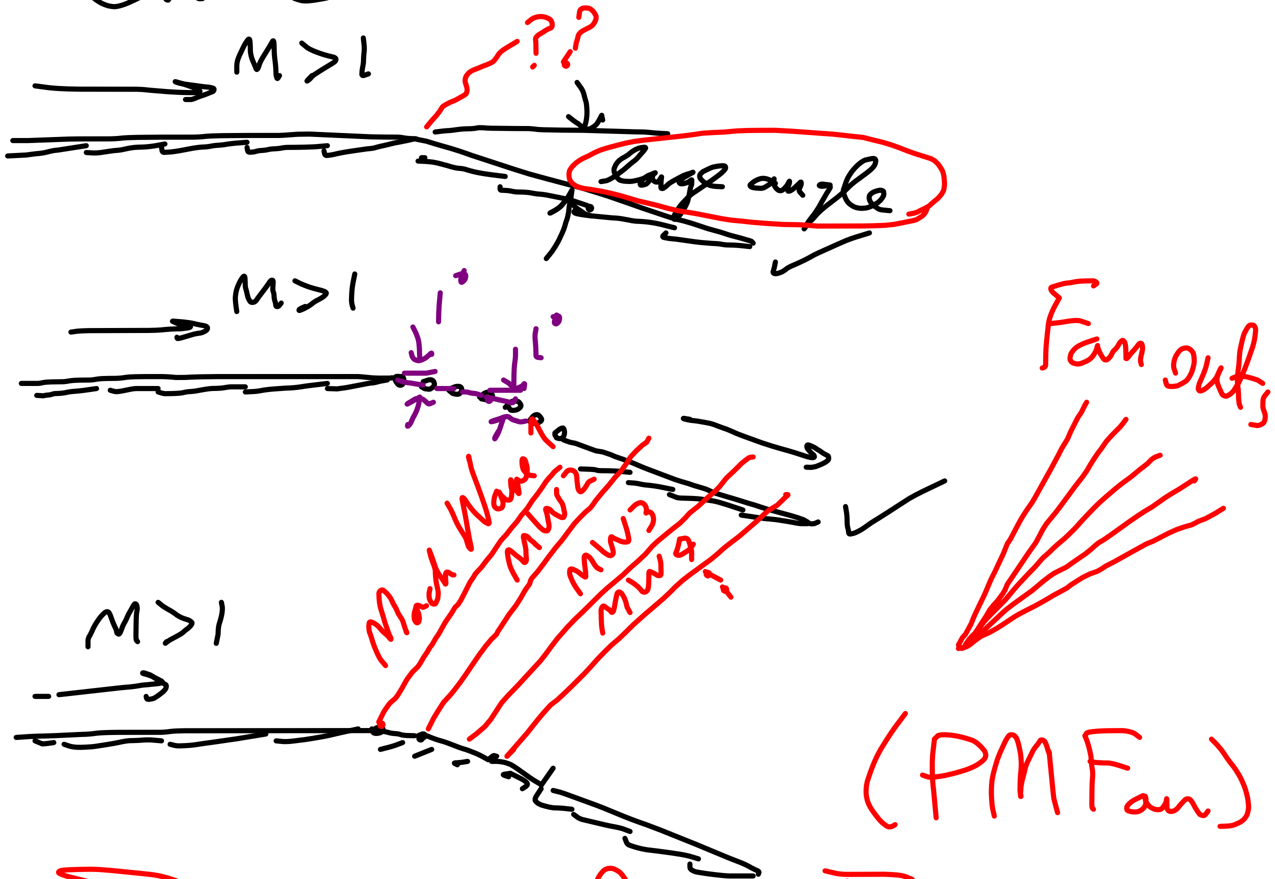
Lecture 20 (Ch. 7)

