

**SOLUTION MANUAL
ENGLISH UNIT PROBLEMS
CHAPTER 16**

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FUNDAMENTALS
of
Thermodynamics
Sixth Edition

CHAPTER 16**CONTENT CHAPTER 16**

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Stagnation properties**16.74E**

Steam leaves a nozzle with a velocity of 800 ft/s. The stagnation pressure is 100 lbf/in², and the stagnation temperature is 500 F. What is the static pressure and temperature?

$$h_1 = h_{o1} - \frac{V_1^2}{2g_c} = 1279.1 - \frac{800^2}{2 \times 32.174} \times 778 = 1266.3 \frac{\text{Btu}}{\text{lbm}}$$

$$s_1 = s_0 = 1.7085 \text{ Btu/lbm R}$$

$$(h, s) \text{ Computer table} \Rightarrow P_1 = \mathbf{88 \text{ lbf/in.}^2}, \quad T = \mathbf{466 \text{ F}}$$

16.75E

Air leaves the compressor of a jet engine at a temperature of 300 F, a pressure of 45 lbf/in², and a velocity of 400 ft/s. Determine the isentropic stagnation temperature and pressure.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2g_c = 400^2/2 \times 32.174 \times 778 = 3.2 \text{ Btu/lbm}$$

$$T_{o1} - T - 1 = (h_{o1} - h_1)/C_p = 3.2/0.24 = 13.3$$

$$T_{o1} = T + \Delta T = 300 + 13.3 = 313.3 \text{ F} = \mathbf{773 \text{ R}}$$

$$P_{o1} = P_1 \left(T_{o1}/T_1 \right)^{\frac{k}{k-1}} = 45(773/759.67)^{3.5} = \mathbf{47.82 \text{ lbf/in}^2}$$

16.76E

A meteorite melts and burn up at temperatures of 5400 R. If it hits air at 0.75 lbf/in.², 90 R how high a velocity should it have to reach such temperature?

Assume we have a stagnation $T = 5400$ R

$$h_1 + V_1^2/2 = h_{\text{stagn.}}$$

Extrapolating from table F.5, $h_{\text{stagn.}} = 1515.6$, $h_1 = 21.4$ Btu/lbm

$$V_1^2/2 = 1515.6 - 21.4 = 1494.2 \text{ Btu/lbm}$$

$$V_1 = \sqrt{2 \times 32.174 \times 778 \times 1494.2} = \mathbf{8649 \text{ ft/s}}$$



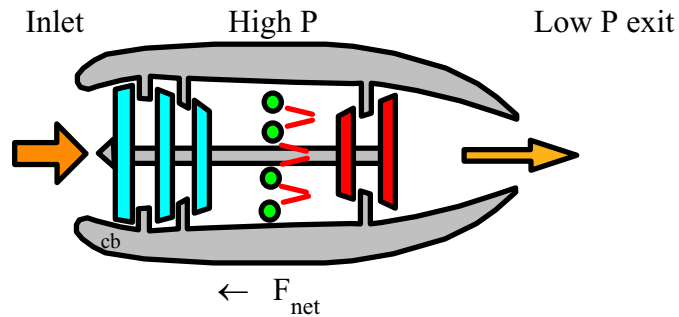
Momentum Equation and Forces**16.77E**

A jet engine receives a flow of 500 ft/s air at 10 lbf/in.², 40 F inlet area of 7 ft² with an exit at 1500 ft/s, 10 lbf/in.², 1100 R. Find the mass flow rate and thrust.

$$\dot{m} = \rho A V; \quad \text{ideal gas} \quad \rho = P/RT$$

$$\dot{m} = (P/RT)AV = \frac{10 \times 144}{53.34 \times 499.7} \times 7 \times 500 = 189.1 \text{ lbm/s}$$

$$F_{\text{net}} = \dot{m} (V_{\text{ex}} - V_{\text{in}}) = 189.1 \times (1500 - 500) / 32.174 = \mathbf{5877 \text{ lbf}}$$



The shaft must have axial load bearings to transmit thrust to aircraft.

