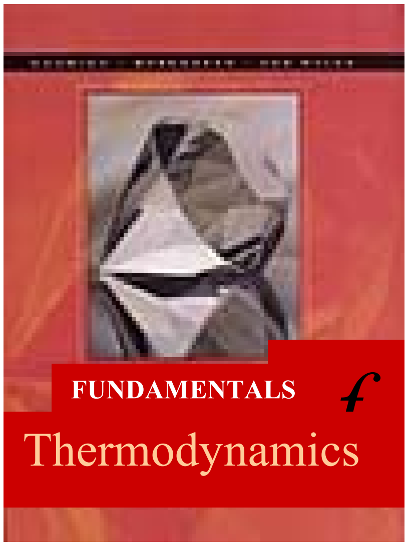
CHAPTER 12
SOLUTION MANUAL

Fundamentals of Thermodynamics 6th Edition
Sonntag, Borgnakke and van Wylen



CONTENT

CHAPTER 12 6th edition Sonntag/Borgnakke/Wylen

The correspondence between this problem set and the 5th edition chapter 12 problem set.

New	5th	New	5th	New	5th	New	5th
21	1	51	23	81	40	111	60
22	new	52	new	82	41	112	66
23	3	53	24	83	50	113	52
24	new	54	new	84	43	114	67
25	4	55	20	85	45	115	72
26	5	56	21	86	46	116	new
27	new	57	22 mod	87	47	117	new
28	2	58	13 mod	88	49	118	new
29	7	59	new	89	53	119	26
30	new	60	15 mod	90	42	120	28
31	6 mod	61	18 mod	91	61	121	30
32	new	62	new	92	65	122	33
33	new	63	18	93	new	123	35
34	8 mod	64	31	94	44	124	51
35	8	65	new	95	48	125	69
36	9 mod	66	19 mod	96	58a,c	126	73
37	new	67	27 mod	97	68	127	80
38	new	68	29 mod	98	new	128	74
39	10 mod	69	32 mod	99	70	129	new
40	11 mod	70	32	100	62	130	75
41	10	71	new	101	64	131	76
42	11	72	34	102	63	132	77
43	new	73	14	103	57	133	78
44	12 mod	74	25	104	new		
45	12	75	new	105	71		
46	new	76	36	106	54		
47	new	77	37	107	55		
48	new	78	new	108	58		
49	16	79	38	109	56		
50	17 mod	80	39 mod	110	59		

The problems that are labeled psychrometric chart only, can be solved without the chart using the formulas and the steam tables. Most of the solutions though, become lengthy and trial and error type due to the implicit connections between the variables shown in the chart. An alternative to this procedure is the use of the computer software, which can be used instead of the chart.

Concept-Study Guide Problems

12.1

Are the mass and mole fractions for a mixture ever the same?

Generally not. If the components all had the same molecular weight the mass and mole fractions would be the same.

12.2

For a mixture how many component concentrations are needed?

A total of N-1 concentrations are needed whether mass or mole fractions. They must sum up to one so the last one is by default.

12.3

Are any of the properties (P, T, v) for oxygen and nitrogen in air the same?

In any mixture under equilibrium T is the same for all components. Each species has its own pressure equal to its partial pressure. The partial volume for a component is: $v_i = V/m_i$ and V is the same for all components so v_i is not.

12.4

If oxygen is 21% by mole of air, what is the oxygen state (P, T, v) in a room at 300 K, 100 kPa of total volume 60 m^3?

The temperature is 300 K,
The partial pressure is $P = yP_{tot} = 0.21 \times 100 = 21$ kPa.
At this T, P: $v_{O2} = RT/P_{O2} = 0.2598 \times 300/21 = 3.711\ m^3/kg$

Remark: If we found the oxygen mass then $m_{O2}v_{O2} = V = 60\ m^3$

12.5

A flow of oxygen and one of nitrogen, both 300 K, are mixed to produce 1 kg/s air at 300 K, 100 kPa. What are the mass and volume flow rates of each line?

For the mixture, $M = 0.21 \times 32 + 0.79 \times 28.013 = 28.85$
For O_2 , $c = 0.21 \times 32 / 28.85 = 0.2329$
For N_2 , $c = 0.79 \times 28.013 / 28.85 = 0.7671$

Since the total flow out is 1 kg/s, these are the component flows in kg/s.
Volume flow of O_2 in is

$$\dot{V} = c\dot{m}v = c\dot{m}\frac{RT}{P} = 0.2329 \times 0.2598 \times 300/100 = 0.1815 \text{ m}^3/\text{s}$$

Volume flow of N_2 in is

$$\dot{V} = c\dot{m}\, v = c\dot{m}\frac{RT}{P} = 0.7671 \times 0.2968 \times 300/100 = 0.6830 \text{ m}^3/\text{s}$$

12.6

A flow of gas A and a flow of gas B are mixed in a 1:1 mole ratio with same T. What is the entropy generation per kmole flow out?

For this each component the mole fraction is one half so,
Eq. 12.19: $\Delta S = -\bar{R}(0.5 \ln 0.5 + 0.5 \ln 0.5) = +0.6931\, \bar{R}$

12.7

A rigid container has 1 kg argon at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any external heat transfer. What is final T, P? Is any s generated?

Energy Eq.: $U_2 - U_1 = 0 = 2mu_2 - mu_{1a} - mu_{1b} = mC_v(2T_2 - T_{1a} - T_{1b})$

$T_2 = (T_{1a} + T_{1b})/2 = \mathbf{350\ K},$

Process Eq.: V = constant =>

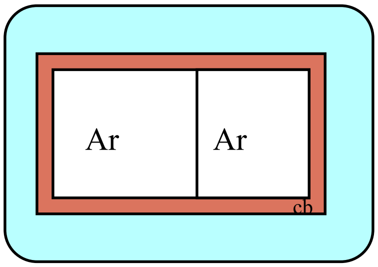
$P_2V = 2mRT_2 = mR(T_{1a} + T_{1b}) = P_1V_{1a} + P_1V_{1b} = P_1V$

$P_2 = P_1 = \mathbf{150\ kPa},$

ΔS due to temperature changes only, not P, internally we have a Q over a ΔT

$$\Delta S = m\,(s_2 - s_{1a}) + m\,(s_2 - s_{1b}) = mC_p\,[\,\ln(T_2/T_{1a}) + \ln(T_2/T_{1b})\,]$$

$$= 1 \times 0.520 \left[\ln\frac{350}{300} + \ln\frac{350}{400}\right] = \mathbf{0.0107\ kJ/K}$$



12.8

A rigid container has 1 kg CO_2 at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any heat transfer. What is final T, P?

No Q, No W so the energy equation gives constant U

Energy Eq.: $U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$

$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$

$= (1\times0.653 + 1\times0.312) \times T_2 - 1\times0.653\times300 - 1\times0.312\times400$

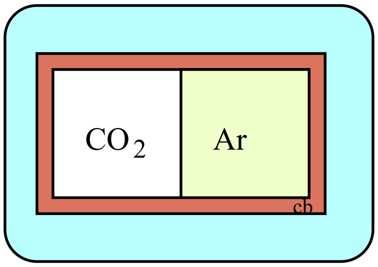
$T_2 =$ **332.3 K**,

$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$

$= 1\times0.1889\times300/150 + 1\times0.2081\times400/150 = 0.932\ 73\ m^3$

Pressure from ideal gas law and Eq.12.15 for R

$P_2 = (1\times0.1889 + 1\times0.2081) \times332.3/0.932\ 73 =$ **141.4 kPa**



12.9

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow. The energy equation becomes

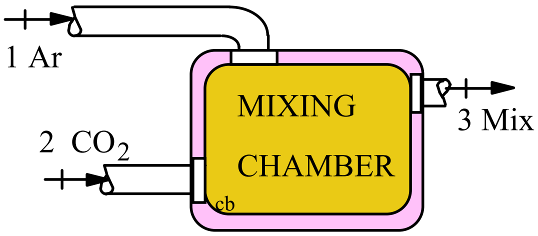
$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$$\Rightarrow \quad \dot{m}_{CO2}C_{p\ CO2}(T_e - T_i)_{CO2} + \dot{m}_{Ar}C_{p\ Ar}(T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \quad \dot{m}_{Ar}C_{p\ Ar}T_i + \dot{m}_{CO2}C_{p\ CO2}T_i = [\dot{m}_{Ar}C_{p\ Ar} + \dot{m}_{CO2}C_{p\ CO2}]\ T_e$$

$$1\times0.520\times300 + 1\times0.842\times1600 = (1\times0.520 + 1\times0.842) \times T_e$$

$$T_e = 1103.7 \text{ K}, \qquad P_e = 150 \text{ kPa}$$



12.10

What is the rate of entropy increase in problem 12.8?

Using Eq. 12.4, the mole fraction of CO_2 in the mixture is 0.4758.

From Eqs. 12.16 and 12.17, from the two inlet states to state 2,

$$\Delta S = 1\times[\ 0.520 \ln(\frac{1103.7}{300}) - 0.2081 \ln(\frac{0.5242\times150}{150})\ ]$$

$$+1\times[\ 0.842 \ln(\frac{1103.7}{1600}) - 0.1889 \ln(\frac{0.4758\times150}{150})\ ] = 0.6394 \text{ kW/K}$$

12.11

If I want to heat a flow of a 4 component mixture from 300 to 310 K at constant P, how many properties and which ones do I need to know to find the heat transfer?

You need to know the flow rate, the four mass fractions, and the component specific heat values.

12.12

For a gas mixture in a tank are the partial pressures important?

Partial pressures are necessary to calculate entropy changes, if the mixture composition changes. Otherwise, they are not needed.

12.13

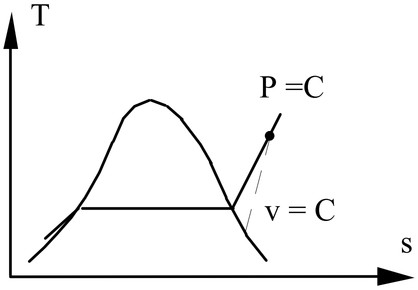
What happens to relative and absolute humidity when moist air is heated?

Relative humidity decreases, while absolute humidity remains constant.
See Figs. 12.8 and 12.9.

12.14

I cool moist air, do I reach the dew first in a constant P or constant V process?

The constant-volume line is steeper than the constant-pressure line,
see Fig. 12.3. Saturation in the constant-P process is at a higher T.



12.15

What happens to relative and absolute humidity when moist air is cooled?

Relative humidity increase, while absolute humidity remains constant.
See Figs. 12.8 and 12.9.

12.16

If I have air at 100 kPa and a) –10°C b) 45°C and c) 110°C what is the maximum absolute humidity I can have?

Humidity is related to relative humidity (max 100%) and the pressures as in Eq.12.28 where from Eq.12.25 $P_v = \Phi\ P_g$ and $P_a = P_{tot} - P_v$.

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{\Phi\ P_g}{P_{tot} - \Phi P_g}$$

a) $P_g = 0.2601$ kPa, $\omega = 0.622 \times 0.2601/99.74 = 0.001\ 62$

b) $P_g = 9.593$ kPa, $\omega = 0.622 \times 9.593/90.407 = 0.0660$

c) $P_g = 143.3$ kPa, no max ω for $P_{tot} = 100$ kPa

12.17

Can moist air below the freezing point, say -5^{o}C, have a dew point?

Yes. At the dew point, water would begin to appear as a solid. It snows.

12.18

Explain in words what the absolute and relative humidity expresses?

Absolute humidity is the ratio of the mass of vapor to the mass of dry air. It says how much water is there per unit mass of dry air.
Relative humidity is the ratio of the mole fraction of vapor to that in a saturated mixture at the same T and P. It expresses how close to the saturated state the water is.

12.19

An adiabatic saturation process changes Φ, ω and T. In which direction?

Relative humidity and absolute humidity increase, and temperature decreases.

12.20

I want to bring air at 35^{o}C, $\Phi = 40\%$ to a state of 25^{o}C, $\omega = 0.01$ do I need to add or subtract water?

Assuming P = 100 kPa,

At 35°C , 40 % : $\omega = 0.622 \times 0.40 \times 5.628/97.749 = 0.014\,32$

To get to $\omega = 0.01$, it is necessary to subtract water.

Mixture composition and properties

12.21

A gas mixture at 20°C, 125 kPa is 50% N_2, 30% H_2O and 20% O_2 on a mole basis. Find the mass fractions, the mixture gas constant and the volume for 5 kg of mixture.

Solution:

The conversion follows the definitions and identities:

From Eq. 12.3: $c_i = y_i M_i / \sum y_j M_j$

From Eq.12.5:

$$M_{mix} = \sum y_j M_j = 0.5\times 28.013 + 0.3\times 18.015 + 0.2\times 31.999$$

$$= 14.0065 + 5.4045 + 6.3998 = 25.811$$

$c_{N2} = 14.0065 / 25.811 = 0.5427$, $c_{H2O} = 5.4045 / 25.811 = 0.2094$

$c_{O2} = 6.3998 / 25.811 = 0.2479$, sums to 1 OK

From Eq.12.14:

$R_{mix} = \bar{R}/M_{mix} = 8.3145 / 25.811 =$ **0.3221 kJ/kg K**

$V = mR_{mix}\, T/P = 5\times 0.3221\times 393.15/125 =$ **$5.065\ m^3$**

12.22

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mass basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mole fractions and the mass of argon.

Solution:

Total mixture $PV = m R_{mix} T$

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.6 \times 0.2968 + 0.3 \times 0.2081 + 0.1 \times 0.2598$$
$$= 0.26629 \text{ kJ/kg K}$$

$m = PV/R_{mix}T = 250 \times 0.5 / 0.26649 \times 310 = 1.513$ kg

$m_{ar} = 0.3\ m =$ **0.454 kg**

From Eq.12.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

	c_i	M_i	c_i/M_i	$\mathbf{y_i}$	
N_2	0.6	28.013	0.02141	**0.668**	
Ar	0.3	39.948	0.00751	**0.234**	
O_2	0.1	31.999	$\underline{0.003125}$	**0.098**	round up
			0.032055		

12.23

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mole basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mass fractions and the mass of argon.