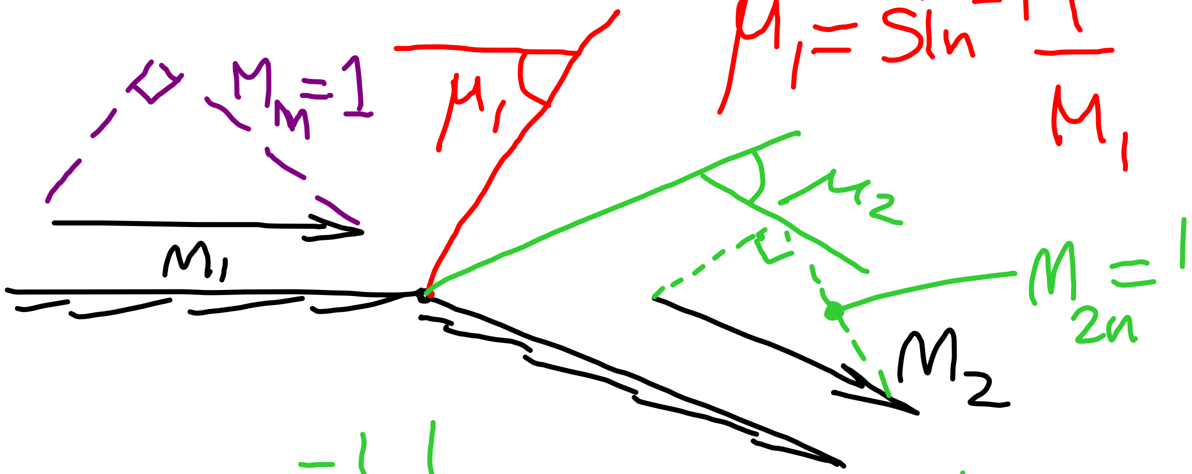
Ch. 6 & Ch. 2.



(PM Fan)

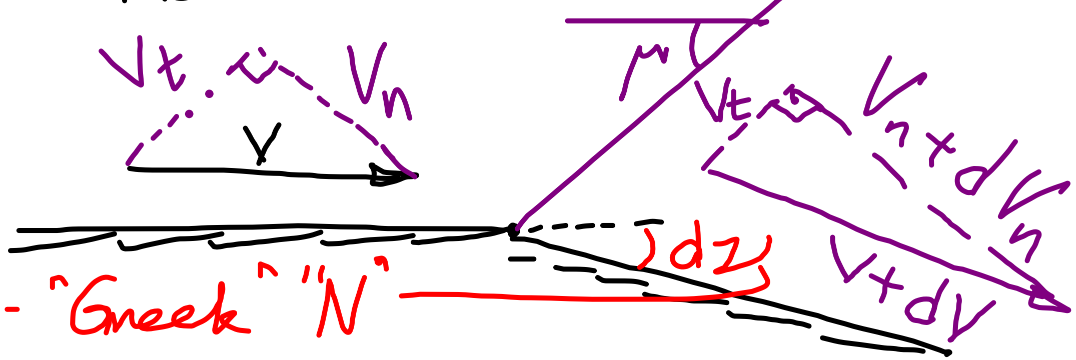
Prandtl-Meyer Expansion Fan

$$\mu_1 = \sin^{-1}\frac{1}{M_1}$$



$$\mu_2 = \sin^{-1}\frac{1}{M_2} \qquad M_{2n} = 1$$

Equations of Motion PMF



ν -- "Greek" "N"

- 2-D flow
- perfect gas with const. C_p and C_v

- Single Mach wave expanding supersonic flow through a small angle ($d\nu$).

$\Rightarrow$ Tangential momentum:

$$V_{t_{upstream}} = V_{t_{downstream}}$$

$$V \cos\mu = (V+dV)\cos(\mu+d\nu)$$

$$'' = (V+dV)(\cos\mu \cos d\nu - \sin\mu \sin d\nu)$$

Angle $d\nu$ is small!

$d\nu$ is small $\Rightarrow$

$\cos d\nu \approx 1.$

$\sin d\nu \approx d\nu$ [radians]

$$V\cos\mu = (V+dV)(\cos\mu - d\nu\sin\mu)$$

multiply through; discard d^2 term on the RHS:

$$\frac{dV}{V} = d\nu\,\frac{\sin\mu}{\cos\mu} = d\nu\tan\mu.$$

$$\sin\mu = \frac{1}{M} \Rightarrow \tan\mu = \frac{1}{\sqrt{M^2-1}}$$

$$\frac{dV}{V} = \frac{1}{\sqrt{M^2-1}}\, d\nu.$$ [small $d\nu$]

On the side $\Rightarrow$

$$V = M\sqrt{\gamma R T}$$

$$\Rightarrow \ln V = \ln M + \frac{1}{2}\ln\gamma + \frac{1}{2}\ln R + \frac{1}{2}\ln T$$

Take derivative:

$$\frac{dV}{V} = \frac{dM}{M} + 0 + 0 + \frac{1}{2}\frac{dT}{T}$$

Difference equation

Stagnation temperature remains constant:

$$T_0 = \text{Const.} = T\left(1+\frac{\gamma-1}{2}M^2\right)$$

Same procedure:

$$\frac{dT}{T} + \frac{(\gamma-1)M\,dM}{1+\frac{\gamma-1}{2}M^2} = 0$$

$$\Rightarrow \frac{dV}{V} = \frac{dM}{M} - \frac{1}{2}\,\frac{(\gamma-1)M\,dM}{1+\frac{\gamma-1}{2}M^2}$$

$$\text{''} = \frac{dM}{M}\left[1-\frac{1}{2}\cdots\cdots\right]$$

$$d\nu = \frac{dM}{M}\left[\frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\right]$$

