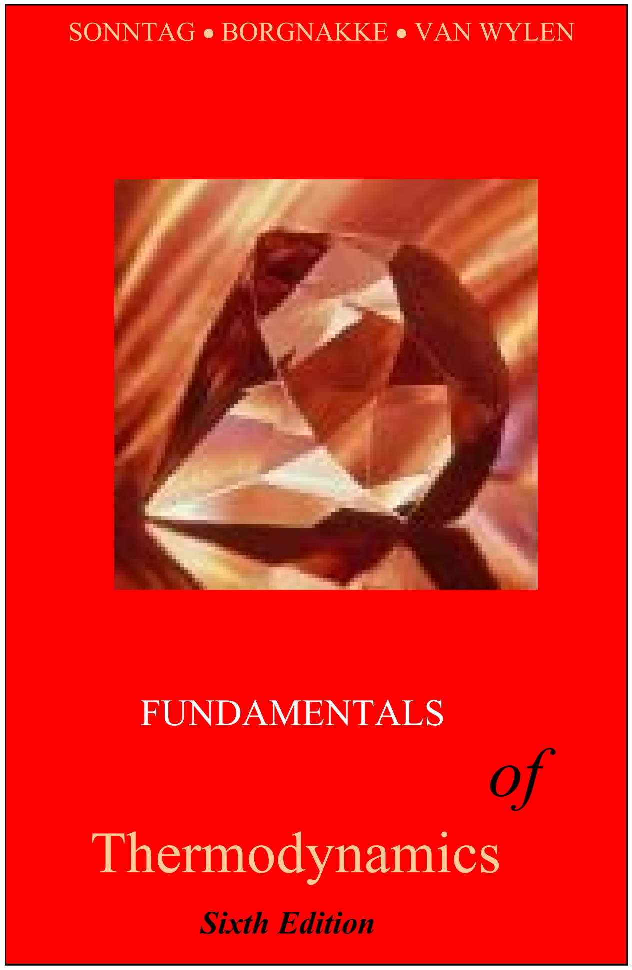
SOLUTION MANUAL
SI UNIT PROBLEMS
CHAPTER 16



Fundamentals of Thermodynamics 6th Edition
Sonntag, Borgnakke and van Wylen

CONTENT CHAPTER 16

CHAPTER 16 6th ed. CORRESPONDANCE TABLE

Notice that most of the solutions are done using the computer tables, which includes the steam tables, air table, compressible flow table and the normal shock table. This significantly reduces the amount of time it will take to solve a problem, so this should be considered in problem assignments and exams.

Changes of problems from the 5th edition Chapter 16 are:
Problems 1-20 are all new

New	5th Ed.	New	5th Ed.	New	5th Ed.
21	1	39	15	57	22
22	2	40	new	58	25b
23	new	41	23	59	32
24	3	42	24	60	new
25	4	43	26	61	30
26	5	44	new	62	31
27	6	45	new	63	33
28	7	46	new	64	34
29	8	47	17	65	37
30	new	48	36	66	38
31	9	49	16	67	35
32	10	50	25a	68	41
33	new	51	27a, b	69	39
34	11	52	27c	70	42
35	12	53	28a, b	71	40
36	new	54	28c	72	19
37	13	55	29	73	43
38	14	56	18		

New	5th	SI	New	5th	SI
74	44E	-	81	52E	43
75	45E	24	82	51E	41
76	46E	25	83	53E	55
77	47E	29	84	55E	53, 54
78	48E	34	85	54E	62
79	49E	37	86	56E	63
80	50E	47	87	57E	71

Concept-Study Guide Problems

16.1

Is stagnation temperature always higher than free stream temperature? Why?

Yes. Since kinetic energy can only be positive we have

$$h_0 = h_1 + \mathbf{V}_1^2/2 \; > \; h_1$$

If it is a gas with constant heat capacity we get

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p$$

16.2

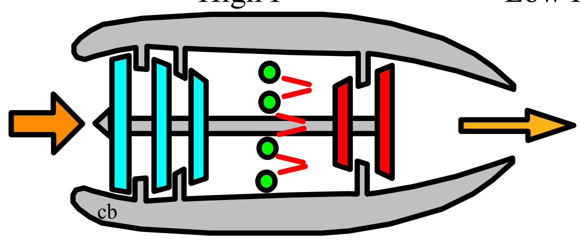
Which temperature does a thermometer or thermocouple measure? Would you ever need to make a correction to that?

Since the probe with the thermocouple in its tip is stationary relative to the moving fluid it will measure something close to the stagnation temperature. If that is high relative to the free stream temperature there will be significant heat transfer (convection and radiation) from the probe and it will measure a little less. For very high accuracy temperature measurements you must make some corrections for these effects.

16.3

The jet engine thrust is found from the overall momentum equation. Where is the actual force acting (it is not a long-range force in the flow)?

The compressor is generating the high pressure flow so the blades pushes hard on the flow and thus a force acts in the forward direction on the shaft holding the rotating blades. The high pressure in the chamber with combustion also has a net force in the forward direction as the flow leaves in the backwards direction so less wall area there. The pressure drop in the turbine means its blades pushes in the other direction but as the turbine exit pressure is higher than the ambient pressure the axial force is less than that of the compressor.



16.4

How large a force must be applied to a squirt gun to have 0.1 kg/s water flow out at 20 m/s? What pressure inside the chamber is needed?

$$F = \frac{d\, m\mathbf{V}}{dt} = \dot{m}\,\mathbf{V} = 0.1 \times 20 \ \text{kg m/s}^2 = \mathbf{2\ N}$$

Eq.16.21: $v\Delta P = 0.5\ \mathbf{V}^2$

$$\Delta P = 0.5\ \mathbf{V}^2/v = 0.5 \times 20^2 / 0.001 = 200\,000\ \text{Pa} = \mathbf{200\ kPa}$$

16.5

By looking at Eq. 16.25, rank the speed of sound for a solid, a liquid, and a gas.

Speed of sound: $\left(\frac{\partial P}{\partial \rho}\right)_s = c^2$

For a solid and liquid phase the density varies only slightly with temperature and constant s is also nearly constant T. We thus expect the derivative to be very high that is we need very large changes in P to give small changes in density.

A gas is highly compressible so the formula reduces to Eq.16.28 which gives modest values for the speed of sound.

16.6

Does speed of sound in an ideal gas depend on pressure? What about a real gas?

No. For an ideal gas the speed of sound is given by Eq.16.28

$$c = \sqrt{kRT}$$

and is only a function of temperature T.

For a real gas we do not recover the simple expression above and there is a dependency on P particularly in the dense gas region above the critical point.

16.7

Can a convergent adiabatic nozzle produce a supersonic flow?

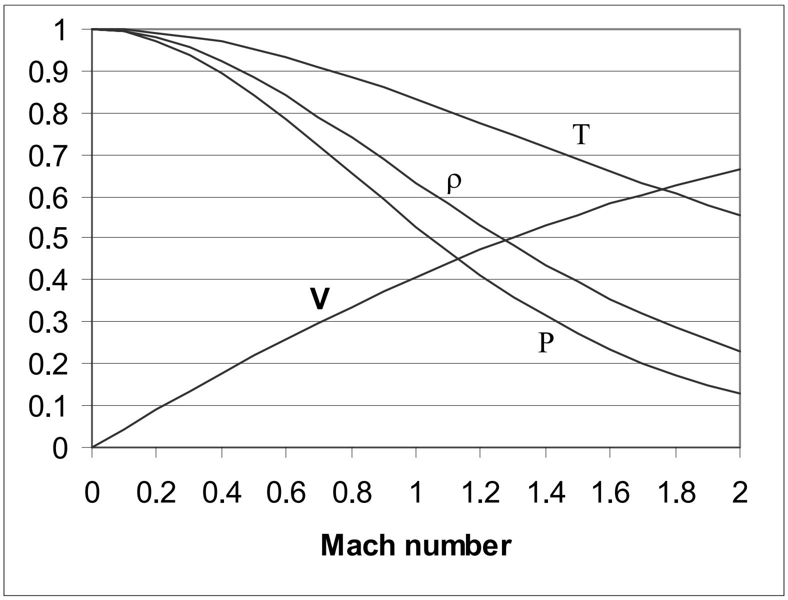
No. From Eq.16.33 and a nozzle so $dP < 0$ it is required to have $dA > 0$ to reach $M > 1$. A convergent nozzle will have $M = 1$ at the exit, which is the smallest area. For lower back pressures there may be a shock standing in the exit plane.

16.8

Sketch the variation in **V**, T, P, ρ and M for a subsonic flow into a convergent nozzle with $M = 1$ at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number M. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.16.34-36. A small spread sheet (M step 0.1) did the calculations.



The curves are plotted as the variables:

T / T_o

ρ / ρ_o

P / P_o

$V / \sqrt{2C_p T_o}$

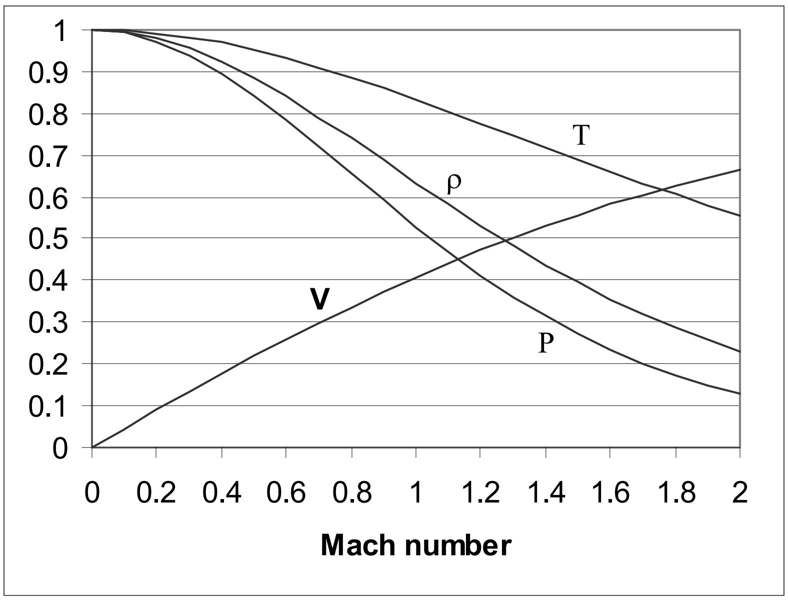
and for k = 1.4

16.9

Sketch the variation in **V**, T, P, ρ and M for a sonic ($M = 1$) flow into a divergent nozzle with $M = 2$ at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number M. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.16.34-36.



The curves are plotted as the variables:

T / T_o

ρ / ρ_o

P / P_o

$V / \sqrt{2C_p T_o}$

and for k = 1.4

16.10

To maximize the mass flow rate of air through a given nozzle, which properties should I try to change and in which direction, higher or lower?

The mass flow rate is given by Eq.16.41 and if we have $M = 1$ at the throat then Eq.16.42 gives the maximum mass flow rate possible.

Max flow for:

Higher upstream stagnation pressure
Lower upstream stagnation temperature

16.11

How do the stagnation temperature and pressure change in an isentropic flow?