Solution:

From Eq. 12.3: $c_i = y_i M_i / \sum y_j M_j$

Eq.12.5:

$M_{mix} = \sum y_j M_j = 0.6 \times 28.013 + 0.3 \times 39.948 + 0.1 \times 31.999 = 31.992$

$c_{N2} = (0.6 \times 28.013) / 31.992 = 0.5254$

$c_{Ar} = (0.3 \times 39.948) / 31.992 = 0.3746$

$c_{O2} = (0.1 \times 31.999) / 31.992 = 0.1$, sums to 1 OK

From Eq.12.14:

$R_{mix} = \bar{R}/M_{MIX} = 8.3145 / 31.992 = 0.260$ kJ/kg K

$m_{mix} = PV/(R_{mix} T) = 250 \times 0.5 / 0.26 \times 310 = 1.551$ kg

$m_{Ar} = c_{Ar} \times m_{mix} = 0.3746 \times 1.551 =$ **0.581 kg**

12.24

A new refrigerant R-407 is a mixture of 23% R-32, 25% R-125 and 52% R-134a on a mass basis. Find the mole fractions, the mixture gas constant and the mixture heat capacities for this new refrigerant.

Solution:

From the conversion in Eq.12.4 we get:

	c_i	M_i	c_i/M_i	y_i
R-32	0.23	52.024	0.004421	0.381
R-125	0.25	120.022	0.002083	0.180
R-134a	0.52	102.03	$\underline{0.0050965}$	0.439
			0.0116005	

Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.23 \times 0.1598 + 0.25 \times 0.06927 + 0.52 \times 0.08149$$

$$= \mathbf{0.09645\ kJ/kg\ K}$$

Eq.12.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.23 \times 0.822 + 0.25 \times 0.791 + 0.52 \times 0.852$$

$$= \mathbf{0.8298\ kJ/kg\ K}$$

Eq.12.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.23 \times 0.662 + 0.25 \times 0.721 + 0.52 \times 0.771$$

$$= \mathbf{0.7334\ kJ/kg\ K} \;\; (= C_{P\ MIX} - R_{MIX})$$

12.25

A carbureted internal combustion engine is converted to run on methane gas (natural gas). The air-fuel ratio in the cylinder is to be 20 to 1 on a mass basis. How many moles of oxygen per mole of methane are there in the cylinder?

Solution:

The mass ratio $m_{AIR}/m_{CH_4} = 20$, so relate mass and mole $n = m/M$

$$\frac{n_{AIR}}{n_{CH_4}} = \left(\frac{m_{AIR}}{m_{CH_4}}\right) \times M_{CH_4}/M_{AIR} = 20 \times 16.04/28.97 = 11.0735$$

$$\rightarrow \frac{n_{O_2}}{n_{CH_4}} = \frac{n_{O_2}}{n_{AIR}} \times \frac{n_{AIR}}{n_{CH_4}} = 0.21 \times 11.0735 = \mathbf{2.325\ mole\ O_2/mole\ CH_4}$$

12.26

Weighing of masses gives a mixture at 60°C, 225 kPa with 0.5 kg O_2, 1.5 kg N_2 and 0.5 kg CH_4. Find the partial pressures of each component, the mixture specific volume (mass basis), mixture molecular weight and the total volume.

Solution:

From Eq.12.4: $y_i = (m_i / M_i) / \sum m_j/M_j$

$n_{tot} = \sum m_j/M_j = (0.5/31.999) + (1.5/28.013) + (0.5/16.04)$

$= 0.015625 + 0.053546 + 0.031172 = 0.100343$

$y_{O2} = 0.015625/0.100343 = 0.1557$,

$y_{N2} = 0.053546/0.100343 = 0.5336$,

$y_{CH4} = 0.031172/0.100343 = 0.3107$

From Eq.12.10:

$P_{O2} = y_{O2}\ P_{tot} = 0.1557 \times 225 = 35$ kPa,

$P_{N2} = y_{N2}\ P_{tot} = 0.5336 \times 225 = 120$ kPa

$P_{CH4} = y_{CH4}\ P_{tot} = 0.3107 \times 225 = 70$ kPa

$V_{tot} = n_{tot}\ \bar{R}T/P = 0.100343 \times 8.31451 \times 333.15 / 225 = \mathbf{1.235\ m^3}$

$v = V_{tot}/m_{tot} = 1.235 / (0.5 + 1.5 + 0.5) = \mathbf{0.494\ m^3/kg}$

From Eq.12.5:

$M_{mix} = \sum y_jM_j = m_{tot}/n_{tot} = 2.5/0.100343 = \mathbf{24.914}$

12.27

A 2 kg mixture of 25% N_2, 50% O_2 and 25% CO_2 by mass is at 150 kPa and 300 K. Find the mixture gas constant and the total volume.

Solution:

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$$

$$= \mathbf{0.2513\ kJ/kg\ K}$$

Ideal gas law: $PV = mR_{mix}T$

$$V = mR_{mix}T/P = 2 \times 0.2513 \times 300/150 = \mathbf{1.005\ m^3}$$

12.28

A 100 m^3 storage tank with fuel gases is at 20°C, 100 kPa containing a mixture of acetylene C_2H_2, propane C_3H_8 and butane C_4H_{10}. A test shows the partial pressure of the C_2H_2 is 15 kPa and that of C_3H_8 is 65 kPa. How much mass is there of each component?

Solution:

Assume ideal gases, then the ratio of partial to total pressure is the mole fraction, $y = P/P_{tot}$

$y_{C2H2} = 15/100 = 0.15, \quad y_{C3H8} = 65/100 = 0.65, \quad y_{C4H10} = 20/100 = 0.20$

$$n_{tot} = \frac{PV}{\bar{R}T} = \frac{100 \times 100}{8.31451 \times 293.15} = 4.1027 \text{ kmoles}$$

$$m_{C2H2} = (nM)_{C2H2} = y_{C2H2}\, n_{tot}\, M_{C2H2}$$

$$= 0.15 \times 4.1027 \times 26.038 = \mathbf{16.024\ kg}$$

$$m_{C3H8} = (nM)_{C3H8} = y_{C3H8}\, n_{tot}\, M_{C3H8}$$

$$= 0.65 \times 4.1027 \times 44.097 = \mathbf{117.597\ kg}$$

$$m_{C4H10} = (nM)_{C4H10} = y_{C4H10}\, n_{tot}\, M_{C4H10}$$

$$= 0.20 \times 4.1027 \times 58.124 = \mathbf{47.693\ kg}$$

12.29

A pipe, cross sectional area 0.1 m^2, carries a flow of 75% O_2 and 25% N_2 by mole with a velocity of 25 m/s at 200 kPa, 290 K. To install and operate a mass flow meter it is necessary to know the mixture density and the gas constant. What are they? What mass flow rate should the meter then show?

Solution:

From Eq.12.3: $c_i = y_i M_i / \sum y_j M_j$

From Eq.12.5

$$M_{mix} = \sum y_j M_j = 0.75 \times 31.999 + 0.25 \times 28.013 = 31.0025$$

Eq.12.14:

$$R_{mix} = \bar{R}/M_{mix} = 8.3145 / 31.0025 = \mathbf{0.2682\ kJ/kg\ K}$$

$$v = R_{mix}\ T/P = 0.2682 \times 290 / 200 = 0.38889\ m^3/kg$$

$$\rho = 1/v = \mathbf{2.5714\ kg/m^3}$$

$$\dot{m} = \rho A \mathbf{V} = 2.5714 \times 0.1 \times 25 = \mathbf{6.429\ kg/s}$$

12.30

A new refrigerant R-410a is a mixture of R-32 and R-125 in a 1:1 mass ratio. What are the overall molecular weight, the gas constant and the ratio of specific heats for such a mixture?

Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.1598 + 0.5 \times 0.06927 = \mathbf{0.1145\ kJ/kg\ K}$$

Eq.12.23:

$$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065 \text{ kJ/kg K}$$

Eq.12.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.5 \times 0.662 + 0.5 \times 0.722 = 0.692 \text{ kJ/kg K}$$

$$(= C_{P\ mix} - R_{mix})$$

$$k_{mix} = C_{P\ mix} / C_{V\ mix} = 0.8065 / 0.692 = \mathbf{1.1655}$$

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\frac{0.5}{52.024} + \frac{0.5}{120.022}} = \mathbf{72.586}$$

Simple processes

12.31

At a certain point in a coal gasification process, a sample of the gas is taken and stored in a 1-L cylinder. An analysis of the mixture yields the following results:

Component	H_2	CO	CO_2	N_2
Percent by mass	2	45	28	25

Determine the mole fractions and total mass in the cylinder at 100 kPa, 20°C. How much heat transfer must be transferred to heat the sample at constant volume from the initial state to 100°C?

Solution:

Determine mole fractions from Eq.12.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

$$\sum c_j / M_j = 0.02 / 2.016 + 0.45 / 28.01 + 0.28 / 44.01 + 0.25 / 28.013$$

$$= 0.009921 + 0.016065 + 0.006362 + 0.00892 = 0.041268$$

$$M_{mix} = 1 / \sum c_j/M_j = 1/0.041268 = 24.232$$

From Eq.12.4

$y_{H2} = 0.009921 \times 24.232 = 0.2404$ $\quad y_{CO} = 0.016065 \times 24.232 = 0.3893$

$y_{CO2} = 0.006362 \times 24.232 = 0.1542$ $\quad y_{N2} = 0.00892 \times 24.232 = 0.2161$

$$R_{mix} = \bar{R}/M_{mix} = 8.3145/24.232 = 0.34312 \text{ kJ/kg/K}$$

$$m = PV/RT = 100 \times 10^{-3}/0.34312 \times 293.15 = \mathbf{9.942 \times 10^{-4}\ kg}$$

$$C_{V0\ MIX} = \sum c_i C_{V0\ i} = 0.02 \times 10.085 + 0.45 \times 0.744$$

$$+ 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056 \text{ kJ/kg K}$$

$${}_1Q_2 = U_2 - U_1 = mC_{V0}(T_2-T_1) = 9.942 \times 10^{-4} \times 0.9056 \times (100-20) = \mathbf{0.0720\ kJ}$$

12.32

The mixture in Problem 12.27 is heated to 500 K with constant volume. Find the final pressure and the total heat transfer needed using Table A.5.

Solution:

C.V. Mixture of constant volume.

Process: $V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = \int P\, dV = 0$

Energy Eq.: ${}_1Q_2 = m(u_2 - u_1) \cong m\, C_{V\text{mix}}\, (T_2 - T_1)$

Ideal gas: $PV = mRT \quad \Rightarrow \quad P_2 = P_1(T_2 / T_1)(V_1/V_2)$

$P_2 = P_1T_2/T_1 = 150 \times 500/300 = \mathbf{250\ kPa}$

From Eq.12.21:

$C_{V\text{mix}} = \sum c_iC_{V\ i} = 0.25 \times 0.745 + 0.5 \times 0.662 + 0.25 \times 0.653$

$= 0.6805$ kJ/kg K

${}_1Q_2 = 2 \times 0.6805(500 - 300) = \mathbf{272.2\ kJ}$

12.33

The mixture in Problem 12.27 is heated up to 500 K in a constant pressure process. Find the final volume and the total heat transfer using Table A.5.

Solution:

C.V. Mixture

Process: $P = \text{constant} \quad \Rightarrow \quad {}_1W_2 = \int P\, dV = P(V_2 - V_1)$

Energy Eq.: $${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1)$$
$$= m(h_2 - h_1) \cong m\, C_{P\ mix}(T_2 - T_1)$$

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$$
$$= 0.2513 \text{ kJ/kg K}$$

From Eq.12.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.25 \times 1.042 + 0.5 \times 0.922 + 0.25 \times 0.842$$
$$= 0.932 \text{ kJ/kg K}$$

$$V_2 = m\, R_{mix}\, T_2/P_2$$
$$= 2 \times 0.2513 \times 500/150 = \mathbf{1.675\ m^3}$$

$${}_1Q_2 = 2 \times 0.932(500 - 300) = \mathbf{372.8\ kJ}$$

12.34

A pipe flows 1.5 kg/s of a mixture with mass fractions of 40% CO_2 and 60% N_2 at 400 kPa, 300 K. Heating tape is wrapped around a section of pipe with insulation added and 2 kW electrical power is heating the pipe flow. Find the mixture exit temperature.

Solution:

C.V. Pipe heating section. Assume no heat loss to the outside, ideal gases.

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_{P\ mix}(T_e - T_i)$

From Eq.12.23

$$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.4 \times 0.842 + 0.6 \times 1.042 = 0.962 \text{ kJ/kg K}$$

Substitute into energy equation and solve for exit temperature

$$T_e = T_i + \dot{Q} / \dot{m}C_{P\ mix} = 300 + 2 / (1.5 \times 0.962) = \mathbf{301.3\ K}$$

12.35

A pipe flows 0.05 kmole a second mixture with mole fractions of 40% CO_2 and 60% N_2 at 400 kPa, 300 K. Heating tape is wrapped around a section of pipe with insulation added and 2 kW electrical power is heating the pipe flow. Find the mixture exit temperature.

Solution:

C.V. Pipe heating section. Assume no heat loss to the outside, ideal gases.

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) = \dot{n}(\bar{h}_e - \bar{h}_i) = \dot{n}\bar{C}_{P\ mix}(T_e - T_i)$

From Eqs. on page 494 (the extension of Eq.12.23)

$$\bar{C}_{P\ mix} = \sum y_i\ \bar{C}_i = 0.4 \times 0.842 \times 44.01 + 0.6 \times 1.042 \times 28.013$$

$$= 32.336\ \text{kJ/kmol}$$

Substitute into energy equation and solve for exit temperature

$$T_e = T_i + \dot{Q} / \dot{n}\bar{C}_{P\ mix} = 300 + 2/(0.05 \times 32.336) = \mathbf{301.2\ K}$$

12.36

A rigid insulated vessel contains 12 kg of oxygen at 200 kPa, 280 K separated by a membrane from 26 kg carbon dioxide at 400 kPa, 360 K. The membrane is removed and the mixture comes to a uniform state. Find the final temperature and pressure of the mixture.

Solution:

C.V. Total vessel. Control mass with two different initial states.

