Lecture 15

- Exam 1 (Graded & Returned)
- Solution (2d,e)

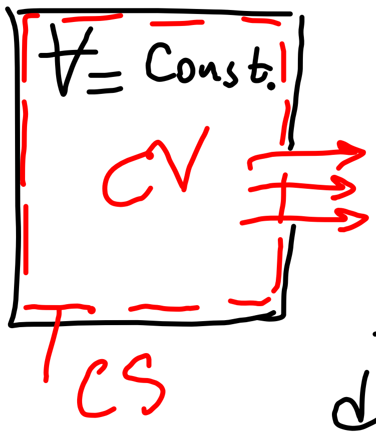
Hi = 99, Low = 33 & Ave ≅ 79.4

Back to lecture on Sudden Discharge

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall = \forall \frac{\partial \rho}{\partial t}.$$

The other term:

$$\oint_{CS} \rho \bar{V} \cdot d\bar{A} \equiv \text{mass flowrate into or out of the CV through CS.}$$



$$\equiv + \dot{m}_e$$

flow discharging

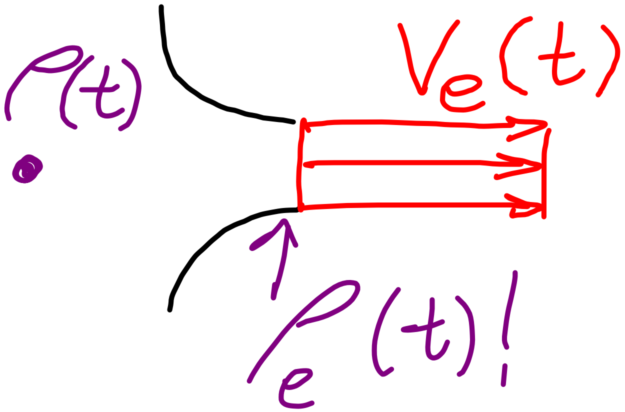
A_e

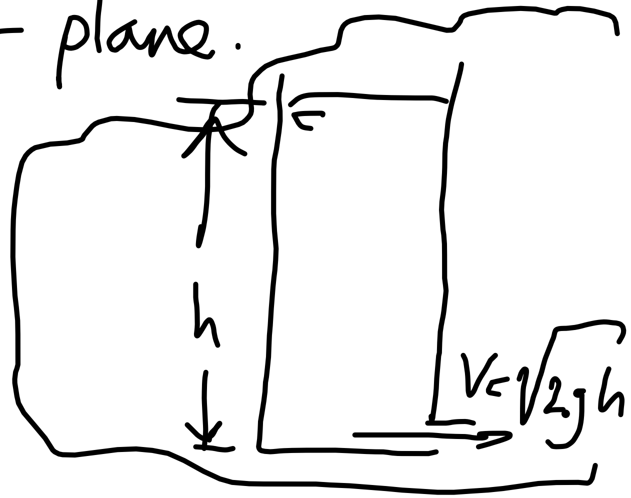
[− sign for a charge problem.]

$$\Rightarrow \boxed{\forall \frac{\partial \rho}{\partial t} + \dot{m}_e = 0}$$

$$\dot{m}_e \equiv \rho_e A_e V_e \equiv \text{mass flow rate leaving the CS.}$$

[properties were taken to be uniform at exit plane.





$$\underset{\text{const.}}{\forall}\,\frac{\partial \rho}{\partial t} + \rho_e \underset{\text{const.}}{A_e} V_e = 0$$