The stagnation temperature and stagnation pressure are constant.

16.12

Can any low enough backup pressure generate an isentropic supersonic flow?

No. Only one back pressure corresponds to a supersonic flow, which is the exit pressure at state d in Figure 16.13. However a pressure lower than that can give an isentropic flow in the nozzle, case e, with a drop in pressure outside the nozzle. This is irreversible leading to an increase in s and therefore not isentropic.

16.13

Is there any benefit to operate a nozzle choked?

Yes. Since the mass flow rate is constant (max value) between points c and d in Fig. 16.12 a small variation in the back pressure will not have any influence. The nozzle then provides a constant mass flow rate free of surges up or down which is very useful for flow calibrations or other measurements where a constant mass flow rate is essential.

16.14

To increase the flow through a choked nozzle, the flow can be heated/cooled or compressed/expanded (four processes) before or after the nozzle. Explain which of these eight possibilities will help and which will not.

The mass flow rate through a choked nozzle is given by Eq.16.42. Since k and R are constant it varies with the upstream stagnation properties P_o and T_o.

After nozzle: Any downstream changes have **no effects**.

Before nozzle: Upstream changes in P_o and T_o has an influence.

a. Heat This lowers mass flow rate (T_o increases)

b. Cool This raises mass flow rate (T_o decreases)

c. Compress. Raises P_o and T_o opposite effects.

Isentropic: $P_{o\,new} = P_o\, r_p$ and $T_{o\,new} = T_o\, (r_p)^{\frac{k-1}{k}}$

$$P_{o\,new} / \sqrt{T_{o\,new}} = (r_p)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o}\right] > \left[P_o / \sqrt{T_o}\right]$$

So the mass flow rate increases

d. Expand. Lowers P_o and T_o opposite effects. Assume isentropic, then mass flow rate decreases.

16.15

Which of the cases in Fig. 16.17 (a-h) have entropy generation and which do not?

a. There is no flow so $s_{gen} = 0$.
b. Subsonic flow, reversible, so $s_{gen} = 0$.
c. Limit for subsonic flow, reversible, so $s_{gen} = 0$.
d. The only supersonic reversible flow solution, so $s_{gen} = 0$.
e. Supersonic reversible in nozzle $s_{gen} = 0$, irreversible outside.
f. Supersonic reversible in nozzle $s_{gen} = 0$, compression outside.
g. Shock stands at exit plane, $s_{gen} > 0$ across shock.
h. Shock is located inside nozzle, $s_{gen} > 0$ across shock.

16.16

A given convergent nozzle operates so it is choked with stagnation inlet flow properties of 400 kPa, 400 K. To increase the flow, a reversible adiabatic compressor is added before the nozzle to increase the stagnation flow pressure to 500 kPa. What happens to the flow rate?

Since the nozzle is choked the mass flow rate is given by Eq.16.42. The compressor changes the stagnation pressure and temperature.

Isentropic: $P_{o\ new} = P_o\ r_p$ and $T_{o\ new} = T_o\ (r_p)^{\frac{k-1}{k}}$

$$P_{o\ new} / \sqrt{T_{o\ new}} = (r_p)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o}\right]$$

so the mass flow rate is multiplied with the factor

$$(r_p)^{\frac{k+1}{2k}} = \left(\frac{500}{400}\right)^{\frac{2.4}{2.8}} = 1.21$$

16.17

How much entropy per kg flow is generated in the shock in Example 16.9?

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.32 - 0.287 \ln 2.4583$$

$$= 0.27874 - 0.25815 = \mathbf{0.0206\ kJ/kg\ K}$$

Notice that could have been tabulated also.

16.18

Suppose a convergent-divergent nozzle is operated as case h in Fig. 16.17. What kind of nozzle could have the same exit pressure but with a reversible flow?

A convergent nozzle, having subsonic flow everywhere assuming the pressure ratio is higher than the critical.

16.19

How does the stagnation temperature and pressure change in an adiabatic nozzle flow with an efficiency of less than 100%?

The stagnation temperature stays constant (energy eq.)
The stagnation pressure drops (s is generated, less kinetic energy).

16.20

How high can a gas velocity (Mach number) be and still treat it as incompressible flow within 2% error?

The relative error in the ΔP versus kinetic energy, Eq.16.66, becomes

$$e = \frac{1}{4}\left(\frac{\mathbf{V}}{c_o}\right)^2 = 0.02 \quad \Rightarrow \quad M = \frac{\mathbf{V}}{c_o} = \sqrt{4 \times 0.02} = \mathbf{0.283}$$

Stagnation Properties

16.21

Steam leaves a nozzle with a pressure of 500 kPa, a temperature of 350°C, and a velocity of 250 m/s. What is the isentropic stagnation pressure and temperature?

Stagnation enthalpy from energy equation and values from steam tables B.1.3

$$h_0 = h_1 + \mathbf{V}_1^2/2 = 3167.7 + \frac{250^2}{2000} = 3198.4 \text{ kJ/kg}$$

$$s_0 = s_1 = 7.6329 \text{ kJ/kg K}$$

It can be linearly interpolated from the printed tables

Computer software: $(h_o, s_o) \Rightarrow T_o$ = **365°C**, P_o = **556 kPa**

16.22

An object from space enters the earth's upper atmosphere at 5 kPa, 100 K, with a relative velocity of 2000 m/s or more . Estimate the object`s surface temperature.

$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 2000^2/2000 = 2000$ kJ/kg

$h_{o1} = h_1 + 2000 = 100 + 2000 = 2100$ kJ/kg => $T =$ **1875 K**

The value for h_1 from ideal gas table A.7 was estimated since the lowest T in the table is 200 K.

16.23

Steam is flowing to a nozzle with a pressure of 400 kPa. The stagnation pressure and temperature are measured to be 600 kPa and 350°C, respectively. What are the flow velocity and temperature?

Stagnation state Table B.1.3: $h_{o1} = 3165.66$ kJ/kg, $s_{o1} = 7.5463$ kJ/kg K

State 1: 400 kPa, $s_1 = s_{o1} = 7.5463$ kJ/kg K

$$T_1 = 250 + (300 - 250)\,\frac{7.5463 - 7.3788}{7.5661 - 7.3788} = \mathbf{294.7^oC}$$

$$h_1 = 2964.16 + \frac{7.5463 - 7.3788}{7.5661 - 7.3788}\,(3066.75 - 2964.16) = 3055.9 \text{ kJ/kg}$$

Energy equation gives

$$\mathbf{V}_1^2/2 = h_{o1} - h_1 = 3165.66 - 3055.9 = 109.76 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2 \times (h_{o1} - h_1)} = \sqrt{2 \times 109.76 \times 1000} = \mathbf{468.5\ m/s}$$

16.24

The products of combustion of a jet engine leave the engine with a velocity relative to the plane of 400 m/s, a temperature of 480°C, and a pressure of 75 kPa. Assuming that $k = 1.32$, $C_p = 1.15$ kJ/kg K for the products, determine the stagnation pressure and temperature of the products relative to the airplane.

Energy Eq.: $h_{o1} - h_1 = \mathbf{V}_1^2/2 = 400^2/2000 = 80$ kJ/kg

$T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 80/1.15 = 69.6$ K

$T_{o1} = 480 + 273.15 + 69.6 =$ **823 K**

Isentropic process relates to the stagnation pressure

$P_{o1} = P_1(T_{o1}/T_1)^{k/(k-1)} = 75(823/753.15)^{4.125} =$ **108 kPa**

16.25

A meteorite melts and burn up at temperatures of 3000 K. If it hits air at 5 kPa, 50 K how high a velocity should it have to experience such a temperature?

Assume we have a stagnation T = 3000 K

$h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Use table A.7, $h_{stagn.} = 3525.36$ kJ/kg, $h_1 = 50$ kJ/kg

$\mathbf{V}_1^2/2 = 3525.36 - 50 = 3475.4$ kJ/kg (remember convert to J/kg = m^2/s^2)

$\mathbf{V}_1 = \sqrt{2 \times 3475.4 \times 1000} = \mathbf{2636\ m/s}$

16.26

I drive down the highway at 110 km/h on a day with 25°C, 101.3 kPa. I put my hand, cross sectional area 0.01 m^2, flat out the window. What is the force on my hand and what temperature do I feel?

The air stagnates on the hand surface : $h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Use constant heat capacity

$$T_{stagn.} = T_1 + \frac{\mathbf{V}_1^2/2}{C_p} = 25 + \frac{0.5 \times 110^2 \times (1000/3600)^2}{1004} = \mathbf{25.465°C}$$

Assume a reversible adiabatic compression

$$P_{stagn.} = P_1\ (T_{stagn.}/T_1)^{k/(k-1)} = 101.3\ (298.615/298.15)^{3.5}$$

$$= \mathbf{101.85\ kPa}$$

16.27

Air leaves a compressor in a pipe with a stagnation temperature and pressure of 150°C, 300 kPa, and a velocity of 125 m/s. The pipe has a cross-sectional area of 0.02 m^2. Determine the static temperature and pressure and the mass flow rate.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 125^2/2000 = 7.8125 \text{ kJ/kg}$$

$$T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 7.8125/1.004 = 7.8 \text{ K}$$

$$T_1 = T_{o1} - \Delta T = 150 - 7.8 = \mathbf{142.2\ °C = 415.4\ K}$$

$$P_1 = P_{o1}(T_1/T_{o1})^{k/(k-1)} = 300(415.4/423.15)^{3.5} = \mathbf{281\ kPa}$$

$$\dot{m} = \rho A\mathbf{V} = \frac{A\mathbf{V}}{v} = \frac{P_1 A\mathbf{V}_1}{RT_1} = \frac{281.2(0.02)(125)}{0.287(415.4)} = \mathbf{5.9\ kg/s}$$

16.28

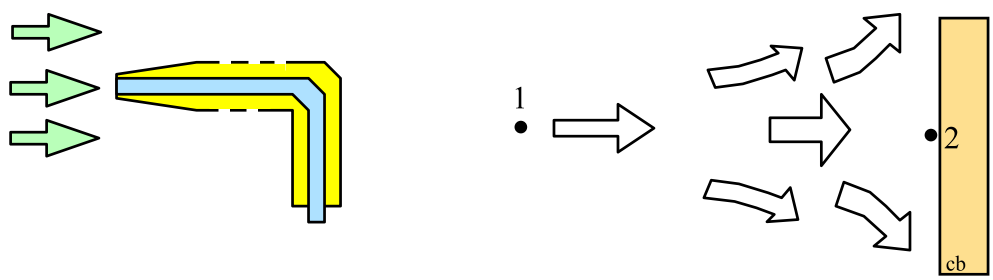
A stagnation pressure of 108 kPa is measured for an airflow where the pressure is 100 kPa and 20°C in the approach flow. What is the incomming velocity?