16.78E

A water turbine using nozzles is located at the bottom of Hoover Dam 575 ft below the surface of Lake Mead. The water enters the nozzles at a stagnation pressure corresponding to the column of water above it minus 20% due to friction. The temperature is 60 F and the water leaves at standard atmospheric pressure. If the flow through the nozzle is reversible and adiabatic, determine the velocity and kinetic energy per kilogram of water leaving the nozzle.

$$\Delta P = \frac{\rho g \Delta Z}{g_c} = g(\Delta Z/v)/g_c = 575/(0.016035 \times 144) = 249 \text{ lbf/in.}^2$$

$$\Delta P_{ac} = 0.8\Delta P = 199.2 \text{ lbf/in.}^2 \quad \text{and Bernoulli} \quad v\Delta P = \mathbf{V_{ex}^2/2}$$

$$\mathbf{V_{ex} = \sqrt{2v\Delta P} = \sqrt{2g\Delta Z} = \sqrt{2 \times 32.174 \times 575} = 192.4 \text{ ft/s}}$$

$$\mathbf{V_{ex}^2/2 = v\Delta P = g\Delta Z/g_c = 575/778 = 0.739 \text{ Btu/lbm}}$$

Velocity of Sound**16.79E**

Find the speed of sound for air at 15 lbf/in.², at the two temperatures of 32 F and 90 F. Repeat the answer for carbon dioxide and argon gases.

From eq. 16.28 we have

$$c_{32} = \sqrt{kRT} = \sqrt{1.4 \times 32.174 \times 53.34 \times 491.7} = \mathbf{1087 \text{ ft/s}}$$

$$c_{90} = \sqrt{1.4 \times 32.174 \times 53.34 \times 549.7} = \mathbf{1149 \text{ ft/s}}$$

For Carbon Dioxide: $R = 35.1$, $k = 1.289$

$$c_{32} = \sqrt{1.289 \times 32.174 \times 35.1 \times 491.7} = \mathbf{846 \text{ ft/s}}$$

$$c_{90} = \sqrt{1.289 \times 32.174 \times 35.1 \times 549.7} = \mathbf{894.5 \text{ ft/s}}$$

For Argon: $R = 38.68$, $k = 1.667$

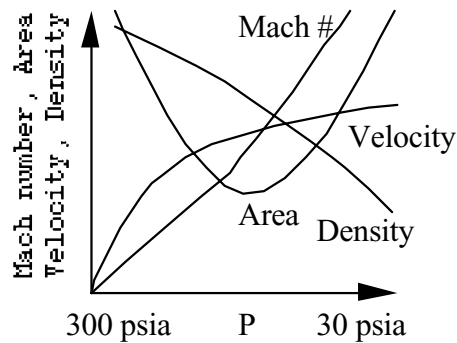
$$c_{32} = \sqrt{1.667 \times 32.174 \times 38.68 \times 491.7} = \mathbf{1010 \text{ ft/s}}$$

$$c_{90} = \sqrt{1.667 \times 32.174 \times 38.68 \times 549.7} = \mathbf{1068 \text{ ft/s}}$$

Flow Through Nozzles, Shocks

16.80E

Air is expanded in a nozzle from 300 lbf/in.², 1100 R to 30 lbf/in.². The mass flow rate through the nozzle is 10 lbm/s. Assume the flow is reversible and adiabatic and determine the throat and exit areas for the nozzle.



$$P^* = P_o \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}}$$

$$= 300 \times 0.5283 = 158.5 \text{ lbf/in.}^2$$

$$T^* = T_o \times 2/(k+1) = 1100 \times 0.8333 = 916.6 \text{ R}$$

$$v^* = RT^*/P^* = 53.34 \times 916.6 / (158.5 \times 144) = 2.1421 \text{ ft}^3/\text{lbm}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 32.174 \times 53.34 \times 916.6} = 1484 \text{ ft/s}$$

$$A^* = \dot{m}v^*/c^* = 10 \times 2.1421/1484 = 0.0144 \text{ ft}^2$$

$$P_2/P_o = 30/300 = 0.1 \quad \text{Table A.11} \Rightarrow M_2^* = 1.701 = V_2/c^*$$

$$V_2 = 1.701 \times 1484 = 2524 \text{ ft/s}$$

$$T_2 = 916.6 \times 0.5176 = 474.4 \text{ R}$$

$$v_2 = RT_2/P_2 = 53.34 \times 474.4 / (30 \times 144) = 5.8579 \text{ ft}^3/\text{lbm}$$

$$A_2 = \dot{m}v_2/V_2 = 10 \times 5.8579 / 2524 = \mathbf{0.0232 \text{ ft}^2}$$