Integrate both sides!

$$\int_1^2 d\nu = \nu_2 - \nu_1 = \int_1^2 \frac{dM}{M}\left[\text{———}\right]$$

$$\nu_2 - \nu_1 = \left[\sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}\right]_{M_1}^{M_2}$$

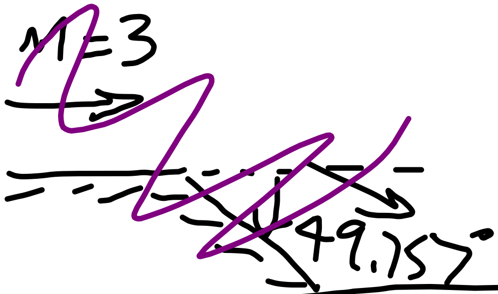
$\nu_1 = \nu_{ref.}$ = reference state!

$\nu_1 = \nu_{ref.} = 0$ at $M_1 = 1.$

$$\boxed{\nu = \left[\sqrt{\frac{\gamma+1}{\gamma-1}}\,\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}\right].}$$

App. E

M	ν	μ
1.0	0	$\sin^{-1}(1/M)$
⋮		
3.0	49.757°	

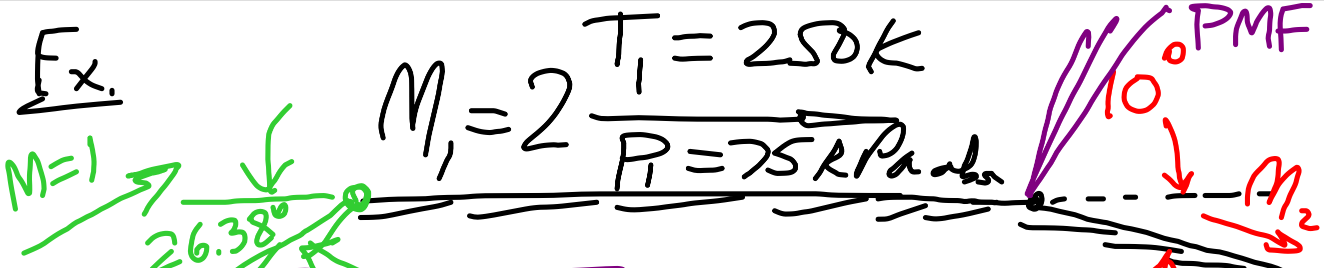


$\Rightarrow$ Pressure & temperature changes follow isentropic relations:

$$\underbrace{P_{01} = P_{02}}_{\text{isentropic}} \quad \& \quad \underline{\underline{T_{01} = T_{02}}}.$$

Mach Waves

Ex.



M_2, P_2 & T_2 = ?