Assume a reversible adiabatic compression

$T_{o1} = T_1 \times (P_{o1}/P_1)^{(k-1)/k} = 293.15 \times (\frac{108}{100})^{0.2857} = 299.67 \text{ K}$

$\mathbf{V}_1^2/2 = h_{o1} - h_1 = C_p (T_{o1} - T_1) = 6.543 \text{ kJ/kg}$

$\mathbf{V}_1 = \sqrt{2 \times 6.543 \times 1000} = \mathbf{114.4\ m/s}$



To the left a Pitot tube, blue inner tube measures stagnation pressure and yellow outer tube with holes in it measures static pressure. To the right is a stagnation point on a wall relative to the free stream flow at state 1.

16.29

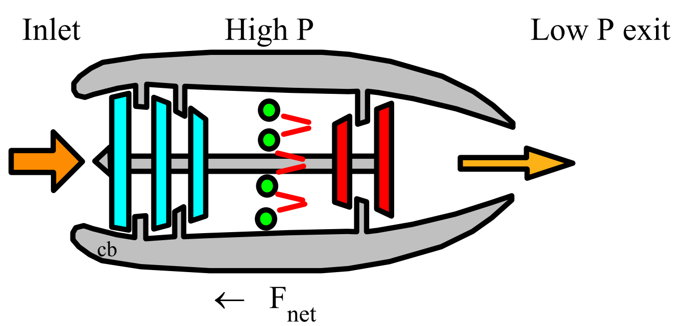
A jet engine receives a flow of 150 m/s air at 75 kPa, 5°C across an area of 0.6 m^2 with an exit flow at 450 m/s, 75 kPa, 600 K. Find the mass flow rate and thrust.

$\dot{m} = \rho A\mathbf{V}$; ideal gas $\rho = P/RT$

$$\dot{m} = (P/RT)A\mathbf{V} = \left(\frac{75}{0.287 \times 278.15}\right) \times 0.6 \times 150 = 0.9395 \times 0.6 \times 150$$

$= \mathbf{84.555\ kg/s}$

$$F_{net} = \dot{m}\,(\mathbf{V}_{ex} - \mathbf{V}_{in}) = 84.555 \times (450 - 150) = \mathbf{25\ 367\ N}$$



The shaft must have axial load bearings to transmit thrust to aircraft.

16.30

A 4-cm inner diameter pipe has an inlet flow of 10 kg/s water at 20°C, 200 kPa. After a 90 degree bend as shown in Fig. P16.30, the exit flow is at 20°C, 190 kPa. Neglect gravitational effects and find the anchoring forces F_x and F_y.

$$D = 0.04 \text{ m} \quad \Rightarrow \quad A = \frac{\pi}{4} D^2 = 0.001257 \text{ m}^2$$

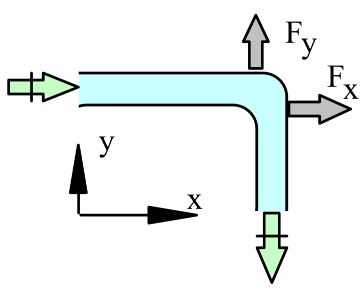
$$\mathbf{V}_{avg} = \frac{\dot{m}}{\rho A} = \frac{10 \times 0.001002}{0.001257} = 7.971 \text{ m/s}$$

Now we can do the x and y direction momentum equations for steady flow and the same magnitude of the velocity, but different directions

X-dir: $\quad 0 = \dot{m}\, \mathbf{V}_{avg\,1} + F_x - \dot{m} \times 0 + (P_1 - P_o)\, A$

Y-dir: $\quad 0 = \dot{m} \times 0 + F_y - \dot{m} \times (-\mathbf{V}_{avg\,2}) + (P_2 - P_o)\, A$

$$F_x = -\, \dot{m}\, \mathbf{V}_{avg\,1} - (P_1 - P_o)\, A$$
$$= -10 \times 7.97 - 100 \times 0.001257 \times 1000 = \mathbf{-205\ N}$$
$$F_y = -\, \dot{m}\, \mathbf{V}_{avg\,2} - (P_2 - P_o)\, A$$
$$= -\,10 \times 7.97 - 90 \times 0.001257 \times 1000 = \mathbf{-193\ N}$$



16.31

A water cannon sprays 1 kg/s liquid water at a velocity of 100 m/s horizontally out from a nozzle. It is driven by a pump that receives the water from a tank at 15°C, 100 kPa. Neglect elevation differences and the kinetic energy of the water flow in the pump and hose to the nozzle. Find the nozzle exit area, the required pressure out of the pump and the horizontal force needed to hold the cannon.

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v \;\Rightarrow\; A = \dot{m}v/\mathbf{V} = 1\,\frac{0.001001}{100} = \mathbf{1.0 \times 10^{-5}\ m^2}$$

$$\dot{W}_p = \dot{m}w_p = \dot{m}v(P_{ex} - P_{in}) = \dot{m}\mathbf{V}_{ex}^2/2$$

$$P_{ex} = P_{in} + \mathbf{V}_{ex}^2/2v = 100 + 100^2/2 \times 1000 \times 0.001 = \mathbf{150\ kPa}$$

$$F = \dot{m}\mathbf{V}_{ex} = 1 \times 100 = \mathbf{100\ N}$$

16.32

An irrigation pump takes water from a lake and discharges it through a nozzle as shown in Fig. P16.32. At the pump exit the pressure is 700 kPa, and the temperature is 20°C. The nozzle is located 10 m above the pump and the atmospheric pressure is 100 kPa. Assuming reversible flow through the system determine the velocity of the water leaving the nozzle.

Assume we can neglect kinetic energy in the pipe in and out of the pump. Incompressible flow so Bernoulli's equation applies ($\mathbf{V}_1 \cong \mathbf{V}_2 \cong \mathbf{V}_3 \cong 0$)

$$v(P_3 - P_2) + (\mathbf{V}_3^2 - \mathbf{V}_2^2)/2 + g(Z_3 - Z_2) = 0$$

$$P_3 = P_2 - \frac{g(Z_3 - Z_2)}{v} = 700 - \frac{9.807(10)}{1000(0.001002)} = 602 \text{ kPa}$$

$$\mathbf{V}_4^2/2 = v(P_3 - P_4)$$

$$\Rightarrow \mathbf{V}_4 = \sqrt{2v(P_3 - P_4)} = \sqrt{2 \times 0.001002 \times 502.1 \times 1000} = \mathbf{31.72\ m/s}$$

16.33

A jet engine at takeoff has air at 20°C, 100 kPa coming at 25 m/s through the 1.0 m diameter inlet. The exit flow is at 1200 K, 100 kPa, through the exit nozzle of 0.4 m diameter. Neglect the fuel flow rate and find the net force (thrust) on the engine.

$$A_1 = \frac{\pi}{4} D^2 = 0.7854 \text{ m}^2; \qquad A_2 = \frac{\pi}{4} D^2 = 0.1257 \text{ m}^2$$

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 293.15}{100} = 0.8409 \text{ m}^3\text{/kg}; \quad v_2 = 3.444 \text{ m}^3\text{/kg}$$

$$\dot{m} = A\mathbf{V}/v = A_1\mathbf{V}_1/v_1 = \frac{0.7854 \times 25}{0.8409} = 48.0 \text{ kg/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}v_2}{A_2} = \frac{48.0 \times 3.444}{0.1257} = 1315 \text{ m/s}$$

Now we can do the x direction momentum equation for steady flow and the same mass flow rate in and out

X-dir: $$0 = \dot{m}\,\mathbf{V}_1 + F_x + (P_1 - P_o)\,A_1 - \dot{m}\,\mathbf{V}_2 - (P_2 - P_o)\,A_2$$

$$F_x = -\,\dot{m}\,\mathbf{V}_1 - (P_1 - P_o)\,A_1 + \dot{m}\,\mathbf{V}_2 + (P_2 - P_o)\,A_2$$
$$= \dot{m}\,(\mathbf{V}_2 - \mathbf{V}_1) - 0 + 0 = 48\,(1315 - 25) = \mathbf{61\ 920\ N}$$

16.34

A water turbine using nozzles is located at the bottom of Hoover Dam 175 m below the surface of Lake Mead. The water enters the nozzles at a stagnation pressure corresponding to the column of water above it minus 20% due to losses. The temperature is 15°C and the water leaves at standard atmospheric pressure. If the flow through the nozzle is reversible and adiabatic, determine the velocity and kinetic energy per kilogram of water leaving the nozzle.

$$\Delta P = \rho g \Delta Z = \frac{g\Delta Z}{v} = \frac{9.807 \times 175}{0.001001 \times 1000} = 1714.5 \text{ kPa}$$

$$\Delta P_{ac} = 0.8\ \Delta P = 1371.6 \text{ kPa}$$

$$v\Delta P = \mathbf{V}_{ex}^2/2 \;\Rightarrow\; \mathbf{V}_{ex} = \sqrt{2v\Delta P}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 0.001001 \times 1000 \times 1371.6} = \mathbf{62.4\ m/s}$$

$$\mathbf{V}_{ex}^2/2 = v\Delta P = \mathbf{1.373\ kJ/kg}$$

16.35

A water tower on a farm holds 1 m^3 liquid water at 20°C, 100 kPa in a tank on top of a 5 m tall tower. A pipe leads to the ground level with a tap that can open a 1.5 cm diameter hole. Neglect friction and pipe losses, and estimate the time it will take to empty the tank for water.

Incompressible flow so we can use Bernoulli Equation.

$$P_e = P_i; \quad \mathbf{V}_i = 0; \quad Z_e = 0; \quad Z_i = H$$

$$\mathbf{V}_e^2/2 = gZ_i \quad \Rightarrow \quad \mathbf{V}_e = \sqrt{2gZ} = \sqrt{2 \times 9.807 \times 5} = 9.9 \text{ m/s}$$

$$\dot{m} = \rho A \mathbf{V}_e = A\mathbf{V}_e/v = \Delta m/\Delta t$$

$$\Delta m = V/v; \quad A = \pi D^2/4 = \pi \times 0.015^2 / 4 = 1.77 \times 10^{-4} \text{ m}^2$$

$$\Rightarrow \quad \Delta t = \Delta m v/A\mathbf{V}_e = V/A\mathbf{V}_e$$

$$\Delta t = \frac{1}{1.77 \times 10^{-4} \times 9.9} = \mathbf{571.6\ sec} = \mathbf{9.53\ min}$$

16.36

Find the expression for the anchoring force R_x for an incompressible flow like in Figure 16.6. Show that it can be written as

$$R_x = \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e}\left[(P_i - P_o)A_i + (P_e - P_o)A_e\right]$$

Apply the X-dir momentum equation for a steady flow

$$0 = R_x + (P_i - P_o)A_i - (P_e - P_o)A_e + \dot{m}\mathbf{V}_i - \dot{m}\mathbf{V}_e$$

Bernoulli equation for the flow is

$$0.5(\mathbf{V}_e^2 - \mathbf{V}_i^2) + v\,(P_e - P_i) = 0 \qquad \Rightarrow \qquad \mathbf{V}_e - \mathbf{V}_i = \frac{2v\,(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e}$$

Continuity equation gives

$$\dot{m} = A_i\mathbf{V}_i/v = A_e\mathbf{V}_e/v$$

Solve for R_x from the momentum equation

$$R_x = \dot{m}\,(\mathbf{V}_e - \mathbf{V}_i) + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{A_i\mathbf{V}_i}{v}\,\frac{2v\,(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e} + (P_e - P_o)A_e - (P_i - P_o)A_i$$

Multiply in and use continuity equation for second term

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\left[P_iA_i\mathbf{V}_i - P_eA_e\mathbf{V}_e\right] + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\Big[P_iA_i\mathbf{V}_i - P_eA_e\mathbf{V}_e + \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i$$
$$+ \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e\Big]$$

Now put the first four terms together

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\Big[\tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i - \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e$$
$$+ \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e\Big]$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\left[\tfrac{1}{2}(P_i - P_o)A_i(\mathbf{V}_i - \mathbf{V}_e) + \tfrac{1}{2}(P_e - P_o)A_e(\mathbf{V}_i - \mathbf{V}_e)\right]$$

$$= \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e}\left[(P_i - P_o)A_i + (P_e - P_o)A_e\right]$$

16.37

Find the speed of sound for air at 100 kPa at the two temperatures 0°C and 30°C. Repeat the answer for carbon dioxide and argon gases.