16.81E

A jet plane travels through the air with a speed of 600 mi/h at an altitude of 20000 ft, where the pressure is 5.75 lbf/in.² and the temperature is 25 F. Consider the diffuser of the engine where air leaves at with a velocity of 300 ft/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$$V = 600 \text{ mi/h} = 880 \text{ ft/s}$$

$$v_1 = 53.34 \times 484.67 / (5.75 \times 144) = 31.223 \text{ ft}^3/\text{lbm},$$

$$h_1 = 115.91 \text{ Btu/lbm},$$

$$h_{o1} = 115.91 + 880^2 / (2 \times 32.174 \times 778) = 131.38 \text{ Btu/lbm}$$

$$\text{Table F.5} \quad \Rightarrow \quad T_{o1} = 549.2 \text{ R},$$

$$P_{o1} = P_1 (T_{o1}/T_1)^{k/(k-1)} = 5.75 \times (549.2/484.67)^{3.5} = 8.9 \text{ lbf/in.}^2$$

$$h_2 = 131.38 - 300^2 / (2 \times 32.174 \times 778) = 129.58 \text{ Btu/lbm}$$

$$T_2 = \mathbf{542 \text{ R}}, \quad =>$$

$$P_2 = P_{o1} (T_2/T_{o1})^{k/(k-1)} = 8.9 \times (542/549.2)^{3.5} = \mathbf{8.5 \text{ lbf/in.}^2}$$

$$v_2 = 53.34 \times 542 / (8.5 \times 144) = 23.62 \text{ ft}^3/\text{lbm}$$

$$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (31.223/23.62)(300/880) = \mathbf{0.45}$$

16.82E

A convergent nozzle has a minimum area of 1 ft² and receives air at 25 lbf/in.², 1800 R flowing with 330 ft/s. What is the back pressure that will produce the maximum flow rate and find that flow rate?

$$\frac{P^*}{P_o} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528 \quad \text{Critical Pressure Ratio}$$

$$\text{Find } P_o: \quad C_p = (463.445 - 449.794)/50 = 0.273 \quad \text{from table C.6}$$

$$h_o = h_i + V_i^2/2 \Rightarrow T_o = T_i + V^2/2C_p$$

$$T_o = 1800 + \frac{330^2/2}{32.174 \times 778 \times 0.273} = 1807.97 \Rightarrow T^* = 0.8333 T_o = 1506.6 \text{ R}$$

$$P_o = P_i (T_o/T_i)^{k/(k-1)} = 25 \times (1807.97/1800)^{3.5} = 25.39 \text{ lbf/in.}^2$$

$$P^* = 0.528 P_o = 0.528 \times 25.39 = \mathbf{13.406 \text{ lbf/in.}^2}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{13.406 \times 144}{53.34 \times 1506.6} = 0.024 \text{ lbm/ft}^3$$

$$V = c = \sqrt{kRT^*} = \sqrt{1.4 \times 53.34 \times 1506.6 \times 32.174} = \mathbf{1902.6 \text{ ft/s}}$$

$$\dot{m} = \rho A V = 0.024 \times 1 \times 1902.6 = \mathbf{45.66 \text{ lbm/s}}$$

16.83E

The products of combustion enter a nozzle of a jet engine at a total pressure of 18 lbf/in.², and a total temperature of 1200 F. The atmospheric pressure is 6.75 lbf/in.². The nozzle is convergent, and the mass flow rate is 50 lbm/s. Assume the flow is adiabatic. Determine the exit area of the nozzle.

$$P_{\text{crit}} = P_2 = 18 \times 0.5283 = 9.5 \text{ lbf/in.}^2 > P_{\text{amb}}$$

The flow is then choked.

$$T_2 = 1660 \times 0.8333 = 1382 \text{ R}$$

$$V_2 = c_2 = \sqrt{1.4 \times 32.174 \times 53.34 \times 1382} = 1822 \text{ ft/s}$$

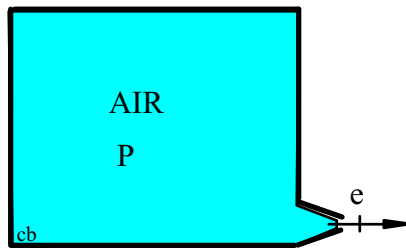
$$v_2 = 53.34 \times 1382 / 9.5 \times 144 = 53.9 \text{ ft}^3/\text{lbm}$$

$$A_2 = \dot{m} v_2 / V_2 = 50 \times 53.9 / 1822 = \mathbf{1.479 \text{ ft}^2}$$

16.84E

A 50-ft³ uninsulated tank contains air at 150 lbf/in.², 1000 R. The tank is now discharged through a small convergent nozzle to the atmosphere at 14.7 lbf/in.² while heat transfer from some source keeps the air temperature in the tank at 1000 R. The nozzle has an exit area of 2×10^{-4} ft².

