2}\rho V^2} = f\left(Re, \varepsilon/D_h\right)$$

roughness

Moody Diagram



$$-dP - \frac{1}{2}\rho V^2 f \frac{dx}{D_h} = \rho V dV$$

Divide by P:

$$\frac{1}{RT}$$

$$-\frac{dP}{P} - \frac{1}{2}\boxed{\frac{\rho}{P}} V^2 f \frac{dx}{D_h} = \boxed{\frac{\rho}{P}} V dV$$

$$\frac{dP}{P} + \frac{1}{2}\gamma M^2 f \frac{dx}{D_h} + \gamma M^2 \frac{dV}{V} = 0$$

Continuity: $\rho V = C \Rightarrow \dfrac{d\rho}{\rho} + \dfrac{dV}{V} = 0$

Perfect Gas: $P = \rho R T$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}.$$ [Difference eq.]

Similarly:

$$V = M\sqrt{\gamma R T} \Rightarrow M = \frac{V}{\sqrt{\gamma R T}}$$

$$\Rightarrow \frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\frac{dT}{T}.$$

$\Rightarrow$ All these go into the momentum equation:

$$\frac{dT}{T} - \frac{1}{2}\frac{dT}{T} - \frac{dM}{M} \cdot \frac{dx}{D_h} f \cdots = 0$$

Since flow is adiabatic:

$$C = T_0 = T\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$\frac{dT}{T} + \frac{(\gamma-1)M^2 \frac{dM}{M}}{1 + \frac{\gamma-1}{2}M^2} = 0$$

$$f\frac{dx}{D_h} = 2\frac{dM}{M}\left[\frac{(1-M^2)}{\left(1+\frac{\gamma-1}{2}M^2\right)\gamma M^2}\right]$$

Integrate with limits;

$$f\,\frac{L_{max}}{D_h} = \int_{M}^{1} \underline{\qquad\qquad} \frac{dM}{M}$$

[Analytic solution; Saad's text]

Fanno line Table (App. F)
($\gamma = 1.4$)

M
0.0

M	T/T^*	P/P^*	P_t/P_t^*	ρ/ρ^*	$f L_{max}/D$
0.0 SUB	1.2	∞	∞	$\left(\frac{V^*}{V}\right)$	∞
1.0 SUP	1.0	1.0	1.0	1.0	0