From eq. 16.28 we have

$$c_0 = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 273.15 \times 1000} = \mathbf{331\ m/s}$$

$$c_{30} = \sqrt{1.4 \times 0.287 \times 303.15 \times 1000} = \mathbf{349\ m/s}$$

For Carbon Dioxide: R = 0.1889 kJ/kg K, k = 1.289

$$c_0 = \sqrt{1.289 \times 0.1889 \times 273.15 \times 1000} = \mathbf{257.9\ m/s}$$

$$c_{30} = \sqrt{1.289 \times 0.1889 \times 303.15 \times 1000} = \mathbf{271.7\ m/s}$$

For Argon: R = 0.2081 kJ/kg K, k = 1.667

$$c_0 = \sqrt{1.667 \times 0.2081 \times 273.15 \times 1000} = \mathbf{307.8\ m/s}$$

$$c_{30} = \sqrt{1.667 \times 0.2081 \times 303.15 \times 1000} = \mathbf{324.3\ m/s}$$

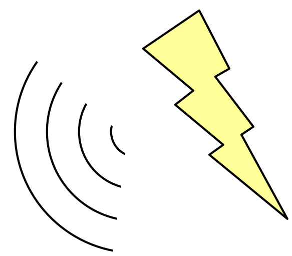
16.38

If the sound of thunder is heard 5 seconds after the lightning is seen and the weather is 20°C. How far away is the lightning taking place?

The sound travels with the speed of sound in air (ideal gas). Use the formula in Eq.16.28

$$L = c \times t = \sqrt{kRT} \times t = \sqrt{1.4 \times 0.287 \times 293.15 \times 1000} \times 5 = \mathbf{1716\ m}$$

For every 3 seconds after the lightning the sound travels about 1 km.



16.39

Estimate the speed of sound for steam directly from Eq. 16.25 and the steam tables for a state of 6 MPa, 400°C. Use table values at 5 and 7 MPa at the same entropy as the wanted state. Eq. 16.25 is then done by finite difference. Find also the answer for the speed of sound assuming steam is an ideal gas.

Eq.16.25: $c^2 = \left(\frac{\delta P}{\delta \rho}\right)_s = \left(\frac{\Delta P}{\Delta \rho}\right)_s$

State 6 MPa, 400°C ⇒ $s = 6.5407$ kJ/kg K

7 MPa, s ⇒ $v = 0.04205\ m^3/kg$; $\rho = 1/v = 23.777\ kg/m^3$

5 MPa, s ⇒ $v = 0.05467\ m^3/kg$; $\rho = 1/v = 18.2909\ kg/m^3$

$$c^2 = \frac{7000 - 5000}{23.777 - 18.2909} = 364.56 \times 1000 \quad \Rightarrow \quad c = \mathbf{603.8\ m/s}$$

From Table A.8: $C_p = \frac{1338.56 - 1235.3}{50} = 2.0652$ kJ/kg K

$C_v = C_p - R = 2.0652 – 0.4615 = 1.6037$ kJ/kg K

$k = C_p/C_v = 1.288$; $R = 0.4615$ kJ/kg K (from A.5)

Now do the speed of sound from Eq.16.28

$$c = \sqrt{kRT} = \sqrt{1.288 \times 0.4615 \times 673.15 \times 1000} = \mathbf{632.6\ m/s}$$

16.40

The speed of sound in liquid water at 25°C is about 1500 m/s. Find the stagnation pressure and temperature for a $M = 0.1$ flow at 25°C, 100 kPa. Is it possible to get a significant mach number flow of liquid water?

$$\mathbf{V} = M\,c = 0.1 \times 1500 = 150 \text{ m/s}$$

$$h_0 = h_1 + \mathbf{V}_1^2/2$$

Bernoulli Eq.: $\Delta P = \mathbf{V}_1^2/2v = \dfrac{150^2}{2 \times 0.001} = 11.25 \times 10^6 \text{ Pa} = 11.25 \text{ MPa}$

$$P_0 = P_1 + \Delta P = 100 + 11\,250 = \mathbf{11\,350\ kPa}$$

$$T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = 25 + \frac{150^2}{2 \times 4180} = \mathbf{27.7^oC}$$

Remark: Notice the very high pressure. To get a higher velocity you need a higher pressure to accelerate the fluid, that is not feasible for any large flow rate.

16.41

A convergent nozzle has a minimum area of 0.1 m^2 and receives air at 175 kPa, 1000 K flowing with 100 m/s. What is the back pressure that will produce the maximum flow rate and find that flow rate?

$$\frac{P^*}{P_o} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528 \qquad \text{Critical Pressure Ratio}$$

Find P_o:

$h_0 = h_1 + \mathbf{V}_1^2/2 = 1046.22 + 100^2/2000 = 1051.22$ kJ/kg

$T_0 = T_i + 4.4 = 1004.4$ K from table A.7

$P_0 = P_i\ (T_0/T_i)^{k/(k-1)} = 175 \times (1004.4/1000)^{3.5} = 177.71$ kPa

The mass flow rate comes from the throat properties

$P^* = 0.528\ P_o = 0.528 \times 177.71 =$ **93.83 kPa**

$T^* = 0.8333\ T_o = 836.97$ K

$$\rho^* = \frac{P^*}{RT^*} = \frac{93.83}{0.287 \times 836.97} = 0.3906 \text{ kg/m}^3$$

$\mathbf{V} = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 836.97} =$ **579.9 m/s**

$\dot{m} = \rho A\mathbf{V} = 0.3906 \times 0.1 \times 579.9 =$ **22.65 kg/s**

16.42

A convergent-divergent nozzle has a throat area of 100 mm^2 and an exit area of 175 mm^2. The inlet flow is helium at a stagnation pressure of 1 MPa, stagnation temperature of 375 K. What is the back pressure that will give sonic condition at the throat, but subsonic everywhere else?

For this flow we have helium with k_{He} = 1.667, so we cannot use the tables for air. We need the solution to the curve labeled c in Fig. 16.13. For critical flow at the throat we have from Table 16.1 last column

$$P^* = 0.4867 \; P_o = 486.7 \text{ kPa}$$

Now we need to find the conditions where the area ratio is

$$A_E/A^* = 175/100 = 1.75$$

that is solve for M in Eq. 16.43 given the area ratio. This is nonlinear so we have to iterate on it. Here (k+1)/2(k-1) = 2 so look also at Fig. 16.10 for the general shape.

$$M = 0.4 \;\Rightarrow\; A/A^* = (1/0.4)\,[\,0.75(1 + 0.3333{*}0.4^2)\,]^2 = 1.5602$$

$$M = 0.3 \;\Rightarrow\; A/A^* = (1/0.3)\,[\,0.75(1 + 0.3333{*}0.3^2)\,]^2 = 1.9892$$

$$M = 0.35 \;\Rightarrow\; A/A^* = (1/0.35)\,[\,0.75(1 + 0.3333{*}0.35^2)\,]^2 = 1.7410$$

$$M = 0.34 \;\Rightarrow\; A/A^* = (1/0.34)\,[\,0.75(1 + 0.3333{*}0.34^2)\,]^2 = 1.7844$$

Now do a linear interpolation for the rest to get $M_E = 0.348$;

Eq. 16.35 $\quad P_E/P_o = [1 + 0.3333{*}0.348^2\,]^{-2.5} = 0.9058$

$$P_E = 0.9058 \times 1000 = \mathbf{906\ kPa}$$

16.43

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is −12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$\mathbf{V}$ = 1000 km/h = 277.8 m/s, $v_1 = RT/P = 0.287 \times 261.15/40 = 1.874\ m^3/kg$

h_1 = 261.48 kJ/kg,

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07$ kJ/kg

$\Rightarrow\ T_{o1} = 299.7$ K,

$P_{o1} = P_1\ (T_{o1}/T_1)^{k/(k-1)} = 40 \times (299.7/261.15)^{3.5} = 64.766$ kPa

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\Rightarrow$ T_2 = **294.7 K**,

$P_2 = P_{o1}\ (T_2/T_{o1})^{k/(k-1)} = 64.766 \times (294.7/299.7)^{3.5}$ = **61 kPa**

$v_2 = 0.287 \times 294.7/61 = 1.386\ m^3/kg$

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.386)(100/277.8)$ = **0.487**

16.44

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K to a backpressure of 1.9 MPa. If the exit cross-sectional area is 0.003 m^2, find the mass flow rate.

This corresponds to case c and is a reversible flow.

$P_E/P_{ox} = 1.9/2.0 = 0.95 \quad \Rightarrow$ Table A.12: $\quad M_E = 0.268$

$T_E = (T/T_o)_E \, T_o = 0.9854 \times 600 = 591.2$ K

$c_E = \sqrt{kRT_E} = \sqrt{1.4 \times 1000 \times 0.287 \times 591.2} = 487.4$ m/s

$\mathbf{V}_E = M_E c_E = 0.268 \times 487.4 = 130.6$ m/s

$v_E = RT/P = 0.287 \times 591.2/1900 = 0.0893$ m^3/kg

$\dot{m} = A_E \mathbf{V}_E / v_E = 0.002435 \times 130.6/0.0893 =$ **3.561 kg/s**

16.45

Air flows into a convergent-divergent nozzle with an exit area of 1.59 times the throat area of 0.005 m^2. The inlet stagnation state is 1 MPa, 600 K. Find the backpressure that will cause subsonic flow throughout the entire nozzle with M = 1 at the throat. What is the mass flow rate?

This corresponds to case c and is a reversible flow.

$A_E/A^* = 1.59$ Look at top in Table A.12 ($M < 1$)

$M_E = 0.4$ and $P_E/P_o = 0.8956$

$P_E = 0.8956\ P_o = 0.8956 \times 1000 =$ **896 kPa**

To find the mass flow rate we need the throat conditions, see Table 16.1,

$$T^* = T\,\frac{2}{k+1} = 600 \times 0.8333 = 500\ \text{K}$$

$$v^* = RT^*/P^* = 0.287 \times 500 \,/\, 528.3 = 0.2716\ \text{m}^3/\text{kg}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 0.287 \times 500 \times 1000} = 448.22\ \text{m/s}$$

$$\dot{m} = A^*c^*/v^* = \frac{0.005 \times 448.22}{0.2716} = \mathbf{8.251\ kg/s}$$

16.46

Air flows into a convergent-divergent nozzle with an exit area of 2.0 times the throat area of 0.005 m^2. The inlet stagnation state is 1 MPa, 600 K. Find the backpressure that will cause a reversible supersonic exit flow with M = 1 at the throat. What is the mass flow rate?