- Find the initial mass flow rate out of the tank.
- Find the mass flow rate when half the mass has been discharged.
- Find the mass of air in the tank and the mass flow rate out of the tank when the nozzle flow changes to become subsonic.



$$P_B/P_o = 14.7/150 = 0.098 < (P^*/P_o)_{\text{crit}} = 0.5283$$

- The flow is choked, max possible flow rate

$$M_E = 1; \quad P_E = 0.5283 \times 150 = 79.245 \text{ lbf/in.}^2$$

$$T_E = T^* = 0.8333 \times 1000 = 833.3 \text{ R}$$

$$V_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 53.34 \times 833.3 \times 32.174} = 1415 \text{ ft/s}$$

$$v_E = RT^*/P_E = 53.34 \times 833.3 / (79.245 \times 144) = 3.895 \text{ ft}^3/\text{lbm}$$

$$\text{Mass flow rate is: } \dot{m}_1 = AV_E/v_E = 2 \times 10^{-4} \times 1415 / 3.895 = \mathbf{0.0727 \text{ lbm/s}}$$

- $m_1 = P_1 V / RT_1 = 150 \times 50 \times 144 / 53.34 \times 1000 = 20.247 \text{ lbm}$

$$m_2 = m_1 / 2 = 10.124 \text{ lbm}, \quad P_2 = P_1 / 2 = 75 \text{ lbf/in.}^2; \quad T_2 = T_1$$

$$P_B/P_2 = 14.7/75 = 0.196 < (P^*/P_o)_{\text{crit}}$$

The flow is choked and the velocity is the same as in a)

$$P_E = 0.5283 \times 75 = 39.623 \text{ lbf/in.}^2; \quad M_E = 1$$

$$\dot{m}_2 = AV_E P_E / RT_E = \frac{2 \times 10^{-4} \times 1415 \times 39.623 \times 144}{53.34 \times 1000} = \mathbf{0.0303 \text{ lbm/s}}$$

- Flow changes to subsonic when the pressure ratio reaches critical.

$$P_B/P_o = 0.5283 \quad P_3 = 27.825 \text{ lbf/in.}^2$$

$$m_3 = m_1 P_3 / P_1 = \mathbf{3.756 \text{ lbm}}; \quad T_3 = T_1 \Rightarrow V_E = 1415 \text{ ft/s}$$

$$\dot{m}_3 = AV_E P_E / RT_E = \frac{2 \times 10^{-4} \times 1415 \times 27.825 \times 144}{53.34 \times 1000} = \mathbf{0.02125 \text{ lbm/s}}$$

Nozzles, Diffusers and Orifices

16.85E

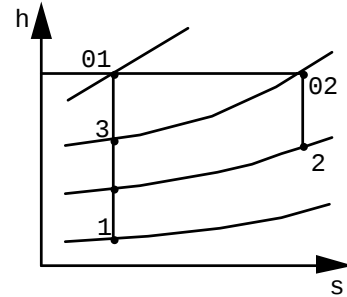
Repeat Problem 16.81 assuming a diffuser efficiency of 80%.

From solution to 16.81

$$h_1 = 115.91 \text{ Btu/lbm}, \quad v_1 = 31.223 \text{ ft}^3/\text{lbm}$$

$$h_{o1} = 115.91 + 880^2 / (2 \times 32.174 \times 778) \\ = 131.38 \text{ Btu/lbm}$$

$$\text{Table F.5} \quad \Rightarrow \quad T_{o1} = 549.2 \text{ R},$$



$$\eta_D = (h_3 - h_1) / (h_{o1} - h_1) = 0.8 \quad \Rightarrow \quad h_3 = 128.29 \text{ Btu/lbm}, \quad T_3 = 536.29 \text{ R}$$

$$P_{o2} = P_3 = P_1 (T_3 / T_1)^{k/(k-1)} = 5.75 \times (536.29 / 484.67)^{3.5} = 8.194 \text{ lbf/in.}^2$$

$$T_{o2} = T_{o1} = 549.2 \text{ R}$$

$$h_2 = 131.38 - 300^2 / (2 \times 32.174 \times 778) = 129.58 \text{ Btu/lbm}$$

$$T_2 = 542 \text{ R}, \quad \Rightarrow$$

$$P_2 = P_{o2} (T_2 / T_{o1})^{k/(k-1)} = 8.194 \times (542 / 549.2)^{3.5} = 7.824 \text{ lbf/in.}^2$$

$$\Rightarrow \quad v_2 = \frac{53.34 \times 542}{7.824 \times 144} = 25.66 \text{ ft}^3/\text{lbm}$$

$$A_1 / A_2 = v_1 V_2 / v_2 V_1 = 31.223 \times 300 / (25.66 \times 880) = 0.415$$