Mass: $m = m_{O2} + m_{CO2} = 12 + 26 = 38$ kg

Process: V = constant (rigid) => W = 0, insulated => Q = 0

Energy: $U_2 - U_1 = 0 - 0 = m_{O2}\, C_{V\,O2}(T_2 - T_{1\,O2}) + m_{CO2}C_{V\,CO2}(T_2 - T_{1\,CO2})$

Initial state from ideal gas Table A.5

$C_{V\,O2} = 0.662$ kJ/kg, $C_{V\,CO2} = 0.653$ kJ/kg K

O_2 : $V_{O2} = mRT_1/P = 12 \times 0.2598 \times 280/200 = 4.3646\ m^3$,

CO_2 : $V_{CO2} = mRT_1/P = 26 \times 0.1889 \times 360/400 = 4.4203\ m^3$

Final state mixture

$R_{MIX} = \sum c_i R_i = [12 \times 0.2598 + 26 \times 0.1889]/38 = 0.2113$ kJ/kg K

The energy equation becomes

$$m_{O2}\, C_{V\,O2}\, T_2 + m_{CO2}C_{V\,CO2}\, T_2$$
$$= m_{O2}\, C_{V\,O2}\, T_{1\,O2} + m_{CO2}C_{V\,CO2}\, T_{1\,CO2}$$
$$(7.944 + 16.978)\, T_2 = 2224.32 + 6112.08 = 8336.4 \text{ kJ}$$
$$\Rightarrow T_2 = \mathbf{334.5\ K}$$

From mixture gas constant and total volume

$P_2 = mR_{mix}T_2/V = 38 \times 0.2113 \times 334.5 / 8.7849 =$ **305.7 kPa**

12.37

A mixture of 40% water and 60% carbon dioxide by mass is heated from 400 K to 1000 K at constant pressure 120 kPa. Find the total change in enthalpy and entropy using Table A.5 values.

Solution:

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = 0.4 \times 0.4615 + 0.6 \times 0.1889 = 0.29794 \text{ kJ/kg K}$$

From Eq.12.23:

$$C_{P\ mix} = 0.4 \times 1.872 + 0.6 \times 0.842 = 1.254 \text{ kJ/kg K}$$

$$h_2 - h_1 \cong C_{P\ mix} (T_2 - T_1) = 1.254 \times (1000 - 400) = \mathbf{752.4\ kJ/kg}$$

From Eq.12.24:

$$s_2 - s_1 = C_{P\ mix} \ln(T_2 / T_1) - R_{\ mix} \ln(P_2 / P_1)$$

$$= C_{P\ mix} \ln(T_2 / T_1) = 1.254 \ln (1000/400) = \mathbf{1.149\ kJ/kg\ K}$$

As the two total pressures are the same the pressure correction term drops out.

12.38

Do Problem 12.37 but with variable heat capacity using values from Table A.8.

A mixture of 40% water and 60% carbon dioxide by mass is heated from 400 K to 1000 K at constant pressure 120 kPa. Find the total change in enthalpy and entropy using Table A.5 values.

Solution:

From Eq.12.12:

$$\begin{aligned} h_2 - h_1 &= c_{H2O}(h_2 - h_1)_{H2O} + c_{CO2}(h_2 - h_1)_{CO2} \\ &= 0.4(1994.13 - 742.3) + 0.6(971.67 - 303.76) \\ &= 500.69 + 400.75 \\ &= \mathbf{901.4\ kJ/kg} \end{aligned}$$

From Eq.12.16 and Eq.12.18:

$$\begin{aligned} s_2 - s_1 &= s^{o}_{T2} - s^{o}_{T1} - R_{MIX} \ln (P_2 / P_1) \\ &= c_{H2O}(s^{o}_{T2} - s^{o}_{T1})_{H2O} + c_{CO2}(s^{o}_{T2} - s^{o}_{T1})_{CO2} \\ &= 0.4(12.9192 - 11.0345) + 0.6(6.119 - 5.1196) \\ &= 0.75388 + 0.59964 = \mathbf{1.3535\ kJ/kg\ K} \end{aligned}$$

As the two total pressures are the same the pressure correction term drops out.

12.39

An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m}C_{P\ mix}(T_i - T_e)$

Properties: From Eqs.12.15 and 12.23

$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$

$= 0.30248$ kJ/kg K

$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.1 \times 0.842 + 0.1 \times 1.872 + 0.8 \times 1.042$

$= 1.105$ kJ/kg K

$PV = mR_{mix}T \;\; \Rightarrow \;\; \dot{m} = P\dot{V} / R_{mix}T$

$\dot{m} = 500 \times 2/(0.30248 \times 1000) = 3.306$ kg/s

$\dot{W}_T = 3.306 \times 1.105\ (1000 - 700) =$ **1096 kW**

12.40

Solve Problem 12.39 using the values of enthalpy from Table A.8.
An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m} \sum c_j (h_i - h_e)_j$

Properties: From Eqs.12.15 and 12.23

$$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$$
$$= 0.30248 \text{ kJ/kg K}$$

$$PV = mR_{mix}T \quad \Rightarrow \quad \dot{m} = P\dot{V} / R_{mix}T$$

$$\dot{m} = 500 \times 2/(0.30248 \times 1000) = 3.306 \text{ kg/s}$$

Now get the h values from Table A.8 (all in kJ/kg)

$$\dot{W}_T = 3.306 \,[\, 0.1 \,(971.67 - 616.22) + 0.1 \,(1994.13 - 1338.56)$$
$$+ 0.8 \,(1075.91 - 735.86) \,]$$

= **1234 kW**

12.41

An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mole basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

C.V. Turbine, Steady flow, 1 inlet, 1 exit flow with an ideal gas mixture, and no heat transfer so $q = 0$.

Energy Eq.: $\dot{W}_T = \dot{m}\,(h_i - h_e) = \dot{n}\,(\bar{h}_i - \bar{h}_e) = \dot{n}\,\bar{C}_{P\,mix}\,(T_i - T_e)$

Ideal gas law: $PV = n\bar{R}T$ =>

$$\dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{500 \times 2}{8.3145 \times 1000} = 0.1203 \text{ kmole/s}$$

The mixture heat capacity becomes

$$\bar{C}_{P\,mix} = \sum y_i\,\bar{C}_i = 0.1 \times 44.01 \times 0.842 + 0.1 \times 18.015 \times 1.872$$
$$+ 0.8 \times 28.013 \times 1.042 = 30.43 \text{ kJ/kmol K}$$

$$\dot{W}_T = 0.1203 \times 30.43\,(1000 - 700) = \mathbf{1098\ kW}$$

12.42

Solve Problem 12.41 using the values of enthalpy from Table A.9.

C.V. Turbine, Steady flow, 1 inlet, 1 exit flow with an ideal gas mixture, and no heat transfer so $q = 0$.

Energy Eq.: $\dot{W}_T = \dot{m}\,(h_i - h_e) = \dot{n}\,(\bar{h}_i - \bar{h}_e) = \dot{n}\left[\,\sum y_j\,(\bar{h}_i - \bar{h}_e)_j\,\right]$

Ideal gas law: $PV = n\bar{R}T$ =>

$$\dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{500 \times 2}{8.3145 \times 1000} = 0.1203 \text{ kmol/s}$$

Read the enthalpies from Table A.9 (they are all in kJ/kmol)

$$\dot{W}_T = 0.1203\left[0.1(33397 - 17754) + 0.1(26000 - 14190) + 0.8(21463 - 11937)\right]$$

$= \mathbf{1247\ kW}$

12.43

A piston/cylinder device contains 0.1 kg of a mixture of 40 % methane and 60 % propane gases by mass at 300 K and 100 kPa. The gas is now slowly compressed in an isothermal (T = constant) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution:

C.V. Mixture of methane and propane, this is a control mass.

Assume methane & propane are ideal gases at these conditions.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Property from Eq.12.15

$$R_{mix} = 0.4\ R_{CH4} + 0.6\ R_{C3H8}$$

$$= 0.4 \times 0.5183 + 0.6 \times 0.1886 = 0.3205 \text{ kJ/kg K}$$

Process: T = constant & ideal gas =>

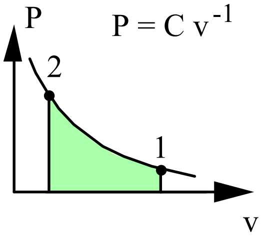
$${}_1W_2 = \int P\ dV = mR_{mix}T \int (1/V)dV = mR_{mix}T \ln (V_2/V_1)$$

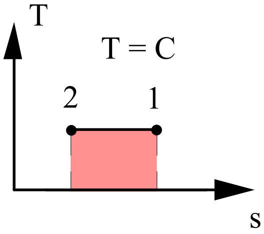
$$= mR_{mix}T \ln (P_1/P_2)$$

$$= 0.1 \times 0.3205 \times 300 \ln (100/250) = \mathbf{-8.81\ kJ}$$

Now heat transfer from the energy equation where we notice that u is a constant (ideal gas and constant T) so

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = {}_1W_2 = \mathbf{-8.81\ kJ}$$





12.44

Consider Problem 12.39 and find the value for the mixture heat capacity, mass basis and the mixture ratio of specific heats, k_{mix}, both estimated at 850 K from values (differences) of *h* in Table A.8. With these values make an estimate for the reversible adiabatic exit temperature of the turbine at 100 kPa.

Solution:

We will find the individual heat capacities by finite differences:

$C_{P\ i} = dh/dT = \Delta h/\Delta T = (h_{900} - h_{800})/(900 - 800)$

Read the h values from Table A.8

$C_{P\ CO2} = (849.72 - 731.02)/100 = 1.187$ kJ/kg K;

$C_{P\ H2O} = (1738.6 - 1550.13)/100 = 1.8847$ kJ/kg K

$C_{P\ N2} = (960.25 - 846.85)/100 = 1.134$ kJ/kg K

Properties: From Eqs.12.15 and 12.23

$$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$$
$$= 0.30248 \text{ kJ/kg K}$$

$$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.1 \times 1.187 + 0.1 \times 1.8847 + 0.8 \times 1.134$$
$$= 1.2144 \text{ kJ/kg K}$$

$$C_{V\ mix} = C_{P\ mix} - R_{mix} = 1.2144 - 0.30248 = 0.9119 \text{ kJ/kg K}$$

$$k = C_{P\ mix}/C_{V\ mix} = \mathbf{1.3317}$$

Reversible adiabatic turbine => Process is s = constant.

Assume constant average heat capacities so Eq.8.32:

$$T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 1000\ (100/500)^{\frac{0.3317}{1.3317}} = 1000\ (0.2)^{0.2491} = \mathbf{670\ K}$$

12.45

Consider Problem 12.41 and find the value for the mixture heat capacity, mole basis and the mixture ratio of specific heats, k_{mix}, both estimated at 850 K from values (differences) of *h* in Table A.9. With these values make an estimate for the reversible adiabatic exit temperature of the turbine at 100 kPa.

We will find the individual heat capacities by finite differences:

$$\bar{C}_{P\,i} = d\bar{h}/dT = \Delta\bar{h}/\Delta T = (\bar{h}_{900} - \bar{h}_{800}) / (900 - 800)$$

Now read the h values from Table A.9 (all in kJ/kmol)

$$\bar{C}_{P\,CO2} = (28030 - 22806)/100 = 52.24 \text{ kJ/kmol K}$$

$$\bar{C}_{P\,H2O} = (21937 - 18002)/100 = 39.35 \text{ kJ/kmol K}$$

$$\bar{C}_{P\,N2} = (18223 - 15046)/100 = 31.77 \text{ kJ/kmol K}$$

$$\bar{C}_{P\,mix} = \sum y_i\,\bar{C}_{P\,i} = 0.1 \times 52.24 + 0.1 \times 39.35 + 0.8 \times 31.77$$

$$= \mathbf{34.575\ kJ/kmol\ K}$$

$$\bar{C}_{V\,mix} = \bar{C}_{P\,mix} - \bar{R} = 34.575 - 8.3145 = 26.26 \text{ kJ/kmol K},$$

$$k = \bar{C}_{P\,mix}/\bar{C}_{V\,mix} = \mathbf{1.3166}$$

Reversible adiabatic turbine => Process is s = constant.

Assume constant average heat capacities:

$$T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 1000\,(100/500)^{\frac{0.3166}{1.3166}} = 1000\,(0.2)^{0.2405} = \mathbf{679\ K}$$

12.46

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 800 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.5 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant => ${}_1W_2 = \int P\,dV = P(V_2 - V_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$$

$$h_2 - h_1 = {}_1Q_2/m \cong C_{P\ mix}(T_2 - T_1)$$

From Eq.12.23 and Table A.5:

$$C_{P\ mix} = (1/2) \times 0.922 + (1/2) \times 1.042 = 0.982 \text{ kJ/kg K}$$

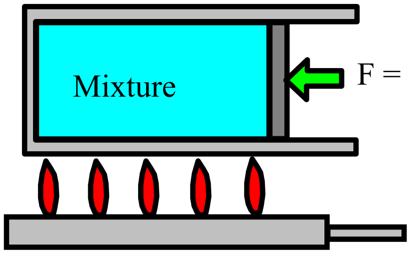
$$T_2 = T_1 + {}_1Q_2/mC_{P\ mix}$$

$$= 300 + 800/(1 \times 0.982) = \mathbf{1114.7\ K}$$

From Eq.12.24:

$$m(s_2 - s_1) = m[C_{P\ mix} \ln(T_2 / T_1) - R \ln(P_2 / P_1)]$$

$$= 1 \times 0.982 \times \ln(1114.7/300) = \mathbf{1.29\ kJ/K}$$



12.47

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 800 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.8 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant => ${}_1W_2 = \int P\,dV = P(V_2 - V_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1)$$

$$= m(h_2 - h_1)$$

$$h_2 - h_1 = {}_1Q_2/m = 800/1 = 800 \text{ kJ/kg}$$

Since T_2 is so high we use Table A.8 values guessing a T_2

$$(h_2 - h_1)_{1100K} = \frac{1}{2}(1193.62 - 311.67) + \frac{1}{2}(1090.62 - 273.15)$$

$$= 849.71 \text{ kJ/kg} \qquad \text{too high}$$

$$(h_2 - h_1)_{1000K} = \frac{1}{2}(1075.91 - 311.67) + \frac{1}{2}(980.95 - 273.15)$$

$$= 736.02 \text{ kJ/kg} \qquad \text{too low}$$

$$T_2 = 1000 + 100[(800 - 736.02)/(849.71 - 736.02)] = \mathbf{1056.3\ K}$$

From Eqs.12.16 and 12.18:

$$s_2 - s_1 = \frac{1}{2}(s^\circ_{T2} - s^\circ_{T1})_{N2} + \frac{1}{2}(s^\circ_{T2} - s^\circ_{T1})_{O2}$$

$$= \frac{1}{2}(8.2082 - 6.8463) + \frac{1}{2}(7.6709 - 6.4168)$$

$$= \mathbf{1.308\ kJ/kg\ K}$$

12.48

New refrigerant R-410a is a mixture of R-32 and R-125 in a 1:1 mass ratio. A process brings 0.5 kg R-410a from 270 K to 320 K at a constant pressure 250 kPa in a piston cylinder. Find the work and heat transfer.