This flow is case d in Fig.16.17 the only reversible supersonic flow.

$$A_E/A^* = 2 \quad \text{see Table A.12} \quad (M > 1)$$

$$\Rightarrow \quad M_E = 2.2 \;\text{ and }\; P_E/P_o = 0.09399$$

$$P_E = 0.09399 \times 1000 = \mathbf{94\ kPa}$$

To find the mass flow rate we need the throat conditions, see Table 16.1,

$$T^* = T \frac{2}{k+1} = 600 \times 0.8333 = 500\text{ K}$$

$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 1000\ (0.833333)^{3.5} = 528.3\text{ kPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{528.3}{0.287 \times 500} = 3.682\text{ kg/m}^3$$

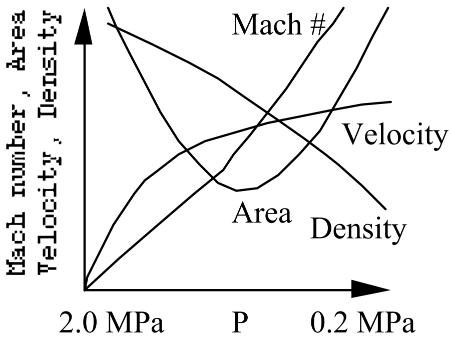
$$\dot{m} = \rho A \mathbf{V} = \rho^* A^* c^* = \rho^* A^* \sqrt{kRT^*}$$

$$= 3.682 \times 0.005 \sqrt{1.4 \times 0.287 \times 500 \times 1000}$$

$$= \mathbf{8.252\ kg/s}$$

16.47

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K, to a static pressure of 200 kPa. The mass flow rate through the nozzle is 5 kg/s. Assume the flow is reversible and adiabatic and determine the throat and exit areas for the nozzle.



$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$
$$= 2 \times 0.5283 = 1.056 \text{ MPa}$$
$$T^* = T_o \times \frac{2}{k+1} = 600 \times 0.8333 = 500 \text{ K}$$
$$v^* = RT^*/P^* = 0.287 \times 500/1056$$
$$= 0.1359 \text{ m}^3\text{/kg}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 500} = 448.2 \text{ m/s}$$

$$A^* = \dot{m}v^*/c^* = 5 \times 0.1359/448.2 = \mathbf{0.00152\ m^2}$$

$$P_2/P_o = 200/2000 = 0.1 \qquad \Rightarrow \qquad M_2^* = 1.701 = \mathbf{V}_2/c^*$$

Column with mach no. based on throat speed of sound.

$$\mathbf{V}_2 = 1.701 \times 448.2 = 762.4 \text{ m/s}$$

$$T_2 = T_o\,(T_2/T_o) = 600 \times 0.5176 = 310.56 \text{ K}$$

$$v_2 = RT_2/P_2 = 0.287 \times 310.56/200 = 0.4456 \text{ m}^3\text{/kg}$$

$$A_2 = \dot{m}v_2/\mathbf{V}_2 = 5 \times 0.4456 / 762.4 = \mathbf{0.00292\ m^2}$$

16.48

Air at 150 kPa, 290 K expands to the atmosphere at 100 kPa through a convergent nozzle with exit area of 0.01 m^2. Assume an ideal nozzle. What is the percent error in mass flow rate if the flow is assumed incompressible?

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}} = 258.28 \text{ K}$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 1.004 (290 - 258.28) = 31.83 \text{ kJ/kg}$$

$$\mathbf{V}_e = 252.3 \text{ m/s}; \quad v_e = \frac{RT_e}{P_e} = \frac{0.287 \times 258.28}{100} = 0.7412 \text{ m}^3\text{/kg}$$

$$\dot{m} = A\mathbf{V}_e / v_e = \frac{0.01 \times 252.3}{0.7413} = 3.4 \text{ kg/s}$$

Incompressible Flow: $v_i = RT/P = 0.287 \times 290/150 = 0.55487 \text{ m}^3\text{/kg}$

$$\mathbf{V}_e^2/2 = v\,\Delta P = v_i (P_i - P_e) = 0.55487 (150 - 100) = 27.74 \text{ kJ/kg}$$

$$\Rightarrow \mathbf{V}_e = 235 \text{ m/s} \quad \Rightarrow \quad \dot{m} = A\mathbf{V}_e / v_i = 0.01 \times 235 / 0.55487 = 4.23 \text{ kg/s}$$

$$\frac{\dot{m}_{incompressible}}{\dot{m}_{compressible}} = \frac{4.23}{3.4} = 1.25$$

about **25% overestimation.**

16.49

A convergent-divergent nozzle has a throat diameter of 0.05 m and an exit diameter of 0.1 m. The inlet stagnation state is 500 kPa, 500 K. Find the back pressure that will lead to the maximum possible flow rate and the mass flow rate for three different gases as: air; hydrogen or carbon dioxide.

There is a maximum possible flow when M = 1 at the throat,

$$T^* = \frac{2}{k+1} T_o; \quad P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}; \quad \rho^* = \rho_o \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

$$\dot{m} = \rho^* A^* V = \rho^* A^* c = P^* A^* \sqrt{k/RT^*}$$

$$A^* = \pi D^2/4 = 0.001963 \text{ m}^2$$

	k	T^*	P^*	c	ρ^*	$\dot{m}$
a)	1.400	416.7	264.1	448.2	2.209	1.944
b)	1.409	415.1	263.4	1704.5	0.154	0.515
c)	1.289	436.9	273.9	348.9	3.318	2.273

$A_E/A^* = (D_E/D^*)^2 = 4$. There are 2 possible solutions corresponding to points c and d in Fig. 16.13 and Fig. 16.17. For these we have

	Subsonic solution		Supersonic solution	
	M_E	P_E/P_o	M_E	P_E/P_o
a)	0.1466	0.985	2.940	0.0298
b)	0.1464	0.985	2.956	0.0293
c)	0.1483	0.986	2.757	0.0367

$P_B = P_E \cong 0.985 \times 500 = 492.5$ kPa all cases point c

a) $P_B = P_E = 0.0298 \times 500 = 14.9$ kPa, point d

b) $P_B = P_E = 0.0293 \times 500 = 14.65$ kPa, point d

c) $P_B = P_E = 0.0367 \times 500 = 18.35$ kPa, point d

16.50

A nozzle is designed assuming reversible adiabatic flow with an exit Mach number of 2.6 while flowing air with a stagnation pressure and temperature of 2 MPa and 150°C, respectively. The mass flow rate is 5 kg/s, and *k* may be assumed to be 1.40 and constant. Determine the exit pressure, temperature, exit area, and the throat area.

From Table A.12: $M_E = 2.6$

$P_E = 2.0 \times 0.05012 = 0.1002$ MPa

Critical properties from Table 16.1

$T^* = 423.15 \times 0.8333 = \mathbf{352.7\ K}$

$P^* = 2.0 \times 0.5283 = \mathbf{1.057\ MPa}$

$c^* = \sqrt{1.4 \times 1000 \times 0.287 \times 352.7} = 376.5$ m/s

$v^* = RT^*/P^* = 0.287 \times 352.7/1057 = 0.0958\ m^3/kg$

$A^* = \dot{m}\, v^* / c^* = 5 \times 0.0958 / 376.5 = \mathbf{1.272 \times 10^{-3} m^2}$

$A_E = A^* (A_E/A^*) = 1.272 \times 10^{-3} \times 2.896 = \mathbf{3.68 \times 10^{-3} m^2}$

$T_E = T_o (T_E / T_o) = 423.15 \times 0.42517 = 179.9$ K

16.51

A 1-m^3 insulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa. The nozzle has an exit area of 2×10^{-5} m^2.

a. Find the initial mass flow rate out of the tank.

b. Find the mass flow rate when half the mass has been discharged.

a. The back pressure ratio:

$$P_B/P_{o1} = 100/1000 = 0.1 < (P^*/P_o)_{crit} = 0.5283$$

so the initial flow is choked with the maximum possible flow rate.

$$M_E = 1\ ;\ \ P_E = 0.5283 \times 1000 = 528.3 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 560 = 466.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$

$$v_E = RT^*/P_E = 0.287 \times 466.7/528.3 = 0.2535 \text{ m}^3\text{/kg}$$

$$\dot{m}_1 = AV_E/v_E = 2 \times 10^{-5} \times 433/0.2535 = 0.0342 \text{ kg/s}$$

b. The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

with a mass at state 2 as $m_2 = m_1/2 = 3.111$ kg.

Assume an adiabatic reversible expansion of the mass that remains in the tank.

$$P_2 = P_1(v_1/v_2)^k = 100 \times 0.5^{1.4} = 378.9 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 560 \times 0.5^{0.4} = 424 \text{ K}$$

The pressure ratio is still less than critical and the flow thus choked.

$$P_B/P_{o2} = 100/378.9 = 0.264 < (P^*/P_o)_{crit}$$

$$M_E = 1\ ;\ \ P_E = 0.5283 \times 378.9 = 200.2 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 424 = 353.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 353.7} = 377 \text{ m/s}$$

$$\dot{m}_2 = A\mathbf{V}_EP_E/RT_E = \frac{2\times10^{-5}(377)(200.2)}{0.287(353.7)} = \mathbf{0.0149\ kg/s}$$

16.52

Assume the same tank and conditions as in Problem 16.51. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

The flow changes to subsonic when the pressure ratio reaches critical.

$$P_B/P_{o3} = 0.5283 \;\Rightarrow\; P_{o3} = 189.3 \text{ kPa}$$

$$v_1/v_3 = (P_{o3}/P_1)^{1/k} = (189.3/1000)^{0.7143} = 0.3046$$

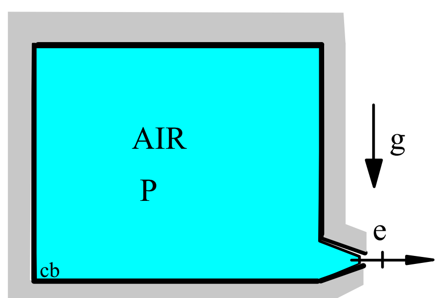
$$m_3 = m_1v_1/v_3 = 6.222 \times 0.3046 = \mathbf{1.895\ kg}$$

$$T_3 = T_1(v_1/v_3)^{k-1} = 560 \times 0.3046^{0.4} = 348 \text{ K}$$

$$P_E = P_B = 100 \text{ kPa} \;;\quad M_E = 1$$

$$T_E = 0.8333 \times 348 = 290 \text{ K} \;;\quad \mathbf{V}_E = \sqrt{kRT_E} = 341.4 \text{ m/s}$$

$$\dot{m}_3 = A\mathbf{V}_EP_E/RT_E = \frac{2\times10^{-5}(341.4)(100)}{0.287(290)} = \mathbf{0.0082\ kg/s}$$



16.53

A 1-m^3 uninsulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa while heat transfer from some source keeps the air temperature in the tank at 560 K. The nozzle has an exit area of 2×10^{-5} m^2.

a. Find the initial mass flow rate out of the tank.

b. Find the mass flow rate when half the mass has been discharged.

a. Same solution as in 16.52 a)

b. From solution 16.52 b) we have $m_2 = m_1/2 = 3.111$ kg

$P_2 = P_1/2 = 500$ kPa ; $T_2 = T_1$; $\quad P_B/P_2 = 100/500 = 0.2 < (P^*/P_o)_{crit}$

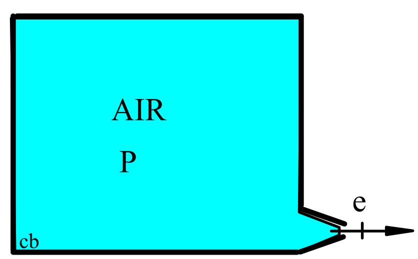
The flow is choked and the velocity is:

$$T_E = T^* = 0.8333 \times 560 = 466.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$

$P_E = 0.5283 \times 500 = 264.2$ kPa ; $M_E = 1$

$$\dot{m}_2 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(433)(264.2)}{0.287(466.7)} = \mathbf{0.01708\ kg/s}$$



16.54

Assume the same tank and conditions as in Problem 16.53. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

Flow changes to subsonic when the pressure ratio reaches critical.