16.86E

Air enters a diffuser with a velocity of 600 ft/s, a static pressure of 10 lbf/in.², and a temperature of 20 F. The velocity leaving the diffuser is 200 ft/s and the static pressure at the diffuser exit is 11.7 lbf/in.². Determine the static temperature at the diffuser exit and the diffuser efficiency. Compare the stagnation pressures at the inlet and the exit.

$$T_{o1} = T_1 + \frac{V_1^2}{2g_c C_p} = 480 + 600^2 / (2 \times 32.174 \times 778 \times 0.24) = 510 \text{ R}$$

$$T_{o2} = T_{o1} \Rightarrow$$

$$T_2 = T_{o2} - V_2^2 / 2C_p = 510 - 200^2 / (2 \times 32.174 \times 0.24 \times 778) = 506.7 \text{ R}$$

$$\frac{T_{o2} - T_2}{T_2} = \frac{k-1}{k} \frac{P_{o2} - P_2}{P_2} \Rightarrow P_{o2} - P_2 = 0.267 \Rightarrow P_{o2} = 11.97 \text{ lbf/in.}^2$$

$$T_{ex,s} = T_1 (P_{o2}/P_1)^{(k-1)/k} = 480 \times 1.0528 = \mathbf{505.3 \text{ R}}$$

$$\eta_D = \frac{T_{ex,s} - T_1}{T_{o1} - T_1} = \frac{505.3 - 480}{51 - 480} = \mathbf{0.844}$$

16.87E

A convergent nozzle with exit diameter of 1 in. has an air inlet flow of 68 F, 14.7 lbf/in.² (stagnation conditions). The nozzle has an isentropic efficiency of 95% and the pressure drop is measured to 20 in. water column. Find the mass flow rate assuming compressible adiabatic flow. Repeat calculation for incompressible flow.

Convert ΔP to lbf/in.²

$$\Delta P = 20 \text{ in H}_2\text{O} = 20 \times 0.03613 = 0.7226 \text{ lbf/in}^2$$

$$T_0 = 68 \text{ F} = 527.7 \text{ R} \quad P_0 = 14.7 \text{ lbf/in}^2$$

$$\text{Assume inlet } V_i = 0 \quad P_e = P_0 - \Delta P = 14.7 - 0.7226 = 13.977 \text{ lbf/in}^2$$

$$T_e = T_0 \left(\frac{P_e}{P_0} \right)^{\frac{k-1}{k}} = 527.7 \times \left(\frac{13.977}{14.7} \right)^{0.2857} = 520.15 \text{ R}$$

$$V_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 0.24 \times (527.7 - 520.15) = 1.812 \text{ Btu/lbm}$$

$$V_{e \text{ ac}}^2/2 = \eta V_e^2/2 = 0.95 \times 1.812 = 1.7214 \text{ Btu/lbm}$$

$$\Rightarrow V_{e \text{ ac}} = \sqrt{2 \times 32.174 \times 1.7214 \times 778} = 293.6 \text{ ft/s}$$

$$T_{e \text{ ac}} = T_i - \frac{V_{e \text{ ac}}^2/2}{C_p} = 527.7 - \frac{1.7214}{0.24} = 520.53 \text{ R}$$

$$\rho_{e \text{ ac}} = \frac{P_e}{RT_{e \text{ ac}}} = \frac{13.977 \times 144}{53.34 \times 520.53} = 0.07249 \text{ lbm/ft}^3$$

$$\dot{m} = \rho A V = 0.07249 \times \frac{\pi}{4} \times \left(\frac{1}{12} \right)^2 \times 293.6 = \mathbf{0.116 \text{ lbm/s}}$$

$$\text{Incompressible: } \rho_i = \frac{P_0}{RT_0} = \frac{14.7 \times 144}{53.34 \times 527.7} = 0.0752 \text{ lbm/ft}^3$$

$$V_e^2/2 = v_i (P_i - P_e) = \frac{\Delta P}{\rho_i} = \frac{0.7226 \times 144}{0.0752 \times 778} = 1.7785 \text{ Btu/lbm}$$

$$V_{e \text{ ac}}^2/2 = \eta V_e^2/2 = 0.95 \times 1.7785 = 1.6896 \text{ Btu/lbm}$$

$$\Rightarrow V_{e \text{ ac}} = \sqrt{2 \times 32.174 \times 1.6896 \times 778} = 290.84 \text{ ft/s}$$

$$\dot{m} = \rho A V = 0.0752 \times \frac{\pi}{4} \times \left(\frac{1}{12} \right)^2 \times 290.84 = \mathbf{0.119 \text{ lbm/s}}$$