$\Rightarrow$ App E. $\Rightarrow$ $M_1 = 2 \Rightarrow \nu_1 = 26.38^\circ$

$$\nu_2 = \nu_1 + 10^\circ = \underline{\underline{36.38^\circ}}$$

$$\boxed{M_2 = 2.385}$$

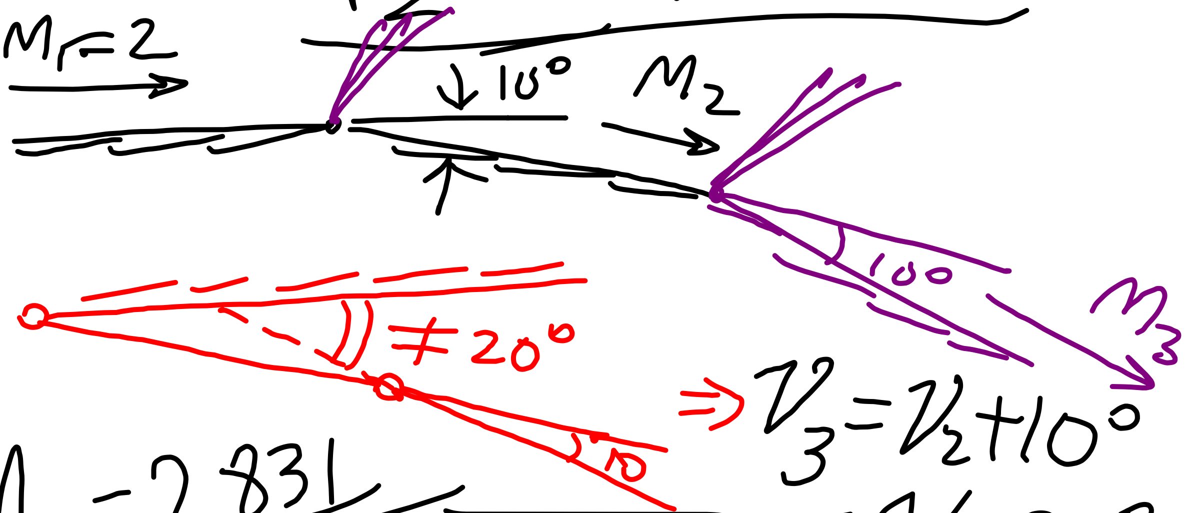
App. B

$$\frac{P_2}{P_1} = \frac{P_2/P_{02}}{P_1/P_{01}} = \frac{P_2/P_0}{P_1/P_0} = \frac{0.0700}{0.1278}$$

$$\frac{P_2}{P_1} = 0.548 \Rightarrow \underline{P_2 = 41.1 \text{ kPa abs}}$$

$$\frac{T_2}{T_1} = \frac{T_2/T_0}{T_1/T_0} = \frac{0.4678}{0.5556} = 0.842$$

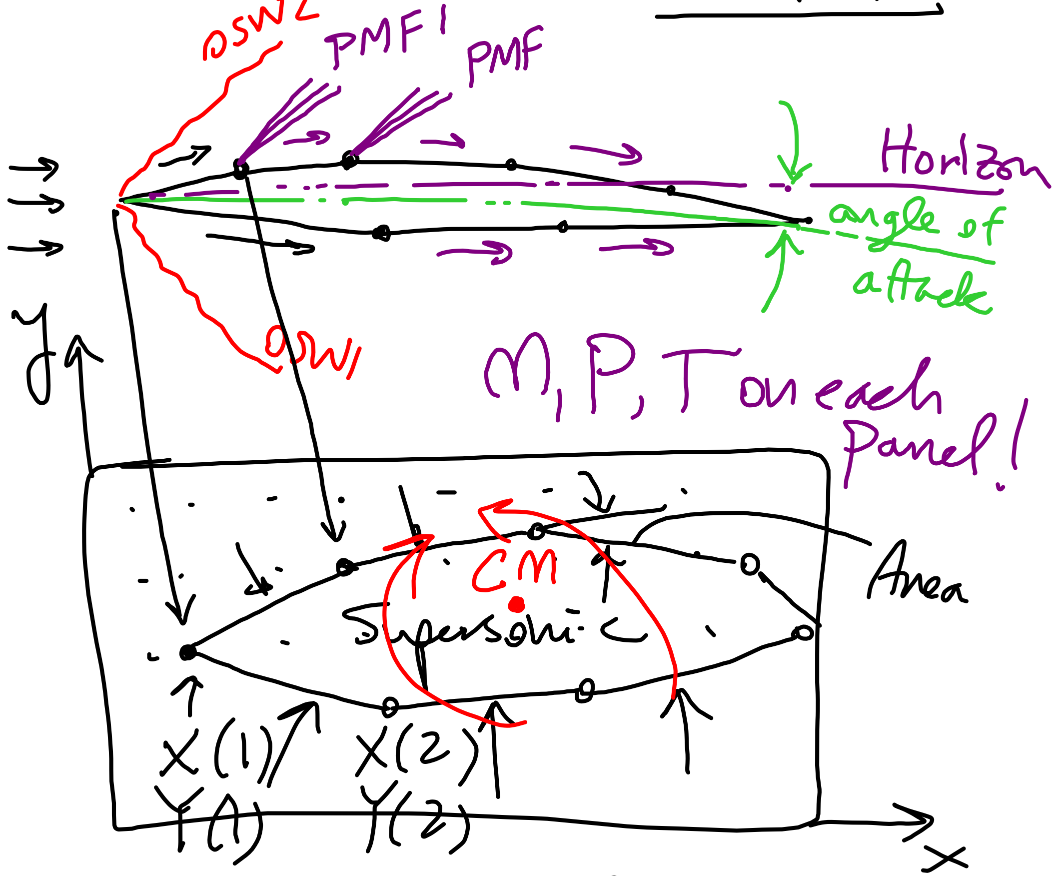
$$\Rightarrow \underline{T_2 = 210.5\,K}$$



$$\Rightarrow \nu_3 = \nu_2 + 10° = 46.38°$$

$$M_3 = 2.831$$

Drag & lift of Supersonic airfoils



⇒ Drag, Lift, Unbalanced Moment

⇒ HW set #9