$P_B/P_o = 0.5283$; $P_3 = P_o = P_B/0.5283 = 100/0.5283 = 189.3$ kPa

$m_3 = m_1P_3/P_1 = \mathbf{1.178\ kg}$;

$T_3 = T_1$

$T_E = T^* = 0.8333 \times 560 = 466.7$ K

$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433$ m/s

$$\dot{m}_3 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(433)(189.3)}{0.287(466.7)} = \mathbf{0.01224\ kg/s}$$

Normal Shocks

16.55

The products of combustion enter a convergent nozzle of a jet engine at a total pressure of 125 kPa, and a total temperature of 650°C. The atmospheric pressure is 45 kPa and the flow is adiabatic, with a rate of 25 kg/s. Determine the exit area of the nozzle.

The critical pressure: $P_{crit} = P_2 = 125 \times 0.5283 = 66 \text{ kPa} > P_{amb}$

The flow is then choked. $T_2 = 923.15 \times 0.8333 = 769.3 \text{ K}$

$\mathbf{V}_2 = c_2 = \sqrt{1.4 \times 1000 \times 0.287 \times 769.3} = 556 \text{ m/s}$

$v_2 = 0.287 \times 769.3/66 = 3.3453 \text{ m}^3\text{/kg}$

$A_2 = \dot{m}v_2 / \mathbf{V}_2 = 25 \times 3.3453/556 = \mathbf{0.1504\ m^2}$

16.56

Consider the nozzle of Problem 16.47 and determine what back pressure will cause a normal shock to stand in the exit plane of the nozzle. This is case g in Fig. 16.17. What is the mass flow rate under these conditions?

We assume reversible flow up to the shock

Table A.12: $P_E/P_o = 200/2000 = 0.1$; $M_E = 2.1591 = M_x$

Shock functions Table A.13: $M_y = 0.5529$; $P_y/P_x = 5.275$

$P_B = P_y = 5.275 \times P_x = 5.275 \times 200 =$ **1055 kPa**

$\dot{m}$ = **5 kg/s** same as in Problem 16.47 since M = 1 at throat.

16.57

At what Mach number will the normal shock occur in the nozzle of Problem 16.49 flowing with air if the back pressure is halfway between the pressures at *c* and *d* in Fig. 16.17?

First find the two pressures that will give exit at c and d. See solution to 16.8 a)

$A_E/A^* = (D_E/D^*)^2 = 4 \quad \Rightarrow \quad P_E = 492.5$ kPa (c) $\quad$ 14.9 kPa (d)

$P_E = (492.5 + 14.9)/2 = 253.7$ kPa

Assume $M_x = 2.4 \Rightarrow \quad M_y = 0.5231 ; \quad P_{oy}/P_{ox} = 0.54015$

$A_x/A_x^* = 2.4031 ; \quad A_x/A_y^* = 1.298$

$A_E/A_y^* = (A_E/A_x^*)\,(A_x/A_y^*)\,/\,(A_x/A_x^*) = 4 \times 1.298/2.4031 = 2.1605$

$\Rightarrow \quad M_E = 0.2807 ; \quad P_E/P_{oy} = 0.94675$

$P_E = (P_E/P_{oy})\,(P_{oy}/P_{ox})\,P_{ox} = 0.94675 \times 0.54015 \times 500 = 255.7$ kPa

Repeat if $M_x = 2.5 \quad \Rightarrow \quad P_E = 233.8$ kPa

Interpolate to match the desired pressure => $M_x =$ **2.41**

16.58

The nozzle in Problem 16.50 will have a throat area of 0.001272 m^2 and an exit area 2.896 times as large. Suppose the back pressure is raised to 1.4 MPa and that the flow remains isentropic except for a normal shock wave. Verify that the shock mach number (M_x) is close to 2 and find the exit mach number, the temperature and the mass flow rate through the nozzle.

(a) From Table A.12: $M_E = 2.6$

$P_E = 2.0 \times 0.05012 = 0.1002$ MPa

$T^* = 423.15 \times 0.8333 = \mathbf{352.7\ K}$

$P^* = 2.0 \times 0.5283 = \mathbf{1.057\ MPa}$

$c^* = \sqrt{1.4 \times 1000 \times 0.287 \times 352.7} = 376.5$ m/s

$v^* = 0.287 \times 352.7/1057 = 0.0958$ m^3/kg

$A^* = 5 \times 0.0958/376.5 = \mathbf{1.272 \times 10^{-3} m^2}$

$A_E = 1.272 \times 10^{-3} \times 2.896 = \mathbf{3.68 \times 10^{-3} m^2}$

$T_E = 423.15 \times 0.42517 = 179.9$ K

Assume $M_x = 2$ then

$M_y = 0.57735$, $P_{oy}/P_{ox} = 0.72088$, $A_E/A^*_x = 2.896$

$A_x/A^*_x = 1.6875$, $A_x/A^*_y = 1.2225$,

$A_E/A^*_y = 2.896 \times 1.2225/1.6875 = 2.098$

$\Rightarrow$ $M_E = \mathbf{0.293}$, $P_E/P_{oy} = 0.94171$

$P_E = 0.94171 \times 0.72088 \times 2.0 = 1.357$ MPa, OK

$T_E = 0.98298 \times 423.15 = \mathbf{416\ K}$, $\dot{m} = \mathbf{5\ kg/s}$

16.59

Consider the diffuser of a supersonic aircraft flying at $M = 1.4$ at such an altitude that the temperature is –20°C, and the atmospheric pressure is 50 kPa. Consider two possible ways in which the diffuser might operate, and for each case calculate the throat area required for a flow of 50 kg/s.

a. The diffuser operates as reversible adiabatic with subsonic exit velocity.

b. A normal shock stands at the entrance to the diffuser. Except for the normal shock the flow is reversible and adiabatic, and the exit velocity is subsonic. This is shown in Fig. P16.59.

a. Assume a convergent-divergent diffuser with $M = 1$ at the throat.

Relate the inlet state to the sonic state

$$P_1/P_o = 0.31424 \; ; \;\; P^*/P_{o1} = 0.5283$$

$$P^* = \frac{0.5283}{0.31424}\,50 = 84 \text{ kPa} \; ; \;\; T^* = \frac{0.8333}{0.71839}; 253.2 = 293.7 \text{ K}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 293.7} = 343.5 \text{ m/s}$$

$$v^* = RT^*/P^* = 0.287 \times 293.7/84 = 1.0035 \text{ m}^3\text{/kg}$$

$$A^* = \dot{m}v^*/c^* = 50 \times 1.0035/343.5 = \mathbf{0.1461 \ m^2}$$

b. Across the shock we have

$$M_y = 0.7397 \; ; \;\; P_y = 50 \times 2.12 = 106 \text{ kPa} \; ;$$

$$T_y = 253.2 \times 1.2547 = 317.7 \text{ K}$$

$$P^* = \frac{0.5283}{0.6952} \times 106 = 80.6 \text{ kPa}$$

$$T^* = \frac{0.8333}{0.9011} \times 317.7 = 293.7 \text{ K}, \quad c^* = 343.5 \text{ m/s}$$

$$v^* = 0.287 \times 293.7/80.6 = 1.0458 \text{ m}^3\text{/kg}$$

$$A^* = 50 \times 1.0458/343.5 = \mathbf{0.1522 \ m^2}$$

16.60

Consider the nozzle in problem 16.42 flowing air. What should the backpressure be for a normal shock to stand at the exit plane? This is case g in Fig.16.17. What is the exit velocity after the shock?

Reversible flow up to the shock with M = 1 at the throat.

$P_{x\,o} = P_o$, $T_{x\,o} = T_o$, $A_E/A^* = 175 / 100 = 1.75$

Table A.12: $M_E = M_x = 2.042$, $P_x/P_{x\,o} = 0.12$, $T_x/ T_{x\,o} = 0.5454$

Now we can do the normal shock from Table A.13

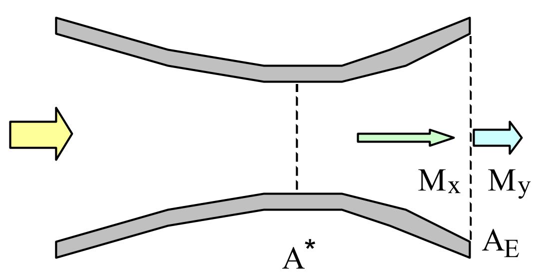
$M_x = 2.042 \Rightarrow M_y = 0.5704$, $P_y/P_x = 4.6984$, $T_y/T_x = 1.7219$

$T_y = 1.7219\ T_x = 1.7219 \times 0.5454\ T_{x\,o} = 1.7219 \times 0.5454 \times 375 = 352.2$ K

$P_y = 4.6984\ P_x = 4.6984 \times 0.12\ P_{x\,o} = 4.6984 \times 0.12 \times 1000 = 563.8$ kPa

$\mathbf{V}_y = M_y\, c_y = M_y \sqrt{kRT_y} = 0.5704 \sqrt{1.4 \times 0.287 \times 352.2 \times 1000}$

$= 0.5704 \times 376.2 =$ **214.6 m/s**



Nozzles, Diffusers, and Orifices

16.61

Air is expanded in a nozzle from 700 kPa, 200°C, to 150 kPa in a nozzle having an efficiency of 90%. The mass flow rate is 4 kg/s. Determine the exit area of the nozzle, the exit velocity, and the increase of entropy per kilogram of air. Compare these results with those of a reversible adiabatic nozzle.