Solution:

C.V. R-410a

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P\,(V_2 - V_1)$

Process: P = constant $\quad {}_1W_2 = P\,(V_2 - V_1) = mR(T_2 - T_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K}$$

FromEq.12.23:

$$C_{P\ mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K}$$

From the process equation

$${}_1W_2 = 0.5 \times 0.1145\,(320 - 270) = \mathbf{2.863\ kJ}$$

From the energy equation

$${}_1Q_2 = 0.5 \times 0.8065\,(320 - 270) = \mathbf{20.16\ kJ}$$

12.49

A piston/cylinder contains 0.5 kg argon and 0.5 kg hydrogen at 300 K, 100 kPa. The mixture is compressed in an adiabatic process to 400 kPa by an external force on the piston. Find the final temperature, the work and the heat transfer in the process.

Solution:

C.V. Mixture in cylinder. Control mass with adiabatic process: ${}_1Q_2 = 0$

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.5.11: $m(u_2 - u_1) = -\,{}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + 0$

Process: adiabatic and assumed reversible gives isentropic.

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.2081 + 0.5 \times 4.1243 = 2.1662 \text{ kJ/kg K}$$

$$C_{P\ mix} = \sum c_i C_{Pi} = 0.5 \times 0.52 + 0.5 \times 14.209 = 7.3645 \text{ kJ/kg K}$$

$$C_V = C_P - R = 7.3645 - 2.1662 = 5.1983 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_P / C_V = 1.417$

The constant s (isentropic) process from Eq.8.32

$$s_2 = s_1 \;\Rightarrow\; T_2 = T_1(P_2/P_1)^{(k-1)/k} = 300\ (400/100)^{0.2943} = \mathbf{451\ K}$$

The energy equation gives the work as

$$ {}_1W_2 = m(u_1 - u_2) = mC_V\,(T_1 - T_2)$$

$$= 1 \times 5.1983(300 - 451) = \mathbf{-784.9\ kJ}$$

12.50

Natural gas as a mixture of 75% methane and 25% ethane by mass is flowing to a compressor at 17°C, 100 kPa. The reversible adiabatic compressor brings the flow to 250 kPa. Find the exit temperature and the needed work per kg flow.

Solution:

C.V. Compressor. Steady, adiabatic $q = 0$, reversible $s_{gen} = 0$

Energy Eq.6.13: $-w = h_{ex} - h_{in}$; Entropy Eq.9.8: $s_i + s_{gen} = s_e$

Process: reversible => $s_{gen} = 0$ => $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.12.15 and 12.23:

$$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785 \text{ kJ/kg K}$$

$$C_{P\ mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132 \text{ kJ/kg K}$$

$$C_V = C_{P\ mix} - R_{mix} = 2.132 - 0.45785 = 1.6742 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_p / C_v = 1.2734$

The isentropic process gives Eq.8.32

$$T_e = T_i (P_e / P_i)^{(k-1)/k} = 290 (250/100)^{0.2147} = \mathbf{353\ K}$$

Work from the energy equation:

$$w_{c\ in} = C_P (T_e - T_i) = 2.132 (353 - 290) = \mathbf{134.3\ kJ/kg}$$

12.51

A mixture of 2 kg oxygen and 2 kg of argon is in an insulated piston cylinder arrangement at 100 kPa, 300 K. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature and the piston work.

Solution:

C.V. Mixture. Control mass, boundary work and no Q, assume reversible.

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = - {}_1w_2$;

Entropy Eq.8.14: $s_2 - s_1 = 0 + 0 = 0$

Process: constant s => Pv^k = constant, $v_2 = v_1/2$,

Assume ideal gases ($T_1 >> T_C$) and use k_{mix} and $C_{v\,mix}$ for properties.

Eq.12.15: $R_{mix} = \Sigma\, c_iR_i = 0.5 \times 0.25983 + 0.5 \times 0.20813 = 0.234$ kJ/kg K

Eq.12.23 $C_{Pmix} = \Sigma\, c_iC_{Pi} = 0.5 \times 0.9216 + 0.5 \times 0.5203 = 0.721$ kJ/kg K

$C_{vmix} = C_{Pmix} - R_{mix} = 0.487$ kJ/kg K

Ratio of specific heats: $k_{mix} = C_{Pmix}/C_{vmix} = 1.4805$

The relations for the polytropic process

Eq.8.34: $P_2 = P_1(v_1/v_2)^k = P_1(2)^k = 100(2)^{1.4805} = \mathbf{279\ kPa}$

Eq.8.33: $T_2 = T_1(v_1/v_2)^{k-1} = T_1(2)^{k-1} = 300(2)^{0.4805} = \mathbf{418.6\ K}$

Work from the energy equation

$${}_1W_2 = m_{tot}\,(u_1 - u_2) = m_{tot}\,C_v(T_1 - T_2) = 4\times 0.487\,(300 - 418.6) = \mathbf{-231\ kJ}$$

12.52

The substance R-410a, see Problem 12.48 is at 100 kPa, 290 K. It is now brought to 250 kPa, 400 K in a reversible polytropic process. Find the change in specific volume, specific enthalpy and specific entropy for the process.

Solution:

Eq.12.15: $R_{mix} = \Sigma\, c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145$ kJ/kg K

Eq.12.23: $C_{Pmix} = \Sigma\, c_i C_{Pi} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065$ kJ/kg K

$v_1 = RT_1/P_1 = 0.1145 \times 290/100 = 0.33205 \text{ m}^3/\text{kg}$

$v_2 = RT_2/P_2 = 0.1145 \times 400/250 = 0.1832 \text{ m}^3/\text{kg}$

$v_2 - v_1 = 0.1832 - 0.33205 =$ **$-0.14885 \text{ m}^3/\text{kg}$**

$h_2 - h_1 = C_{Pmix} (T_2 - T_1) = 0.8065\,(400 - 290) =$ **88.72 kJ/kg**

From Eq.12.24

$s_2 - s_1 = C_{Pmix} \ln(T_2 / T_1) - R_{mix} \ln(P_2 / P_1)$

$= 0.8065 \ln(400/290) - 0.1145 \ln(250/100) =$ **0.154 kJ/kg K**

12.53

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 1 m^3 and initially contains argon at 300 kPa, 10°C. Tank B has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C. The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Determine the final pressure and temperature.

Solution:

C.V. Tanks A + B. Control mass no W, no Q.

Energy Eq.5.11: $U_2 - U_1 = 0 = m_{Ar}C_{V0}(T_2 - T_{A1}) + m_{C_2H_6}C_{VO}(T_2 - T_{B1})$

$m_{Ar} = P_{A1}V_A/RT_{A1} = (300 \times 1) / (0.2081 \times 283.15) = 5.0913$ kg

$m_{C_2H_6} = P_{B1}V_B/RT_{B1} = (200 \times 2) / (0.2765 \times 323.15) = 4.4767$ kg

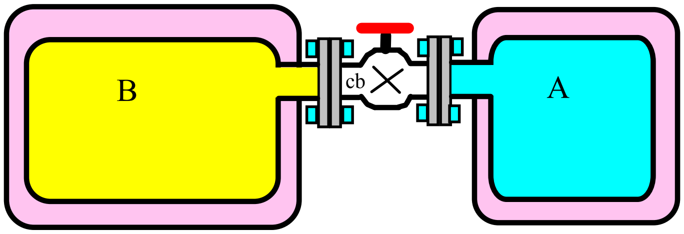
Continuity Eq.: $m_2 = m_{Ar} + m_{C_2H_6} = 9.568$ kg

Energy Eq.: $5.0913 \times 0.312\ (T_2 - 283.2)$
$+ 4.4767 \times 1.490\ (T_2 - 323.2) = 0$

Solving, $T_2 =$ **315.5 K**

$$R_{mix} = \Sigma\ c_i R_i = \frac{5.0913}{9.568} \times 0.2081 + \frac{4.4767}{9.568} \times 0.2765 = 0.2401 \text{ kJ/kg K}$$

$P_2 = m_2 R T_2 / (V_A + V_B) = 9.568 \times 0.2401 \times 315.5/3 =$ **242 kPa**



12.54

A compressor brings R-410a (see problem 12.48) from –10 ^{o}C, 125 kPa up to 500 kPa in an adiabatic reversible compression. Assume ideal gas behavior and find the exit temperature and the specific work.

Solution:

C.V. Compressor

Process: $q = 0$; adiabatic and reversible.

Energy Eq.6.13: $w = h_i - h_e$;

Entropy Eq.9.8: $s_e = s_i + s_{gen} + \int dq/T = s_i + 0 + 0 = s_i$

From Eq.12.15:

$$R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K}$$

FromEq.12.23:

$$C_{P\ mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K}$$

$$R_{mix}/\ C_{P\ mix} = 0.1145/0.8065 = 0.14197$$

For constant s, ideal gas and use constant specific heat as in Eq.8.29

$$T_e/T_i = (P_e/P_i)^{R/Cp}$$

$$T_e = 263.15 \times (500/125)^{0.14197} = \mathbf{320.39\ K}$$

$$w \cong C_{P\ mix}(\ T_i - T_e) = 0.8065\ (263.15 - 320.39)$$

$$= \mathbf{-46.164\ kJ/kg}$$

12.55

A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Process Pv^n = constant leading to

$$n \ln(v_2/v_1) = \ln(P_1/P_2); \quad v = RT/P$$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Eq.12.15: $R_{mix} = \Sigma\, c_i R_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252$ kJ/kg K

Eq.12.23: $C_{P\,mix} = \Sigma\, c_i C_{Pi} = 0.5 \times 0.8418 + 0.5 \times 1.872 = 1.3569$ kJ/kg K

Work is from Eq.9.19:

$$w = -\int v dP = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i) = \mathbf{476.4\ kJ/kg}$$

Heat transfer from the energy equation

$$q = h_e - h_i + w = C_P(T_e - T_i) + w = \mathbf{-880.5\ kJ/kg}$$

12.56

Solve Problem 12.55 using specific heats $C_P = \Delta h/\Delta T$, from Table A.8 at 1000 K. A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Using values from Table A.8 we estimate the heat capacities

$$C_{P\,CO_2} = \frac{1096.36 - 849.72}{1100 - 900} = 1.2332 \text{ kJ/kg K}$$

$$C_{P\,H_2O} = \frac{2226.73 - 1768.6}{1100 - 900} = 2.2906 \text{ kJ/kg K}$$

Eq.12.23: $C_{P\,mix} = \Sigma\, c_i C_{Pi} = 0.5 \times 1.2332 + 0.5 \times 2.2906 = 1.7619$ kJ/kg K

Eq.12.15: $R_{mix} = \Sigma\, c_i R_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252$ kJ/kg K

Process $Pv^n = C$ => $n = \ln(P_1/P_2) / \ln(v_2/v_1)$ and use $Pv = RT$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Work is from Eq.9.19

$$w = -\int v dP = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i) = \mathbf{476.4 \text{ kJ/kg}}$$

Heat transfer from energy equation

$$q = h_e - h_i + w = 1.7619(500 - 1500) + 476.4 = \mathbf{-1285.5 \text{ kJ/kg}}$$

12.57

A 50/50 (by mass) gas mixture of methane CH_4 and ethylene C_2H_4 is contained in a cylinder/piston at the initial state 480 kPa, 330 K, 1.05 m^3. The piston is now moved, compressing the mixture in a reversible, polytropic process to the final state 260 K, 0.03 m^3. Calculate the final pressure, the polytropic exponent, the work and heat transfer and entropy change for the mixture.

Solution:

Ideal gas mixture: CH_4, C_2H_4, 50% each by mass => $c_{CH_4} = c_{C_2H_4} = 0.5$

$$R_{mix} = \Sigma\, c_iR_i = 0.5 \times 0.5183 + 0.5 \times 0.2964 = 0.40735 \text{ kJ/kg K}$$

$$C_{v\ mix} = \sum c_i\, C_{vi} = 0.5 \times 1.736 + 0.5 \times 1.252 = 1.494 \text{ kJ/kg K}$$

State 1: $m = P_1V_1/R_{mix}T_1 = 480 \times 1.05 / (0.40735 \times 330) = 3.7493$ kg

State 2: $T_2 = 260$ K, $V_2 = 0.03$ m^3, Ideal gas PV = mRT so take ratio

$$=> \quad P_2 = P_1 \frac{V_1}{V_2}\frac{T_2}{T_1} = \mathbf{13\ 236\ kPa}$$

Process: PV^n = constant and PV = mRT gives $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$,

$$\ln\frac{T_2}{T_1} = (n\text{-}1)\ \ln\frac{V_1}{V_2} \quad => \quad \mathbf{n = 0.933}$$

also for this process we get Eq.8.38 or Eq.4.4

$$=> \qquad {}_1W_2 = \int P\, dV = \frac{1}{1\text{-}n}(P_2V_2 - P_1V_1) = \mathbf{-1595.7\ kJ}$$

Energy Eq.: $\quad {}_1Q_2 = U_2 - U_1 + {}_1W_2 = m\, C_{v\ mix}\,(T_2 - T_1) + {}_1W_2$

$$= 3.7493 \times 1.494(260 - 330) - 1595.7 = \mathbf{-1988\ kJ}$$

Change of entropy from Eq.8.26

$$s_2 - s_1 = C_{v\ mix} \ln (T_2 / T_1) + R_{mix} \ln (V_2 / V_1)$$

$$= 1.494 \ln(260 / 330) + 0.40735 \ln(0.03 / 1.05)$$

$$= \mathbf{-1.8045\ kJ/kg\ K}$$

and

$$S_2 - S_1 = m(s_2 - s_1) = 3.7493\ (-1.8045) = \mathbf{-\ 6.7656\ kJ/K}$$

12.58

The gas mixture from Problem 12.31 is compressed in a reversible adiabatic process from the initial state in the sample cylinder to a volume of 0.2 L. Determine the final temperature of the mixture and the work done during the process.