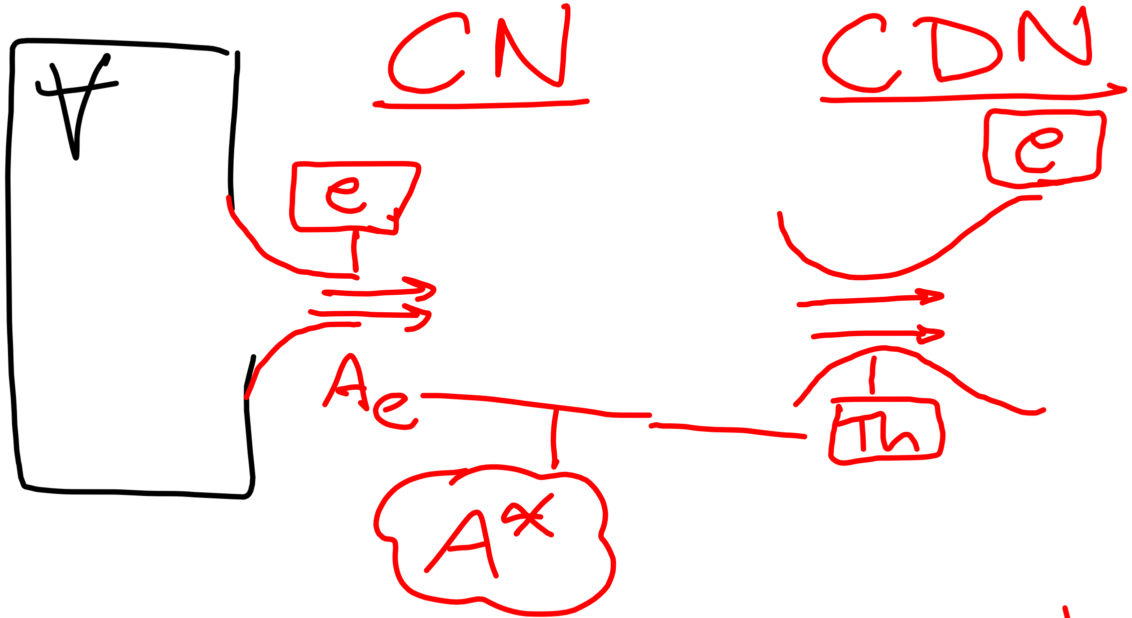
Fliegner's formula:

$$\frac{\dot{m}\sqrt{RT_0}}{P_0 A_e} = f(M_e, \gamma)$$

Assume Choked Nozzle:

$$RHS = \sqrt{\gamma}\left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} = 0.6847$$

for $\gamma = 1.4$



$A_e = A^*$ for CN!

$A_{th} = A^*$ for CDN!

Fliegner's $\Rightarrow$ $\dot{m}_e = \dfrac{KPA^*}{\sqrt{RT}}$

- 0 - subscripts are dropped since properties are the container are stagnation condition!

$$\Rightarrow \forall \frac{\partial \rho}{\partial t} + \frac{KPA^*}{\sqrt{RT}} = 0$$

divide by: $\forall$

$$\frac{\partial \rho}{\partial t} + \frac{KPA^*}{\forall \sqrt{RT}} = 0$$

$\rho, P \& T$ are container properties!

Nondimensionalize: $\rho^+ = \frac{\rho}{\rho_i}$

$$\frac{d}{dt}\left(\frac{\rho}{\rho_i}\right) + \frac{K \check{P} A^*}{V\sqrt{RT}} \frac{RT}{P_i} = 0$$

$$\frac{P}{P_i} = P^+ \qquad \frac{\rho}{\rho_i} = \rho^+$$

$$\frac{d\rho^+}{dt} + \underbrace{\frac{K A^* RT}{V\sqrt{RT}}}_{1/t_{char}} P^+ = 0$$

characteristic

$$\frac{d\rho^+}{dt} + \frac{1}{t_{char}} P^+ = 0 \quad \left[\frac{1}{time}\right]$$

with: $t_{char} \equiv \frac{V}{KA^* \sqrt{RT}}$.

$$t^+ = \frac{t}{t_{char}}$$

$$\Rightarrow \frac{d\rho^+}{dt^+} + P^+ = 0$$

We showed: $\underline{\underline{T^+ = 1 ; \; P^+ = \rho^+}}$

$$\Rightarrow \frac{dP^+}{dt^+} + P^+ = 0$$

Separate & integrate:

$$\frac{dP^+}{P^+} = -dt^+ \Rightarrow \boxed{P^+ = \rho^+ = e^{-t^+}}$$

$$\boxed{T^+ = 1}$$

Example:

Hemisphere

Space Colony 20XY

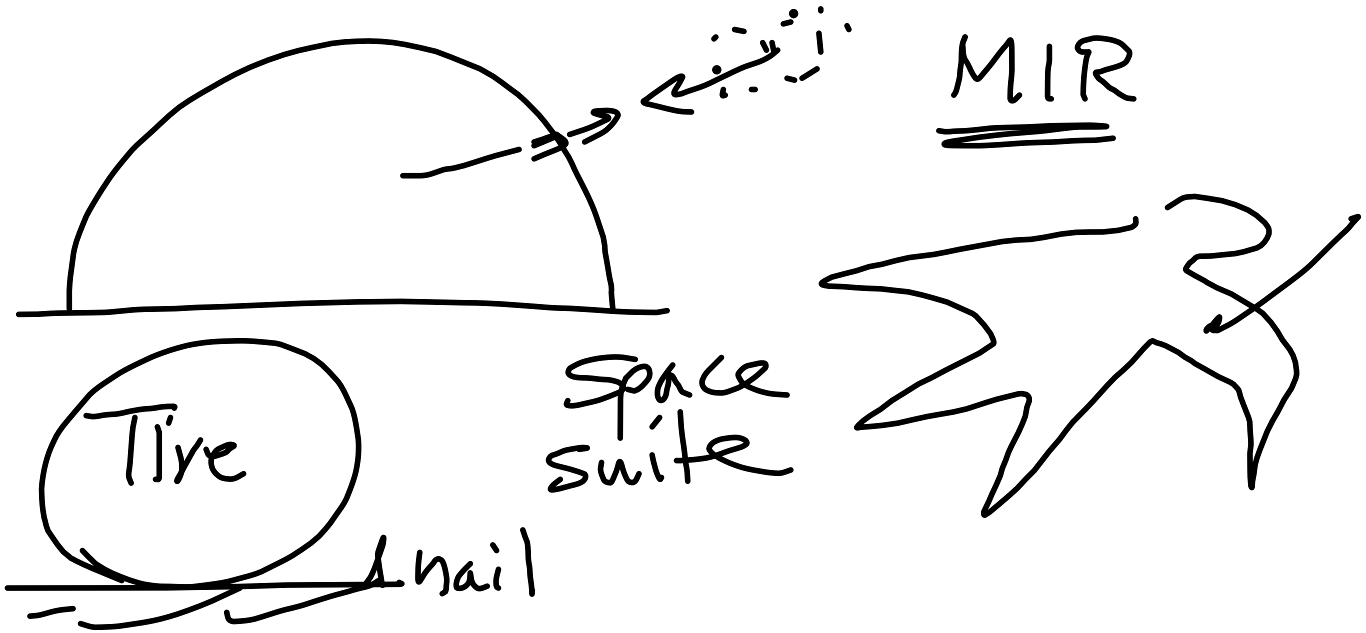
(a) Mass of air?

$$\forall = \frac{1}{2}\left[\frac{4}{3}\pi R^3\right] = 16,755\ m^3$$

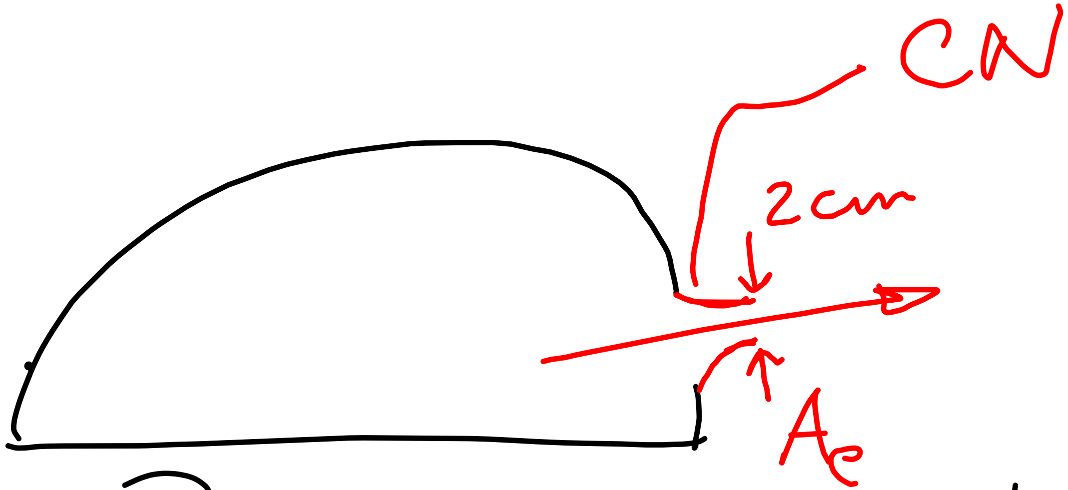
$$\rho = \frac{P}{RT} = \frac{101,000}{(287)(300)} = 1.225\ kg/m^3$$

$$m = \rho \forall = 20,520\ kg$$

(b) Survivability of this unit in the event of meteorite showers.



Assume: Hole with diameter of 2cm that will be modelled at CN.



⇒ People can survive with ρ as low as 40% standard.

⇒ Determine how long does it take ρ_{air} to reach 40% of the initial value.

Adiabatic Analysis:

⇒ Verify that flow will remains choked!

$$\left.\frac{P_B}{P} = \frac{0}{101{,}000}\right|_{t=0} = 0$$

$$0 < 0.5283$$

Always Choked!