$T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 473.2\ (150/700)^{0.286} = 304.6\text{ K}$

$\mathbf{V}_{2s}^{\,2} = 2 \times 1000 \times 1.004(473.2 - 304.6) = 338400\ \text{ J/kg}$

$\mathbf{V}_2^{\,2} = 0.9 \times 338400 \;\Rightarrow\; \mathbf{V}_2 = \mathbf{552\ m/s}$

$h_2 + \mathbf{V}_2^2/2 = h_1 \;\Rightarrow\; T_2 = T_1 - \mathbf{V}_2^2/2C_p$

$T_2 = 473.2 - 552^2/(2 \times 1000 \times 1.004) = 321.4\text{ K}$;

$v_2 = 0.287 \times 321.4/150 = 0.6149\ \text{ m}^3\text{/kg}$

$A_2 = 4 \times 0.6149/552 = 0.00446\text{ m}^2 = \mathbf{4460\ mm^2}$

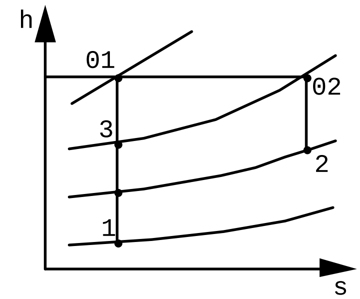
$s_2 - s_1 = 1.0035 \ln\left(\frac{321.4}{473.2}\right) - 0.287 \ln\left(\frac{150}{700}\right) = \mathbf{0.0539\ kJ/kg\ K}$

16.62

Repeat Problem 16.43 assuming a diffuser efficiency of 80%.

Same as problem 16.43, except

$\eta_D = 0.80$. We thus have from 16.43



$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$\Rightarrow h_3 = 292.35$ kJ/kg, $T_3 = 291.9$ K

$P_{o2} = P_3 = P_1\,(T_3/T_1)^{k/(k-1)}$

$= 40\,(291.9/261.15)^{3.5} = 59.06$ kPa

$T_{o2} = T_{o1} = 299.7$ K

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\quad \Rightarrow T_2 =$ **294.7 K**,

$P_2 = P_{o2}\,(T_2/T_{o1})^{k/(k-1)} = 59.06 \times (294.7/299.7)^{3.5} =$ **55.68 kPa**

$v_2 = 0.287 \times 294.7/55.68 = 1.519$ m^3/kg

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.519)\,(100/277.8) =$ **0.444**

16.63

Air enters a diffuser with a velocity of 200 m/s, a static pressure of 70 kPa, and a temperature of −6°C. The velocity leaving the diffuser is 60 m/s and the static pressure at the diffuser exit is 80 kPa. Determine the static temperature at the diffuser exit and the diffuser efficiency. Compare the stagnation pressures at the inlet and the exit.

$$T_{o1} = T_1 + \mathbf{V}_1^2/2C_p = 267.15 + 200^2/(2000 \times 1.004) = 287.1 \text{ K}$$

$$T_{o2} = T_{o1} \Rightarrow T_2 = T_{o2} - \mathbf{V}_2^2/2C_p = 287.1 - 60^2/(2000 \times 1.004) = 285.3 \text{ K}$$

$$\frac{T_{o1} - T_1}{T_1} = \frac{k-1}{k}\frac{P_{o1} - P_1}{P_1} \quad \Rightarrow \quad P_{o1} - P_1 = 18.25 \Rightarrow P_{o1} = 88.3 \text{ kPa}$$

$$\frac{T_{o2} - T_2}{T_2} = \frac{k-1}{k}\frac{P_{o2} - P_2}{P_2} \quad \Rightarrow \quad P_{o2} - P_2 = 1.77 \Rightarrow P_{o2} = 81.8 \text{ kPa}$$

$$T_s^{ex} = T_1\,(P_{o2}/P_1)^{k-1/k} = 267.15 \times 1.0454 = \mathbf{279.3\ K}$$

$$\eta_D = \frac{T_s^{ex} - T_1}{T_{o1} - T_1} = \frac{279.3 - 267.15}{287.1 - 267.15} = \mathbf{0.608}$$

16.64

Steam at a pressure of 1 MPa and temperature of 400°C expands in a nozzle to a pressure of 200 kPa. The nozzle efficiency is 90% and the mass flow rate is 10 kg/s. Determine the nozzle exit area and the exit velocity.

First do the ideal reversible adiabatic nozzle

$s_{2s} = s_1 = 7.4651$ kJ/kg K, $h_1 = 3263.9$ kJ/kg

$\Rightarrow$ $T_{2s} = 190.4°C$; $h_{2s} = 2851$ kJ/kg

Now the actual nozzle can be calculated

$h_1 - h_{2ac} = \eta_D(h_1 - h_{2s}) = 0.9(3263.9 - 2851) = 371.6$ kJ/kg

$h_{2ac} = 2892.3$ kJ/kg, $T_2 = 210.9°C$, $v_2 = 1.1062$ m^3/kg

$\mathbf{V}_2 = \sqrt{2000(3263.9 - 2892.3)} = \mathbf{862\ m/s}$

$A_2 = \dot{m}v_2/\mathbf{V}_2 = 10 \times 1.1062/862 = \mathbf{0.01283\ m^2}$

16.65

A sharp-edged orifice is used to measure the flow of air in a pipe. The pipe diameter is 100 mm and the diameter of the orifice is 25 mm. Upstream of the orifice, the absolute pressure is 150 kPa and the temperature is 35°C. The pressure drop across the orifice is 15 kPa, and the coefficient of discharge is 0.62. Determine the mass flow rate in the pipeline.

$$\Delta T = T_i\left(\frac{k-1}{k}\right)\frac{\Delta P}{P_i} = 308.15 \times \frac{0.4}{1.4} \times \frac{15}{150} = 8.8 \text{ K}$$

$v_i = RT_i/P_i = 0.5896 \text{ m}^3/\text{kg}$

$P_e = 135 \text{ kPa}$, $T_e = 299.35 \text{ K}$, $v_e = 0.6364 \text{ m}^3/\text{kg}$

$\dot{m}_i = \dot{m}_e \;\Rightarrow\; \mathbf{V}_i / \mathbf{V}_e = (D_e/D_i)^2\, v_i/v_e = 0.0579$

$h_i - h_e = \mathbf{V}_e^2(1 - 0.0579^2)/2 = C_p(T_i - T_e)$

$\mathbf{V}_{e\,s} = \sqrt{2 \times 1000 \times 1.004 \times 8.8/(1 - 0.0579)^2} = 133.1 \text{ m/s}$

$\dot{m} = C_D A\mathbf{V}/v = 0.62\ (\pi/4)\ (0.025)^2\ 133.1 / 0.6364 =$ **0.06365 kg/s**

16.66

A critical nozzle is used for the accurate measurement of the flow rate of air. Exhaust from a car engine is diluted with air so its temperature is 50°C at a total pressure of 100 kPa. It flows through the nozzle with throat area of 700 mm^2 by suction from a blower. Find the needed suction pressure that will lead to critical flow in the nozzle and the mass flow rate.

$P^* = 0.5283\ P_o =$ **52.83 kPa**, $T^* = 0.8333\ T_o = 269.3$ K

$v^* = RT^*/P^* = 0.287 \times 269.3/52.83 = 1.463\ m^3/kg$

$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 269.3} = 328.9$ m/s

$\dot{m} = Ac^*/v^* = 700 \times 10^{-6} \times 328.9/1.463 =$ **0.157 kg/s**

16.67

Steam at 800 kPa, 350°C flows through a convergent-divergent nozzle that has a throat area of 350 mm^2. The pressure at the exit plane is 150 kPa and the exit velocity is 800 m/s. The flow from the nozzle entrance to the throat is reversible and adiabatic. Determine the exit area of the nozzle, the overall nozzle efficiency, and the entropy generation in the process.

h_{o1} = 3161.7 kJ/kg, s_{o1} = 7.4089 kJ/kg K

$P^*/P_{o1} = (2/(k+1))^{k/(k-1)} = 0.54099 \Rightarrow P^* = 432.7$ kPa

At *: $(P^*, s^* = s_{o1}) \Rightarrow h^* = 2999.3$ kJ/kg, $v^* = 0.5687$ m^3/kg

$\Delta h = \mathbf{V}^2/2 \Rightarrow \mathbf{V}^* = \sqrt{2000(3161.7\text{-}2999.3)} = 569.9$ m/s

$\dot{m} = A\mathbf{V}^*/v^* = 350 \times 10^{-6} \times 569.9/0.5687 = 0.3507$ kg/s

$h_e = h_{o1} - \mathbf{V}_e^2/2 = 3161.7 - 800^2/2 \times 1000 = 2841.7$ kJ/kg

Exit: P_e, h_e: $v_e = 1.395$ m^3/kg, $s_e = 7.576$ kJ/kg K

$A_e = \dot{m}v_e/\mathbf{V}_e = 0.3507 \times 1.395/800 =$ $\mathbf{6.115 \times 10^{-4}\ m^2}$

$s_{gen} = s_e - s_{o1} = 7.576 - 7.4089 =$ **0.167 kJ/kg K**

16.68

Steam at 600 kPa, 300°C is fed to a set of convergent nozzles in a steam turbine. The total nozzle exit area is 0.005 m^2 and they have a discharge coefficient of 0.94. The mass flow rate should be estimated from the measurement of the pressure drop across the nozzles, which is measured to be 200 kPa. Determine the mass flow rate.

Inlet B.1.3 $h_i = 3061.6$ kJ/kg, $s_i = 7.3724$ kJ/kg K

Exit: $(P_e, s_{e,s})$ $P_e = P_i - 200 = 400$ kPa, $s_{e,s} = s_i = 7.3724$ kJ/kg K

$\Rightarrow h_{e,s} = 2961$ kJ/kg and $v_{e,s} = 0.5932$ m^3/kg,

$$\mathbf{V}_{e,s} = \sqrt{2 \times 1000(3061.6 - 2961)} = 448.55 \text{ m/s}$$

$$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 0.005 \times 448.55/0.5932 = 3.781 \text{ kg/s}$$

$$\dot{m}_a = C_D\dot{m}_s = 0.94 \times 3.781 = \mathbf{3.554 \text{ kg/s}}$$

16.69

A convergent nozzle is used to measure the flow of air to an engine. The atmosphere is at 100 kPa, 25°C. The nozzle used has a minimum area of 2000 mm^2 and the coefficient of discharge is 0.95. A pressure difference across the nozzle is measured to 2.5 kPa. Find the mass flow rate assuming incompressible flow. Also find the mass flow rate assuming compressible adiabatic flow.