Solution:

From Eq.12.15

$$R_{mix} = \sum c_i R_i = 0.02 \times 4.1243 + 0.45 \times 0.2968 + 0.28 \times 0.1889$$
$$+ 0.25 \times 0.2968 = 0.34314 \text{ kJ/kg K}$$

$$m = PV/R_{mix}T = 100\times10^{-3}/(0.34314\times 293.15) = \mathbf{9.941\times10^{-4}\ kg}$$

$$C_{V0\,MIX} = \sum c_i\, C_{V0\,i} = 0.02 \times 10.085 + 0.45 \times 0.744$$
$$+ 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056 \text{ kJ/kg K}$$

$$C_{P0\,MIX} = C_{V0\,MIX} + R_{mix} = 0.9056 + 0.34314 = 1.2487 \text{ kJ/kg K}$$

$$\rightarrow k = C_{P0}/C_{V0} = 1.2487/0.9056 = 1.379$$

The process (adiabatic and reversible) is isentropic expressed in Eq.8.32

$$\rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 293.15\left(\frac{1}{0.2}\right)^{0.379} = \mathbf{539.5\ K}$$

$${}_1W_2 = -\Delta U_{12} = -mC_{V0}(T_2 - T_1)$$
$$= -9.941\times10^{-4} \times 0.9056 \times (539.5 - 293.15) = \mathbf{-\ 0.22\ kJ}$$

Entropy generation

12.59

A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process.

Solution:

C.V. Heat exchanger $\quad w = 0$

Energy Eq.6.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2\ [\tfrac{1}{2} \times (971.67 - 303.76) + \tfrac{1}{2} \times (980.95 - 366.03)] = \mathbf{1282.8\ kW}$$

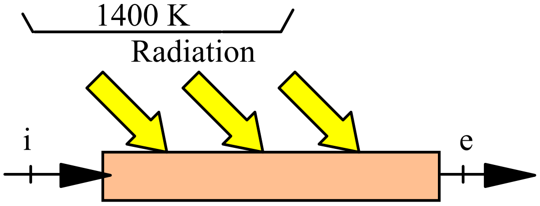
Entropy Eq.9.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

As the pressure is constant the pressure correction in Eq.8.28 drops out to give the generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$

$$= 2\ [\tfrac{1}{2} \times (6.119 - 5.1196) + \tfrac{1}{2} \times (7.6121 - 6.6838)] - 1282.8/1400$$

$$= \mathbf{1.01\ kW/K}$$



12.60

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are at 100 kPa and the mass ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kg of the exit mixture.

Solution:

CV mixing chamber. The inlet ratio is so $\dot{m}_{CO_2} = 2\,\dot{m}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity Eq.6.9: $\dot{m}_{N_2} + 2\dot{m}_{N_2} = \dot{m}_{ex} = 3\dot{m}_{N_2}$;

Energy Eq.6.10: $\dot{m}_{N_2}(h_{N_2} + 2\,h_{CO_2}) = 3\dot{m}_{N_2}h_{mix\ ex}$

Take 300 K as reference and write $h = h_{300} + C_{Pmix}(T - 300)$.

$$C_{P\,N_2}(T_{i\,N_2} - 300) + 2C_{P\,CO_2}(T_{i\,CO_2} - 300) = 3C_{P\,mix}(T_{mix\ ex} - 300)$$

$$C_{P\,mix} = \sum c_iC_{P\,i} = \frac{2}{3} \times 0.842 + \frac{1}{3} \times 1.042 = 0.9087 \text{ kJ/kg K}$$

$$3C_{P\,mix}T_{mix\ ex} = C_{P\,N_2}T_{i\,N_2} + 2C_{P\,CO_2}T_{i\,CO_2} = 830.64 \text{ kJ/kg}$$

$T_{mix\ ex} =$ **304.7 K**;

To find the entropies we need the partial pressures, which assuming ideal gas are equal to the mole fractions times the total pressure:

$$y_i = [c_i/\,M_i] \,/ \sum c_j/M_j$$

$$y_{N_2} = [0.3333 \,/\, 28.013] \,/\, [\frac{0.3333}{28.013} + \frac{0.6666}{44.01}] = 0.44$$

$$y_{CO_2} = 1 - y_{N_2} = 0.56$$

$$\dot{S}_{gen} = \dot{m}_{ex}s_{ex} - (\dot{m}s)_{iCO_2} - (\dot{m}s)_{iN_2} = \dot{m}_{N_2}(s_e - s_i)_{N_2} + 2\dot{m}_{N_2}(s_e - s_i)_{CO_2}$$

$$\frac{\dot{S}_{gen}}{3\dot{m}_{N_2}} = \frac{1}{3}\,[C_{PN_2}\ln\frac{T_{ex}}{T_{iN_2}} - R_{N_2}\ln y_{N_2}\,] + \frac{2}{3}\,[C_{PCO_2}\ln\frac{T_{ex}}{T_{iCO_2}} - R_{CO_2}\ln y_{CO_2}]$$

$$= \frac{1}{3}\,[\,1.042\,\ln(\frac{304.7}{280}) - 0.2968\,\ln 0.44\,]$$

$$+ \frac{2}{3}\,[\,0.842\,\ln(\frac{304.7}{320}) - 0.1889\,\ln 0.56\,]$$

$$= \ 0.110585 + 0.068275$$

= **0.1789 kJ/kg mix K**

12.61

Take Problem 12.60 with inlet temperature of 1400 K for the carbon dioxide and 300 K for the nitrogen. First estimate the exit temperature with the specific heats from Table A.5 and use this to start iterations using A.8 to find the exit temperature.

Solution:

CV mixing chamber. The inlet ratio is so $\dot{m}_{CO_2} = 2\ \dot{m}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity Eq.6.9: $\dot{m}_{N_2} + 2\dot{m}_{N_2} = \dot{m}_{ex} = 3\dot{m}_{N_2}$;

Energy Eq.6.10: $\dot{m}_{N_2}(h_{N_2} + 2\ h_{CO_2}) = 3\dot{m}_{N_2}h_{mix\ ex}$

$$C_{P\ mix} = \sum c_i C_{P\ i} = \frac{2}{3} \times 0.842 + \frac{1}{3} \times 1.042 = 0.9087 \text{ kJ/kg K}$$

Take 300 K as reference and write $h = h_{300} + C_{Pmix}(T - 300)$.

$$C_{P\ N_2}(T_{i\ N_2} - 300) + 2C_{P\ CO_2}(T_{i\ CO_2} - 300) = 3C_{P\ mix}(T_{mix\ ex} - 300)$$

$$3C_{P\ mix}T_{mix\ ex} = C_{P\ N_2}T_{i\ N_2} + 2C_{P\ CO_2}T_{i\ CO_2}$$

$$= 1.042 \times 300 + 2 \times 0.842 \times 1400 = 2669.6 \text{ kJ/kg}$$

⇨ $\mathbf{T_{mix\ ex} = 979.3\ K}$

A more accurate answer results from using the ideal gas Tables.

From Table A.8: $\Sigma\dot{m}_{in}\,h_{in} = \dot{m}_{N_2}\,[2 \times 1482.87 + 1 \times 311.67] = \dot{m}_{N_2} \times 3277.4$

@ 1000K : $\Sigma\dot{m}_{ex}\,h_{ex} = \dot{m}_{N_2}\,[2 \times 971.67 + 1075.91] = \dot{m}_{N_2} \times 3019.3$

@ 1100K : $\Sigma\dot{m}_{ex}\,h_{ex} = \dot{m}_{N_2}\,[2 \times 1096.36 + 1193.62] = \dot{m}_{N_2} \times 3386.34$

Now linear interpolation between 1000 K and 1100 K

$$T_{ex} = 1000 + 100 \times \frac{3277.4 - 3019.3}{3386.34 - 3019.3} = \mathbf{1070\ K}$$

12.62

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are coming in at 100 kPa and the mole ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kmole of the exit mixture.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\,\dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity: $\dot{n}_{CO_2} + 2\dot{n}_{N_2} = \dot{n}_{ex} = 3\dot{n}_{N_2}$;

Energy Eq.: $\dot{n}_{N_2}(\bar{h}_{N_2} + 2\bar{h}_{CO_2}) = 3\dot{n}_{N_2}\bar{h}_{mix\ ex}$

Take 300 K as reference and write $\bar{h} = \bar{h}_{300} + \bar{C}_{Pmix}(T - 300)$.

$$\bar{C}_{P\ N_2}(T_{i\ N_2} - 300) + 2\bar{C}_{P\ CO_2}(T_{i\ CO_2} - 300) = 3\bar{C}_{P\ mix}(T_{mix\ ex} - 300)$$

Find the specific heats in Table A.5 to get

$$\bar{C}_{P\ mix} = \sum y_i\bar{C}_{P\ i} = (1.042 \times 28.013 + 2 \times 0.842 \times 44.01)/3$$

$$= 34.43 \text{ kJ/kmol K}$$

$$3\bar{C}_{P\ mix}T_{mix\ ex} = \bar{C}_{P\ N_2}T_{i\ N_2} + 2\bar{C}_{P\ CO_2}T_{i\ CO_2} = 31889 \text{ kJ/kmol}$$

$$T_{mix\ ex} = \mathbf{308.7\ K}$$

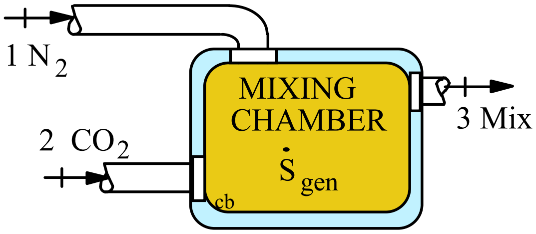
Partial pressures are total pressure times molefraction

$$P_{ex\ N_2} = P_{tot}/3; \quad P_{ex\ CO_2} = 2P_{tot}/3$$

$$\dot{S}_{gen} = \dot{n}_{ex}\bar{s}_{ex} - (\dot{n}\bar{s})_{iCO_2} - (\dot{n}\bar{s})_{iN_2} = \dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{N_2} + 2\dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{CO_2}$$

$$\dot{S}_{gen}/3\dot{n}_{N_2} = [\bar{C}_{PN_2}\ln\frac{T_{ex}}{T_{iN_2}} - \bar{R}\ln y_{N_2} + 2\bar{C}_{PCO_2}\ln\frac{T_{ex}}{T_{iCO_2}} - 2\,\bar{R}\ln y_{CO_2}]/3$$

$$= [2.8485 + 9.1343 - 2.6607 + 6.742]/3 = \mathbf{5.35\ kJ/kmol\ mix\ K}$$



12.63

Take Problem 12.62 with inlet temperature of 1400 K for the carbon dioxide and 300 K for the nitrogen. First estimate the exit temperature with the specific heats from Table A.5 and use this to start iterations using A.9 to find the exit temperature.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\ \dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

$\bar{C}_{P\ CO_2} = 44.01 \times 0.842 = 37.06$; $\bar{C}_{P\ N_2} = 28.013 \times 1.042 = 29.189$ kJ/kmol K

Continuity Equation: $0 = \Sigma\dot{n}_{in} - \Sigma\dot{n}_{ex}$;

Energy Equation: $0 = \Sigma\dot{n}_{in}\ \bar{h}_{in} - \Sigma\dot{n}_{ex}\ \bar{h}_{ex}$

$$0 = 2\dot{n}_{N_2}\ \bar{C}_{P\ CO_2}(T_{in} - T_{ex})_{\ CO_2} + \dot{n}_{N_2}\ \bar{C}_{P\ N_2}\ (T_{in} - T_{ex})_{\ N_2}$$

$$0 = 2 \times 37.06 \times (1400 - T_{ex}) + 29.189 \times (300 - T_{ex})$$

$$0 = 103768 + 8756.7 - 103.309\ T_{ex} \quad \Rightarrow \quad \mathbf{T_{ex} = 1089\ K}$$

From Table A.9: $\Sigma\dot{n}_{in}\ \bar{h}_{in} = \dot{n}_{N_2}\ [2 \times 55895 + 1 \times 54] = \dot{n}_{N_2} \times 111844$

@ 1000K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 33397 + 21463] = \dot{n}_{N_2} \times 88257$

@ 1100K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 38885 + 24760] = \dot{n}_{N_2} \times 102530$

@ 1200K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 44473 + 28109] = \dot{n}_{N_2} \times 117055$

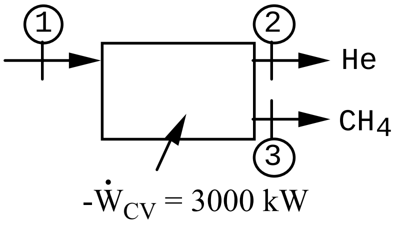
Now linear interpolation between 1100 K and 1200 K

$$T_{ex} = 1100 + 100 \times \frac{111844 - 102530}{117055 - 102530} = \mathbf{1164\ K}$$

12.64

The only known sources of helium are the atmosphere (mole fraction approximately 5×10^{-6}) and natural gas. A large unit is being constructed to separate 100 m^3/s of natural gas, assumed to be 0.001 He mole fraction and 0.999 CH_4. The gas enters the unit at 150 kPa, 10°C. Pure helium exits at 100 kPa, 20°C, and pure methane exits at 150 kPa, 30°C. Any heat transfer is with the surroundings at 20°C. Is an electrical power input of 3000 kW sufficient to drive this unit?