Assume $\mathbf{V}_i \cong 0$, $\quad v_i = RT_i/P_i = 0.287 \times 298.15/100 = 0.8557\ m^3/kg$

$\mathbf{V}_{e,s}{}^2/2 = h_i - h_{e,s} = v_i(P_i - P_e) = 2.1393$ kJ/kg

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1393} = 65.41$ m/s

$\dot{m}_s = A\mathbf{V}_{e,s}/v_i = 2000 \times 10^{-6} \times 65.41/0.8557 = 0.153$ kg/s

$\dot{m}_a = C_D\dot{m}_s = 0.1454$ kg/s

$T_{e,s} = T_i\ (P_e/P_i)^{(k-1)/k} = 298.15(97.5/100)^{0.2857} = 296$ K

$\Delta h = C_p\Delta T = 1.0035 \times 2.15 = 2.1575 = \mathbf{V}_{e,s}{}^2/2$

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1575} = 65.69$ m/s

$v_{e,s} = 0.287 \times 296/97.5 = 0.8713\ m^3/kg$

$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 2000 \times 10^{-6} \times 65.69/0.8713 =$ **0.1508 kg/s**

$\dot{m}_a = C_D\dot{m}_s =$ **0.1433 kg/s**

16.70

The coefficient of discharge of a sharp-edged orifice is determined at one set of conditions by use of an accurately calibrated gasometer. The orifice has a diameter of 20 mm and the pipe diameter is 50 mm. The absolute upstream pressure is 200 kPa and the pressure drop across the orifice is 82 mm of mercury. The temperature of the air entering the orifice is 25°C and the mass flow rate measured with the gasometer is 2.4 kg/min. What is the coefficient of discharge of the orifice at these conditions?

$$\Delta P = 82 \times 101.325/760 = 10.93 \text{ kPa}$$

$$\Delta T = T_i \left(\frac{k-1}{k}\right) \Delta P/P_i = 298.15 \times \frac{0.4}{1.4} \times 10.93/200 = 4.66$$

$$v_i = RT_i/P_i = 0.4278 \text{ m}^3\text{/kg}, \quad v_e = RT_e/P_e = 0.4455 \text{ m}^3\text{/kg}$$

$$\mathbf{V}_i = \mathbf{V}_e A_e v_i / A_i v_e = 0.1536\ \mathbf{V}_e$$

$$(\mathbf{V}_e^2 - \mathbf{V}_i^2)/2 = \mathbf{V}_e^2 (1 - 0.1536^2)/2 = h_i - h_e = C_p \Delta T$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 1.004 \times 4.66/(1 - 0.1536^2)} = 97.9 \text{ m/s}$$

$$\dot{m} = A_e \mathbf{V}_e / v_e = \frac{\pi}{4} \times 0.02^2 \times 97.9/0.4455 = 0.069 \text{ kg/s}$$

$$C_D = 2.4/60 \times 0.069 = \mathbf{0.58}$$

16.71

A convergent nozzle with exit diameter of 2 cm has an air inlet flow of 20°C, 101 kPa (stagnation conditions). The nozzle has an isentropic efficiency of 95% and the pressure drop is measured to 50 cm water column. Find the mass flow rate assuming compressible adiabatic flow. Repeat calculation for incompressible flow.

Convert ΔP to kPa:

$\Delta P = 50 \text{ cm } H_2O = 0.5 \times 9.8064 = 4.903 \text{ kPa}$

$T_0 = 20°C = 293.15 \text{ K}$ $\quad\quad P_0 = 101 \text{ kPa}$

Assume inlet $V_i = 0$ $\quad\quad P_e = P_0 - \Delta P = 101 - 4.903 = 96.097 \text{ kPa}$

$$T_e = T_0 \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}} = 293.15 \times \left(\frac{96.097}{101}\right)^{0.2857} = 289.01$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 1.004 \times (293.15 - 289.01)$$

$$= 4.1545 \text{ kJ/kg} = 4254.5 \text{ J/kg} \quad\quad => \quad \mathbf{V}_e = 91.15 \text{ m/s}$$

$$\mathbf{V}_{e\,ac}^{\,2}/2 = \eta\, \mathbf{V}_{e\,s}^{\,2}/2 = 0.95 \times 4154.5 = 3946.78 \quad\quad \Rightarrow \quad \mathbf{V}_{e\,ac} = 88.85 \text{ m/s}$$

$$T_{e\,ac} = T_i - \frac{V_{e\,ac}^{\,2}/2}{C_p} = 293.15 - \frac{3.9468}{1.0035} = 289.2 \text{ K}$$

$$\rho_{e\,ac} = \frac{P_e}{RT_p} = \frac{96.097}{0.287 \times 289.2} = 1.158 \text{ kg/m}^3$$

$$\dot{m} = \rho A\mathbf{V} = 1.158 \times \frac{\pi}{4} \times 0.02^2 \times 88.85 = \mathbf{0.0323 \text{ kg/s}}$$

Review Problems

16.72

At what Mach number will the normal shock occur in the nozzle of Problem 16.47 if the back pressure is 1.4 MPa? (trial and error on M_x)

Relate the inlet and exit conditions to the shock conditions with reversible flow before and after the shock. It becomes trial and error.

Assume $M_x = 1.8 \Rightarrow M_y = 0.6165$; $P_{oy}/P_{ox} = 0.8127$

$A_E/A^*_x = A_2/A^* = 0.002435/0.001516 = 1.6062$

$A_x/A^*_x = 1.439$; $A_x/A^*_y = 1.1694$

$A_E/A^*_y = (A_E/A^*_x)(A_x/A^*_y)/(A_x/A^*_x) = \dfrac{1.6062(1.1694)}{1.439} = 1.3053$

$\Rightarrow M_E = 0.5189$; $P_E/P_{oy} = 0.8323$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.8323 \times 0.8127 \times 2000 = 1353 \text{ kPa} < 1.4 \text{ MPa}$

So select the mach number a little less

$M_x = 1.7 \Rightarrow M_y = 0.64055$; $P_{oy}/P_{ox} = 0.85573$

$A_x/A^*_x = 1.3376$; $A_x/A^*_y = 1.1446$

$A_E/A^*_y = (A_E/A^*_x)(A_x/A^*_y)/(A_x/A^*_x) = \dfrac{1.6062(1.1446)}{1.3376} = 1.3744$

$\Rightarrow M_E = 0.482$; $P_E/P_{oy} = 0.853$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.853 \times 0.85573 \times 2000 = 1459.9 \text{ kPa}$

Now interpolate between the two

$M_x = 1.756 \Rightarrow M_y = 0.6266$; $P_{oy}/P_{ox} = 0.832$

$A_x/A^*_x = 1.3926$; $A_x/A^*_y = 1.1586$

$A_E/A^*_y = 1.6062 \times 1.1586/1.3926 = 1.3363$

$\Rightarrow M_E = 0.5$; $P_E/P_{oy} = 0.843$

$P_E = 0.843 \times 0.832 \times 2000 = 1402.7 \text{ kPa}$ **OK**

16.73

Atmospheric air is at 20°C, 100 kPa with zero velocity. An adiabatic reversible compressor takes atmospheric air in through a pipe with cross-sectional area of 0.1 m^2 at a rate of 1 kg/s. It is compressed up to a measured stagnation pressure of 500 kPa and leaves through a pipe with cross-sectional area of 0.01 m^2. What are the required compressor work and the air velocity, static pressure, and temperature in the exit pipeline?

C.V. compressor out to standing air and exit to stagnation point.

$\dot{m}\, h_{o1} + \dot{W}?_c = \dot{m}(h + \mathbf{V}^2/2)_{ex} = \dot{m}h_{o,ex}$

$\dot{m}s_{o1} = \dot{m}s_{o,ex} \;\Rightarrow\; P_{r,o,ex} = P_{r,o1}$

$(P_{st,ex}/P_{o1}) = 1.028(500/100) = 5.14$

$\Rightarrow\; T_{o,ex} = 463$ K, $\quad h_{o,ex} = 465.38$ kJ/kg, $\quad h_{o1} = 209.45$ kJ/kg

$\dot{W}_c = \dot{m}(h_{o,ex} - h_{o1}) = 1(465.38 - 209.45) =$ **255.9 kW**

$P_{ex} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/(k-1)} \qquad T_{ex} = T_{o,ex} - \mathbf{V}_{ex}^2/2C_p$

$\dot{m} = 1 \text{ kg/s} = (\rho A\mathbf{V})_{ex} = P_{ex}AV_{ex}/RT_{ex}$

Now select 1 unknown amongst P_{ex}, T_{ex}, V_{ex} and write the continuity eq. $\dot{m}$ and solve the nonlinear equation. Say, use T_{ex} then

$\mathbf{V}_{ex} = \sqrt{2C_p(T_{o,ex} - T_{ex})}$

$\dot{m} = 1 \text{ kg/s} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/k-1}A\sqrt{2C_p(T_{o,ex} - T_{ex})}/RT_{ex}$

solve for $T_{ex}/T_{o,ex}$ (close to 1)

$T_{ex} =$ **462.6 K** $\;\Rightarrow\;$ $\mathbf{V}_{ex} =$ **28.3 m/s**, $\;P_{ex} =$ **498.6 kPa**

Solution using the Pr or vr functions

16.43

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is −12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$\mathbf{V} = 1000 \text{ km/h} = 277.8 \text{ m/s}, \quad v_1 = \frac{RT}{P} = \frac{0.287 \times 261.15}{40} = 1.874 \text{ m}^3\text{/kg}$

$h_1 = 261.48 \text{ kJ/kg}, \;\; P_{r1} = 0.6862$

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07 \text{ kJ/kg}$

$\Rightarrow \;\; T_{o1} = 299.7 \text{ K}, \quad P_{ro1} = 1.1107$

The ratio of the pressures equals the ratio of the Pr functions when s = constant

$P_{o1} = P\, P_{ro1}/\, P_{r1} = 40 \times 1.1107/0.6862 = 64.74 \text{ kPa}$

$h_2 = 300.07 - 100^2/2000 = 295.07 \quad \Rightarrow \quad T_2 = \mathbf{294.7\ K}, \quad P_{r2} = 1.0462$

$P_2 = 64.74 \times 1.0462/1.1107 = \mathbf{61\ kPa}$

$v_2 = 0.287 \times 294.7/61 = 1.386 \text{ m}^3\text{/kg}$

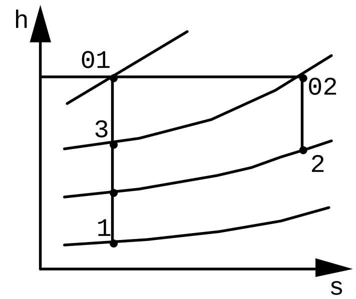
$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.386)(100/277.8) = \mathbf{0.487}$

16.62

Repeat Problem 16.43 assuming a diffuser efficiency of 80%.

Same as problem 16.43, except

$\eta_D = 0.80$. We thus have from 16.43



$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$\Rightarrow$ $h_3 = 292.35$ kJ/kg, $P_{r3} = 1.0129$

$P_{o2} = P_3 = 40 \times 1.0129/0.6862 = 59.04$ kPa

$P_{ro2} = P_{ro1} = 1.1107$

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\Rightarrow$ $T_2 =$ **294.7 K**, $P_{r2} = 1.0462$

$P_2 = P_{o2}\, P_{r2} / P_{ro2} = 59.04 \times 1.0462/1.1107 =$ **55.6 kPa**

$v_2 = 0.287 \times 294.7/55.6 = 1.521\ m^3/kg$

$A_1/A_2 = (1.874/1.521)\ (100/277.8) =$ **0.444**