0.999 CH_4
0.001 He
at 150 kPa, 10 °C
$\dot{V}_1 = 100 \text{ m}^3/\text{s}$



$P_2 = 100$ kPa
$T_2 = 20$ °C

$P_3 = 140$ kPa
$T_3 = 30$ °C

$$\dot{n}_1 = P_1\dot{V}_1/RT_1 = 150\times100/(8.3145\times283.2) = 6.37 \text{ kmol/s}$$

$$=> \quad \dot{n}_2 = 0.001; \;\; \dot{n}_1 = 0.006\,37; \;\; \dot{n}_3 = 6.3636 \text{ kmol/s}$$

$$\bar{C}_{P\,He} = 4.003\times5.193 = 20.7876 \text{ kJ/kmol K},$$

$$\bar{C}_{P\,CH_4} = 16.043\times2.254 = 36.1609 \text{ kJ/kmol K}$$

$$\dot{Q}_{CV} = \dot{n}_2\bar{h}_2 + \dot{n}_3\bar{h}_3 - \dot{n}_1\bar{h}_1 + \dot{W}_{CV} = \dot{n}_2\bar{C}_{P0\,He}(T_2-T_1) + \dot{n}_3\bar{C}_{P0\,CH_4}(T_3-T_1) + \dot{W}_{CV}$$

$$= 0.00637\times20.7876(20-10)+6.3636\times36.1609(30-10)+(-3000) = +1600 \text{ kW}$$

$$\dot{S}_{gen} = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3 - \dot{n}_1\bar{s}_1 - \dot{Q}_{CV}/T_0$$

$$= 0.00637\left[20.7876 \ln\frac{293.2}{283.2} - 8.3145 \ln\frac{100}{0.001\times150}\right]$$

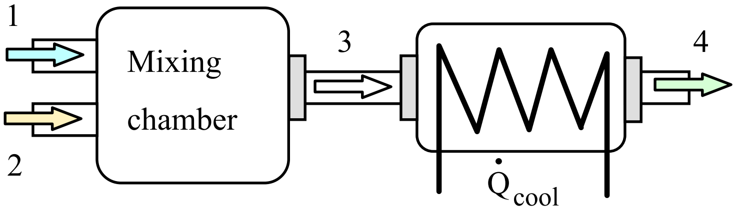
$$+ 6.3636\left[36.1609 \ln\frac{303.2}{283.2} - 8.3145 \ln\frac{140}{0.999\times150}\right] - 1600/293.2$$

$$= \mathbf{+13.5\ kW/K\ > 0}$$

12.65

A flow of 1 kg/s carbon dioxide at 1600 K, 100 kPa is mixed with a flow of 2 kg/s water at 800 K, 100 kPa and after the mixing it goes through a heat exchanger where it is cooled to 500 K by a 400 K ambient. How much heat transfer is taken out in the heat exchanger? What is the entropy generation rate for the whole process?

Solution:



C.V. Total mixing section and heat exchanger. Steady flow and no work. To do the entropy at the partial pressures we need the mole fractions.

$$\dot{n}_{H2O} = \dot{m}_{H2O}/M_{H2O} = 2 / 18.015 = 0.11102 \text{ kmol/s}$$

$$\dot{n}_{CO2} = \dot{m}_{CO2}/M_{CO2} = 1 / 44.01 = 0.022722 \text{ kmol/s}$$

$$y_{H2O} = \frac{0.11102}{0.11102 + 0.022722} = 0.8301, \quad y_{CO2} = 1 - y_{H2O} = 0.1699$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{CO2}\, h_2 = \dot{Q}_{cool} + \dot{m}_{H2O}\, h_{4\,H2O} + \dot{m}_{CO2}\, h_{4\,CO2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{CO2}\, s_2 + \dot{S}_{gen} = \frac{\dot{Q}_{cool}}{T_{amb}} + \dot{m}_{H2O}\, s_{4\,H2O} + \dot{m}_{CO2}\, s_{4\,CO2}$

As T is fairly high we use Table A.8 for properties on a mass basis.

	1	2	4 H2O	4 CO2
h [kJ/kg]	1550.13	1748.12	935.12	401.52
s^o_T [kJ/kg K]	12.4244	6.7254	11.4644	5.3375

$$\dot{Q}_{cool} = \dot{m}_{H2O}\,(h_1 - h_{4\,H2O}) + \dot{m}_{CO2}\,(h_2 - h_{4\,CO2})$$

$$= 2\,(1550.13 - 935.12) + 1\,(1748.12 - 401.52) = \mathbf{2577\ kW}$$

$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{4\,H2O} - s_1) + \dot{m}_{CO2}\,(s_{4\,CO2} - s_2) + \frac{\dot{Q}_{cool}}{T_{amb}}$$

$$= 2\,[11.4644 - 12.4244 - 0.4615 \ln(0.8301)\,]$$

$$+ 1\,[5.3375 - 6.7254 - 0.1889 \ln(0.1699)\,] + \frac{2577}{400}$$

$$= -1.74813 - 1.05307 + 6.4415 = \mathbf{3.64\ kW/K}$$

12.66

A mixture of 60% helium and 40% nitrogen by mass enters a turbine at 1 MPa, 800 K at a rate of 2 kg/s. The adiabatic turbine has an exit pressure of 100 kPa and an isentropic efficiency of 85%. Find the turbine work.

Solution:

Assume ideal gas mixture and take CV as turbine.

Energy Eq.6.13: $w_{T\,s} = h_i - h_{es}$,

Entropy Eq.9.8: $s_{es} = s_i$, adiabatic and reversible

Process Eq.8.32: $T_{es} = T_i(P_e/P_i)^{(k-1)/k}$

Properties from Eq.12.23, 12.15 and 8.30

$C_{P\,mix} = 0.6\times 5.193 + 0.4\times 1.042 = 3.5326$ kJ/kg K

$R_{mix} = 0.6\times 2.0771 + 0.4\times 0.2968 = 1.365$ kJ/kg K

$(k-1)/k = R/C_{P\,mix} = 1.365/3.5326 = 0.3864$

$T_{es} = 800(100/1000)^{0.3864} = 328.6$ K

$w_{Ts} = C_P(T_i - T_{es}) = 3.5326(800 - 328.6) = 1665$ kJ/kg

$w_{T\,ac} = \eta w_{Ts} = 1415.5$ kJ/kg

$\dot{W}_{T\,ac} = \dot{m} w_{T\,ac} =$ **2831 kW**

12.67

Repeat Problem 12.50 for an isentropic compressor efficiency of 82%.

Solution:

C.V. Compressor. Steady, adiabatic q = 0, reversible $s_{gen} = 0$

Energy Eq.6.13: $-w = h_{ex} - h_{in}$; Entropy Eq.9.8: $s_i + s_{gen} = s_i = s_e$

Process: reversible => $s_{gen} = 0$ => $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.12.15 and 12.23:

$$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785 \text{ kJ/kg K}$$

$$C_{P\,mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132 \text{ kJ/kg K}$$

$$C_V = C_{P\,mix} - R_{mix} = 2.132 - 0.45785 = 1.6742 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_p/ C_v = 1.2734$

The isentropic process gives Eq.8.32

$$T_e = T_i\,(P_e/ P_i)^{(k-1)/k} = 290\,(250/100)^{\,0.2147} = 353 \text{ K}$$

Work from the energy equation:

$$w_{c\,in} = C_P\,(T_e - T_i) = 2.132\,(353 - 290) = 134.3 \text{ kJ/kg}$$

The actual compressor requires more work

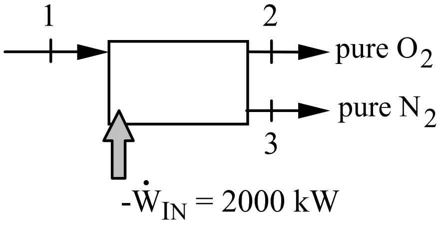
$w_{c\,actual} = w_{c\,in}/\eta = 134.3/0.82 = \mathbf{163.8\ kJ/kg} = C_p\,(T_{e\,actual} - T_i)$

=> $T_{e\,actual} = T + w_{c\,actual}/C_P = 290 + 163.8 / 2.132 = \mathbf{366.8\ K}$

12.68

A large air separation plant takes in ambient air (79% N_2, 21% O_2 by mole) at 100 kPa, 20°C, at a rate of 25 kg/s. It discharges a stream of pure O_2 gas at 200 kPa, 100°C, and a stream of pure N_2 gas at 100 kPa, 20°C. The plant operates on an electrical power input of 2000 kW. Calculate the net rate of entropy change for the process.

Air 79 % N_2
21 % O_2
$P_1 = 100$ kPa
$T_1 = 20$ °C
$\dot{m}_1 = 25$ kg/s



$P_2 = 200$ kPa
$T_2 = 100$ °C
$P_3 = 100$ kPa
$T_3 = 20$ °C

Solution:

To have the flow terms on a mass basis let us find the mass fractions

From Eq. 12.3: $c_i = y_i M_i / \sum y_j M_j$

$c_{O_2} = 0.21 \times 32 / [0.21 \times 32 + 0.79 \times 28.013] = 0.23293$;

$c_{N_2} = 1 - c_{O_2} = 0.76707$

$\dot{m}_2 = c_{O_2}\dot{m}_1 = 5.823$ kg/s ; $\dot{m}_3 = c_{N_2}\dot{m}_1 = 19.177$ kg/s

The energy equation, Eq.6.10 gives the heat transfer rate as

$$\dot{Q}_{CV} = \Sigma\, \dot{m}\Delta h_i + \dot{W}_{CV} = \dot{m}_{O_2}C_{P0\,O_2}(T_2 - T_1) + \dot{m}_{N_2}C_{P0\,N_2}(T_3 - T_1) + \dot{W}_{CV}$$

$$= 5.823 \times 0.922 \times (100\text{-}20) + 0 - 2000 = -1570.5 \text{ kW}$$

The entropy equation, Eq.9.7 gives the generation rates as

$$\dot{S}_{gen} = \Sigma\, \dot{m}_i\Delta s_i - \dot{Q}_{CV}/T_0 = (\dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1) - \dot{Q}_{CV}/T_0$$

Use Eq.8.25 for the entropy change

$$\Sigma\, \dot{m}_i\Delta s_i = 5.823 \left[\, 0.922 \ln\frac{373.2}{293.2} - 0.2598 \ln\frac{200}{21} \right]$$

$$+\ 19.177\, [0 - 0.2968 \ln(100/79)] = -3.456 \text{ kW/K}$$

$\dot{S}_{gen} = 1570.5/293.2 - 3.456 =$ **1.90 kW/K**

12.69

A steady flow of 0.3 kg/s of 50% carbon dioxide and 50% water by mass at 1200K and 200 kPa is used in a heat exchanger where 300 kW is extracted from the flow. Find the flow exit temperature and the rate of change of entropy using Table A.8.

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = c_{H2O} = 0.5$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i)$ => $h_e = h_i + \dot{Q}/\dot{m}$

Inlet state: Table A.8 $h_i = 0.5 \times 1223.34 + 0.5 \times 2466.25 = 1844.8$ kJ/kg

Exit state: $h_e = h_i + \dot{Q}/\dot{m} = 1844.8 - 300/0.3 = 844.8$ kJ/kg

Trial and error for T with h values from Table A.8

@500 K $h_e = 0.5(401.52 + 935.12) = 668.32$ kJ/kg

@600 K $h_e = 0.5(506.07 + 1133.67) = 819.87$ kJ/kg

@650 K $h_e = 0.5(560.51 + 1235.30) = 897.905$ kJ/kg

Interpolate to have the right h: **T = 616 K**

Entropy Eq.9.8: $\dot{m}s_e = \dot{m}s_i + \dot{Q}/T + \dot{S}_{gen}$

The rate of change of entropy for the flow is (P is assumed constant)

$$\dot{m}(s_e - s_i) = \dot{m}(s^o_{Te} - s^o_{Ti})$$

$$= 0.3[\ 0.5(5.5558 - 6.3483) + 0.5(11.8784 - 13.3492)\ ]$$

$$= \mathbf{-0.339\ kW/K}$$

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

12.70

A steady flow of 0.01 kmol/s of 50% carbon dioxide and 50% water at 1200K and 200 kPa is used in a heat exchanger where 300 kW is extracted from the flow. Find the flow exit temperature and the rate of change of entropy using Table A.9.

C.V. Heat exchanger, Steady flow, 1 inlet, 1 exit, no work.

Continuity Eq.: $y_{CO2} = y_{H2O} = 0.5$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) = \dot{n}(\bar{h}_e - \bar{h}_i) \quad => \quad \bar{h}_e = \bar{h}_i + \dot{Q}/\dot{n}$

Inlet state: Table A.9 $\bar{h}_i = 0.5 \times 44473 + 0.5 \times 34506 = 39489.5$ kJ/kmol

Exit state: $\bar{h}_e = \bar{h}_i + \dot{Q}/\dot{n} = 39489.5 - 300/0.01 = 9489.5$ kJ/kmol

Trial and error for T with h values from Table A.9

@500 K $\bar{h}_e = 0.5(8305 + 6922) = 7613.5$ kJ/kmol

@600 K $\bar{h}_e = 0.5(12906 + 10499) = 11702.5$ kJ/kmol

Interpolate to have the right h: **T = 545.9 K**

12.71

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{O2}\, h_2 = \dot{m}_{H2O}\, h_{3\ H2O} + \dot{m}_{O2}\, h_{3\ O2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{O2}\, s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\, s_{3\ H2O} + \dot{m}_{O2}\, s_{3\ O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\,(h_{3\ H2O} - h_1) + \dot{m}_{O2}\,(h_{3\ O2} - h_2) = 0$$

$$\dot{m}_{H2O}\, C_{P\ H2O}(T_3 - T_1) + \dot{m}_{O2}\, C_{P\ O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872\,(T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \mathbf{545.6\ K}$$

$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{3\ H2O} - s_1) + \dot{m}_{O2}\,(s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O}\,[\, C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O}\,] + \dot{m}_{O2}\,[\, C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2}\,]$$

$$= 1.8\,[\,1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5\,]$$

$$+ 3.2\,[\,0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5\,]$$

$$= -0.132 + 1.492 = \mathbf{1.36\ kW/K}$$

12.72

A tank has two sides initially separated by a diaphragm. Side A contains 1 kg of water and side B contains 1.2 kg of air, both at 20°C, 100 kPa. The diaphragm is now broken and the whole tank is heated to 600°C by a 700°C reservoir. Find the final total pressure, heat transfer and total entropy generation.

C.V. Total tank out to reservoir.

Energy Eq.5.11: $U_2 - U_1 = m_a(u_2 - u_1)_a + m_v(u_2 - u_1)_v = {}_1Q_2$

Entropy Eq.8.14 and 8.18:

$$S_2 - S_1 = m_a(s_2 - s_1)_a + m_v(s_2 - s_1)_v = {}_1Q_2/T_{res} + S_{gen}$$

Volume: $V_2 = V_A + V_B = m_v v_{v1} + m_a v_{a1} = 0.001 + 1.009 = 1.01 \text{ m}^3$

$v_{v2} = V_2/m_v = 1.01,\ T_2 \ \Rightarrow\ P_{2v} = 400 \text{ kPa}$

$v_{a2} = V_2/m_a = 0.8417,\ T_2 \ \Rightarrow\ P_{2a} = mRT_2/V_2 = 297.7 \text{ kPa}$

$P_{2tot} = P_{2v} + P_{2a} =$ **697.7 kPa**

Water table B.1: $u_1 = 83.95$ kJ/kg, $u_2 = 3300$ kJ/kg,

$s_1 = 0.2966$ kJ/kg K, $s_2 = 8.4558$ kJ/kg K

Air table A.7: $u_1 = 293$ kJ/kg, $u_2 = 652.3$ kJ/kg,

$s_{T1} = 2.492$ kJ/kg K, $s_{T2} = 3.628$ kJ/kg K

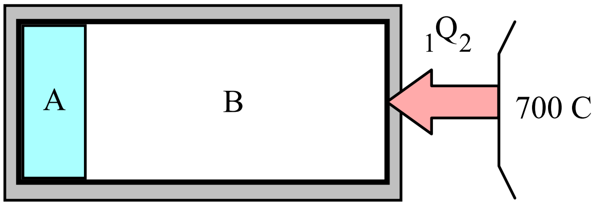
From energy equation we have

${}_1Q_2 = 1(3300 - 83.95) + 1.2(652.3 - 293) =$ **3647.2 kJ**

From the entropy equation we have

$S_{gen} = 1(8.4558 - 0.2966) + 1.2[3.628 - 2.492 - 0.287\times\ln(301.6/100)]$

$- 3647.2 / 973.2 =$ **5.4 kJ/K**



12.73

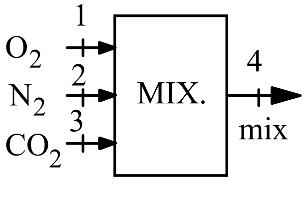
Three steady flows are mixed in an adiabatic chamber at 150 kPa. Flow one is 2 kg/s of O_2 at 340 K, flow two is 4 kg/s of N_2 at 280 K and flow three is 3 kg/s of CO_2 at 310 K. All flows are at 150 kPa the same as the total exit pressure. Find the exit temperature and the rate of entropy generation in the process.

Solution:

C.V. Mixing chamber, no heat transfer, no work.

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$



Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_4 s_4$

Assume ideal gases and since T is close to 300 K use heat capacity from A.5 in the energy equation as

$$\dot{m}_1 C_{P\,O2}(T_1 - T_4) + \dot{m}_2 C_{P\,N2}(T_2 - T_4) + \dot{m}_3 C_{P\,CO2}(T_3 - T_4) = 0$$

$$2 \times 0.922 \times 340 + 4 \times 1.042 \times 280 + 3 \times 0.842 \times 310$$

$$= (2 \times 0.922 + 4 \times 1.042 + 3 \times 0.842\ )\ T_4$$

$$\Rightarrow \quad 2577.06 = 8.538\ T_4 \qquad \Rightarrow \qquad \mathbf{T_4 = 301.83\ K}$$

State 4 is a mixture so the component exit pressure is the partial pressure. For each component $s_e - s_i = C_P \ln(T_e / T_i) - R \ln(P_e / P_i)$ and the pressure ratio is $P_e / P_i = y\ P_4 / P_i = y$ for each.

$$n = \sum \frac{m}{M} = \frac{2}{32} + \frac{4}{28.013} + \frac{3}{44.01} = 0.0625 + 0.1428 + 0.06817 = 0.2735$$

$$y_{O2} = \frac{0.0625}{0.2735} = 0.2285, \quad y_{N2} = \frac{0.1428}{0.2735} = 0.5222, \quad y_{CO2} = \frac{0.06817}{0.2735} = 0.2493$$

The entropy generation becomes

$$\dot{S}_{gen} = \dot{m}_1(s_4 - s_1) + \dot{m}_2(s_4 - s_2) + \dot{m}_3(s_4 - s_3)$$

$$= \quad 2\ [\ 0.922 \ln(301.83/340) - 0.2598 \ln(0.2285)]$$

$$+ 4\ [\ 1.042 \ln(301.83/280) - 0.2968 \ln(0.5222)]$$

$$+ 3\ [\ 0.842 \ln(301.83/310) - 0.1889 \ln(0.2493)]$$

$$= 0.5475 + 1.084 + 0.2399 = \mathbf{1.871\ kW/K}$$

12.74

Reconsider the Problem 12.53, but let the tanks have a small amount of heat transfer so the final mixture is at 400 K. Find the final pressure, the heat transfer and the entropy change for the process.

C.V. Both tanks. Control mass with mixing and heating of two ideal gases.

$$n_{Ar} = P_{A1}V_A/\bar{R}T_{A1} = \frac{300\times1}{8.3145\times283.2} = 0.1274 \text{ kmol}$$

$$n_{C_2H_6} = P_{B1}V_B/\bar{R}T_{B1} = \frac{200\times2}{8.3145\times323.2} = 0.1489 \text{ kmol}$$

Continuity Eq.: $n_2 = n_{Ar} + n_{C_2H_6} = 0.2763$ kmol

Energy Eq.: $U_2\text{-}U_1 = n_{Ar}\bar{C}_{V0}(T_2\text{-}T_{A1}) + n_{C_2H_6}\bar{C}_{VO}(T_2\text{-}T_{B1}) = {}_1Q_2$

$P_2 = n_2\bar{R}T_2/(V_A+V_B) = 0.2763\times8.3145\times400 / 3 =$ **306.3 kPa**

${}_1Q_2 = 0.1274\times39.948\times0.312(400 - 283.15)$

$+ 0.1489\times30.07\times1.49(400 - 323.15) =$ **698.3 kJ**

$\Delta S_{SURR} = -{}_1Q_2/T_{SURR}$; $\Delta S_{SYS} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6}$

$y_{Ar} = 0.1274/0.2763 = 0.4611$

$$\Delta\bar{S}_{Ar} = \bar{C}_{P\,Ar} \ln\frac{T_2}{T_{A1}} - \bar{R}\ln\frac{y_{Ar}P_2}{P_{A1}}$$

$$= 39.948\times0.520 \ln\frac{400}{283.15} - 8.3145 \ln\frac{0.4611\times306.3}{300}$$

$= 13.445$ kJ/kmol K

$$\Delta\bar{S}_{C_2H_6} = \bar{C}_{C_2H_6} \ln\frac{T_2}{T_{B1}} - \bar{R}\ln\frac{y_{C_2H_6}P_2}{P_{B1}}$$

$$= 30.07\times1.766 \ln\frac{400}{323.15} - 8.3145 \ln\frac{0.5389\times306.3}{200}$$

$= 12.9270$ kJ/kmol K

Assume the surroundings are at 400 K (it heats the gas)

$\Delta S_{NET} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6} + \Delta S_{SURR}$

$= 0.1274\times13.445 + 0.1489\times12.9270 - 698.3/400$

$=$ **1.892 kJ/K**

Air- water vapor mixtures

12.75

Atmospheric air is at 100 kPa, 25°C and relative humidity 75%. Find the absolute humidity and the dew point of the mixture. If the mixture is heated to 30°C what is the new relative humidity?

Solution:

Eq.12.25: $P_v = \phi\ P_g = 0.75 \times 3.169 = 2.377$ kPa

Eq.12.28: $w = 0.622\ P_v/(P_{tot} - P_v) = 0.622 \times 2.377/(100 - 2.377) = 0.01514$

T_{dew} is the T such that $P_g(T) = P_v = 2.377$ kPa

B.1.1 => $T \cong 20.2$ °C

Heating => w is constant => P_v is constant

From Table B.1.1: $P_g(30°C) = 4.246$ kPa

$\phi = P_v/P_g = 2.377/4.246 =$ **0.56 or 56 %**

12.76

Consider 100 m^3 of atmospheric air which is an air–water vapor mixture at 100 kPa, 15°C, and 40% relative humidity. Find the mass of water and the humidity ratio. What is the dew point of the mixture?

Solution:

Air-vapor $P = 100$ kPa, $T = 15\ ^oC$, $\phi = 40\%$

Use Table B.1.1 and then Eq.12.25

$P_g = P_{sat15} = 1.705$ kPa => $P_v = \phi\ P_g = 0.4 \times 1.705 = 0.682$ kPa

$$m_v = \frac{P_v V}{R_v T} = \frac{0.682 \times 100}{0.461 \times 288.15} = \mathbf{0.513\ kg}$$

$P_a = P_{tot} - P_{v1} = 100 - 0.682 = 99.32$ kPa

$$m_a = \frac{P_a V}{R_a T} = \frac{99.32 \times 100}{0.287 \times 288.15} = 120.1 \text{ kg}$$

$$w_1 = \frac{m_v}{m_a} = \frac{0.513}{120.1} = \mathbf{0.0043}$$

T_{dew} is T when $P_v = P_g = 0.682$ kPa;

Table B.1.2 gives $T = \mathbf{1.4\ ^oC}$

12.77

The products of combustion are flowing through a heat exchanger with 12% CO_2, 13% H_2O and 75% N_2 on a volume basis at the rate 0.1 kg/s and 100 kPa. What is the dew-point temperature? If the mixture is cooled 10°C below the dew-point temperature, how long will it take to collect 10 kg of liquid water?

Solution:

Volume basis is the same as mole fraction

$y_{H_2O} = 0.13; \quad P_{H_2O} = 0.13 \times 100 = 13$ kPa,

Table B.1.2 $\quad T_{DEW} = \mathbf{50.95\ ^oC}$

Cool to 40.95 $^oC < T_{DEW}$ so saturated $\quad \rightarrow P_G = 7.805$ kPa

$y_{H_2O} = 7.805/100 = n_{H_2O(v)}/(n_{H_2O(v)} + 0.87)$

$n_{H_2O(v)} = 0.07365$ per kmol mix in

$\rightarrow n_{LIQ} = 0.13 - 0.07365 = 0.05635$

$M_{MIX\ IN} = 0.12 \times 44.01 + 0.13 \times 18.015 + 0.75 \times 28.013 = 28.63$ kg/kmol

$\dot{n}_{MIX\ IN} = \dot{m}_{TOTAL}/M_{MIX\ IN} = 0.1/28.63 = 0.003493$ kmol/s

$\dot{n}_{LIQ\ COND} = 0.003\ 493 \times 0.05635 = 0.000\ 197$ kmol/s

or $\dot{m}_{LIQ\ COND} = 0.000\ 197 \times 18.015 = 0.003\ 55$ kg/s

For 10 kg, takes ~ **47 minutes**

12.78

A flow of 1 kg/s saturated moist air (relative humidity 100%) at 100 kPa, 10^oC goes through a heat exchanger and comes out at 25^oC. What is the exit relative humidity and how much power is needed?

Solution:

State 1 : $\phi_1 = 1$; $P_v = P_g = 1.2276$

Eq.12.28: $w = 0.622\ P_v/P_a = 0.622 \times 1.2276/(100 - 1.2276) = 0.00773$

State 2 : No water added => $w_2 = w_1$ => $P_{v2} = P_{v1}$

$\phi_2 = P_{v2}/P_{g2} = 1.2276/3.169 = 0.387$ or 39 %

Energy Eq.6.10

$$\dot{Q} = \dot{m}_2 h_2 - \dot{m}_1 h_1 = \dot{m}_a(h_2 - h_1)_{air} + w\dot{m}_a(h_2 - h_1)_{vapor}$$

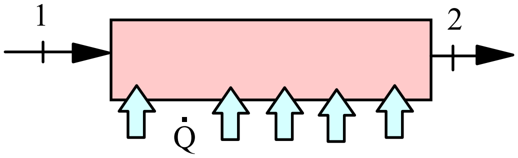
$$\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + w_1)$$

Energy equation with $C_{P\ air}$ from A.5 and h's from B.1.1

$$\dot{Q} = \frac{\dot{m}_{tot}}{1 + w_1} C_{P\ air}\,(25 - 10) + \frac{\dot{m}_{tot}}{1 + w_1} w\,(h_{g2} - h_{g1})$$

$$= \frac{1}{1.00773} \times 1.004(25 - 10) + \frac{1 \times 0.00773}{1.00773} (2547.17 - 2519.74)$$

$$= 14.9445 + 0.210407 = \mathbf{15.15\ kW}$$



12.79

A new high-efficiency home heating system includes an air-to-air heat exchanger which uses energy from outgoing stale air to heat the fresh incoming air. If the outside ambient temperature is −10°C and the relative humidity is 30%, how much water will have to be added to the incoming air, if it flows in at the rate of 1 m^3/s and must eventually be conditioned to 20°C and 40% relative humidity?

Solution:

Outside ambient air: $P_{V1} = \phi_1 P_{G1} = 0.30 \times 0.2602 = 0.078$ kPa

Assuming $P_1 = P_2 = 100$ kPa, => $P_{A1} = 100 - 0.078 = 99.922$ kPa

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.922 \times 1}{0.287 \times 263.2} = 1.3228 \text{ kg/s}$$

From Eq.12.28: $w_1 = 0.622 \times \frac{0.078}{99.922} = 0.00049$

Conditioned to : $T_2 = 20\ ^oC$, $\phi_2 = 0.40$

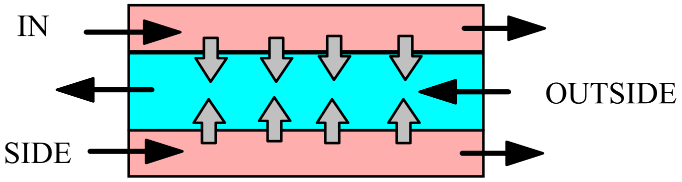
Eq.12.25: $P_{V2} = \phi_2 P_{G2} = 0.40 \times 2.339 = 0.9356$ =>

Eq.12.28: $w_2 = 0.622 \times \frac{0.9356}{99.064} = 0.00587$

Continuity equation for water,

$$\dot{m}_{LIQ\ IN} = \dot{m}_A(w_2 - w_1) = 1.3228(0.00587 - 0.00049)$$

$= 0.00712$ kg/s = **25.6 kg/h**



12.80

Consider a 1 m^3/s flow of atmospheric air at 100 kPa, 25°C, and 80% relative humidity. Assume this flows into a basement room where it cools to 15°C, 100 kPa. How much liquid water will condense out?

Solution:

State 1: $P_g = P_{sat25} = 3.169 \text{ kPa} \;\Rightarrow\; P_v = \phi\, P_g = 0.8 \times 3.169 = 2.535 \text{ kPa}$

$$\dot{m}_{v1} = \frac{P_v \dot{V}}{R_v T} = \frac{2.535 \times 1}{0.461 \times 298.15} = 0.0184 \text{ kg/s}$$

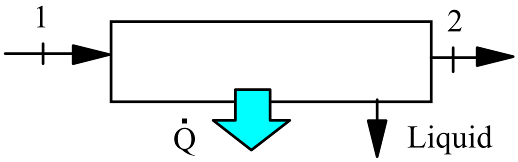
$$w_1 = \frac{\dot{m}_{v1}}{\dot{m}_{A1}} = 0.622\,\frac{P_{v1}}{P_{A1}} = 0.622\,\frac{2.535}{100 - 2.535} = 0.0162$$

$$\dot{m}_{A1} = \frac{\dot{m}_{v1}}{w_1} = \frac{0.0184}{0.0162} = 1.136 \text{ kg/s} = \dot{m}_{A2} \qquad \text{(continuity for air)}$$

Check for state 2:

$P_{g15°C} = 1.705 \text{ kPa} < P_{v1}$

so liquid water out.



State 2 is saturated $\phi_2 = 100\%$, $P_{v2} = P_{g2} = 1.705$ kPa

$$w_2 = 0.622\,\frac{P_{v2}}{P_{A2}} = 0.622\,\frac{1.705}{100 - 1.705} = 0.0108$$

$$\dot{m}_{v2} = w_2 \dot{m}_{A2} = 0.0108 \times 1.136 = 0.0123 \text{ kg/s}$$

$$\dot{m}_{liq} = \dot{m}_{v1} - \dot{m}_{v2} = 0.0184 - 0.0123 = \mathbf{0.0061 \text{ kg/s}}$$

Note that the given volume flow rate at the inlet is not that at the exit. The mass flow rate of dry air is the quantity that is the same at the inlet and exit.

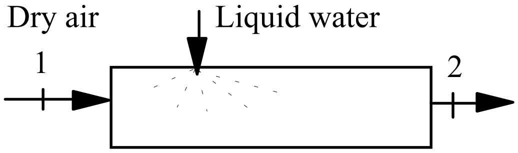
12.81

A flow of 2 kg/s completely dry air at T_1, 100 kPa is cooled down to 10°C by spraying liquid water at 10°C, 100 kPa into it so it becomes saturated moist air at 10°C. The process is steady state with no external heat transfer or work. Find the exit moist air humidity ratio and the flow rate of liquid water. Find also the dry air inlet temperature T_1.

Solution:

2: saturated $\quad P_v = P_g = 1.2276$ kPa and h_{fg} (10°C)= 2477.7 kJ/kg

Eq.12.25: $\quad w_2 = 0.622 \times 1.2276/ (100 - 1.2276) = \mathbf{0.00773}$



C.V. Box

Continuity Eq.: $\quad \dot{m}_a + \dot{m}_{liq} = \dot{m}_a(1 + w_2) \quad =>$

$\dot{m}_{liq} = w_2\, \dot{m}_a = \mathbf{0.0155\ kg/s}$

Energy Eq.: $\quad \dot{m}_a\, h_{a1} + \dot{m}_{liq}\, h_f = \dot{m}_a\, (h_{a2} + w_2\, h_{g2})$

$h_{a1} - h_{a2} = C_{pa}\, (T_1 - T_2) = w_2\, h_{g2} - w_2\, h_f = w_2\, h_{fg} = \mathbf{19.15\ kJ/kg\ dry\ air}$

$=> \quad T_1 = \mathbf{29.1°C}$

12.82

A piston/cylinder has 100 kg of saturated moist air at 100 kPa, 5°C. If it is heated to 45°C in an isobaric process, find ${}_1Q_2$ and the final relative humidity. If it is compressed from the initial state to 200 kPa in an isothermal process, find the mass of water condensing.

Solution:

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2,$

Initial state 1: $\phi_1 = 100\%$, Table B.1.1: $P_{v1} = 0.8721$ kPa, $h_{v1} = 2510.54$

Eq.12.28 $$w_1 = 0.622\ \frac{P_{v1}}{P_{tot} - P_{v1}} = \frac{0.8721}{100 - 0.8721} = 0.005472$$

Eq.12.26 with $m_a = m_{tot} - m_{v1} = m_{tot} - w_1 m_a$ gives

$$m_a = m_{tot}/(1+ w_1) = 99.456 \text{ kg},$$

Eq.12.26 $m_{v1} = w_1 m_a = 0.544$ kg

Case a: $P = \text{constant} \Rightarrow {}_1W_2 = mP(v_2-v_1) \Rightarrow$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = m_aC_p(T_2 - T_1) + m(h_{v2} - h_{v1})$$

State 2: $w_2 = w_1$, $T_2 \Rightarrow P_{v2} = P_{v1}$ and

Table B.1.1: $h_{v2} = 2583.19$ kJ/kg, $P_{g2} = 9.593$ kPa

Eq.12.25 $$\phi_2 = \frac{P_{v2}}{P_{g2}} = \frac{0.8721}{9.593} = 0.091 \quad \text{or} \quad \phi_2 = \mathbf{9.1\%}$$

From the energy equation

$${}_1Q_2 = 99.456 \times 1.004(45 - 5) + 0.544(2583.19 - 2510.54) = \mathbf{4034\ kJ}$$

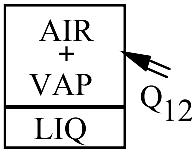
Case b: $T = \text{constant}\ \&\ \phi_2 = 100\% \Rightarrow P_v = P_g = 0.8721$ kPa

$$w_2 = 0.622\frac{P_{v2}}{P_{a2}} = 0.622\frac{P_{v2}}{P_{tot2} - P_{v2}} = \frac{0.8721}{200 - 0.8721} = 0.002724$$

$m_{v2} = w_2\, m_a = 0.271$ kg, $m_{liq} = m_{v1} - m_{v2} = \mathbf{0.273\ kg}$

12.83

A saturated air-water vapor mixture at 20 °C, 100 kPa, is contained in a 5-m^3 closed tank in equilibrium with 1 kg of liquid water. The tank is heated to 80°C. Is there any liquid water in the final state? Find the heat transfer for the process.



a) Since $V_{LIQ} = m_{LIQ}v_F \approx 0.001$ m^3, $V_{GAS} \approx V$

$\phi_1 = 1.00 \rightarrow P_{v1} = P_{G1} = 2.339$ kPa

$w_1 = 0.622\times 2.339/(100 - 2.339) = 0.0149$

$$m_a = \frac{P_{a1}V}{R_aT_1} = \frac{97.661\times 4.999}{0.287\times 293.2} = 5.802 \text{ kg} \quad => \quad m_{v1} = w_1m_a = 0.086 \text{ kg}$$

At state 2: $$P_{a2} = 97.661\times\frac{353.2}{293.2}\times\frac{4.999}{5} = 117.623 \text{ kPa}$$

$$w_{MAX\,2} = 0.622 \times 47.39 / 117.623 = 0.2506$$

But $$w_{2\,ACTUAL} = \frac{0.086 + 1.0}{5.802} = 0.1872 < w_{MAX\,2}$$ → **No liquid at 2**

$$Q_{12} = m_a(u_{a2} - u_{a1}) + m_{v2}u_{v2} - m_{v1}u_{v1} - m_{liq\,1}u_{liq\,1}$$

$$= 5.802 \times 0.717(80 - 20) + 1.086 \times 2482.2 - 0.086 \times 2402.9 - 1 \times 84.0$$

$$= 249.6 + 2695.7 - 206.65 - 84 = \mathbf{2655\ kJ}$$

12.84

Ambient moist air enters a steady-flow air-conditioning unit at 102 kPa, 30°C, with a 60% relative humidity. The volume flow rate entering the unit is 100 L/s. The moist air leaves the unit at 95 kPa, 15°C, with a relative humidity of 100%. Liquid condensate also leaves the unit at 15°C. Determine the rate of heat transfer for this process.

Solution:

State 1: $P_{V1} = \phi_1 P_{G1} = 0.60\times4.246 = 2.5476$

$w_1 = 0.622 \times 2.5476/(102 - 2.5476) = 0.01593$

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.45\times0.1}{0.287\times303.2} = 0.1143 \text{ kg/s}$$

$P_{V2} = P_{G2} = 1.705, \quad w_2 = 0.622 \times 1.705/(95 - 1.705) = 0.01137$

Energy Eq.6.10: $\dot{Q}_{CV} + \dot{m}_A h_{A1} + \dot{m}_{V1} h_{V1} = \dot{m}_A h_{A2} + \dot{m}_{V2} h_{A2} + \dot{m}_3 h_{L3}$

$\dot{Q}_{CV}/\dot{m}_A = C_{P0A}(T_2-T_1) + w_2 h_{V2} - w_1 h_{V1} + (w_1-w_2)h_{L3}$

$= 1.004(15-30) + 0.01137\times2528.9 - 0.01593\times2556.2$

$+ 0.00456\times63.0 = -26.732$ kJ/kg air

$\dot{Q}_{CV} = 0.1143(-26.73) =$ **-3.055 kW**

12.85

Consider a 500-L rigid tank containing an air–water vapor mixture at 100 kPa, 35°C, with a 70% relative humidity. The system is cooled until the water just begins to condense. Determine the final temperature in the tank and the heat transfer for the process.

Solution:

$P_{v1} = \phi P_{G1} = 0.7 \times 5.628 = 3.9396$ kPa

Since m_v = const & V = const & also $P_v = P_{G2}$:

$P_{G2} = P_{v1} \times T_2/T_1 = 3.9396 \times T_2/308.2 = 0.01278\ T_2$

Assume $T_2 = 30^oC$: $0.01278 \times 303.2 = 3.875 \neq 4.246 = P_{G\ 30C}$

Assume $T_2 = 25^oC$: $0.01278 \times 298.2 = 3.811 \neq 3.169 = P_{G\ 25C}$

interpolating → T_2 = **28.2 °C**

$$w_2 = w_1 = 0.622 \frac{3.9396}{(100\text{-}3.9369)} = 0.025\ 51$$

$m_a = P_{a1}V/R_aT_1 = (100\text{-}3.94)\times 0.5/(0.287\times 308.2) = 0.543$ kg

1st law Eq.5.11: ${}_1Q_2 = U_2 - U_1 = m_a(u_{a2} - u_{a1}) + m_v(u_{v2} - u_{v1})$

$= 0.717(28.2 - 35) + 0.02551\ (2414.2 - 2423.4) = -5.11$ kJ/kg

$\rightarrow {}_1Q_2 = 0.543(-5.11) =$ **-2.77 kJ**

12.86

Air in a piston/cylinder is at 35°C, 100 kPa and a relative humidity of 80%. It is now compressed to a pressure of 500 kPa in a constant temperature process. Find the final relative and specific humidity and the volume ratio V_2/V_1.

Solution:

Check to see if the second state is saturated or not. First assume no water is condensed

$1:\quad w_1 = 0.029 \qquad 2:\quad w_2 = 0.622\, P_{v2}/(P_2 - P_{v2})$

$w_2 = w_1 \;\Rightarrow\; P_{v2} = 22.568 > P_g = 5.628 \text{ kPa}$

Conclusion is state 2 is saturated

$\boldsymbol{\phi_2 = 100\%}$, $w_2 = 0.622\, P_g/(P_2 - P_g) = \mathbf{0.00699}$

To get the volume ratio, write the ideal gas law for the vapor phases

$$V_2 = V_{a2} + V_{v2} + V_{f2} = (m_a R_a + m_{v2} R_v)T/P_2 + m_{liq}\, v_f$$

$$V_1 = V_{a1} + V_{v1} = (m_a R_a + m_{v1} R_v)T/P_1$$

Take the ratio and divide through with $m_a R_a T/P_2$ to get

$$V_2/V_1 = (P_1/P_2)[1 + 0.622 w_2 + (w_1 - w_2)P_2 v_f/R_a T] \,/\, [1 + 0.622\, w_1]$$

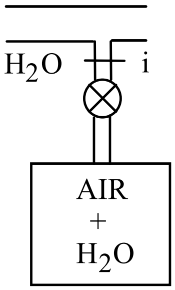
$$= \mathbf{0.1973}$$

The liquid contribution is nearly zero (=0.000126) in the numerator.

12.87

A 300-L rigid vessel initially contains moist air at 150 kPa, 40°C, with a relative humidity of 10%. A supply line connected to this vessel by a valve carries steam at 600 kPa, 200°C. The valve is opened, and steam flows into the vessel until the relative humidity of the resultant moist air mixture is 90%. Then the valve is closed. Sufficient heat is transferred from the vessel so the temperature remains at 40°C during the process. Determine the heat transfer for the process, the mass of steam entering the vessel, and the final pressure inside the vessel.

Solution:



$P_{v1} = \phi_1 P_{G1} = 0.1 \times 7.384 = 0.7384$ kPa

$P_{v2} = 0.9 \times 7.384 = 6.6456$ kPa

$P_{a2} = P_{a1} = 150 - 0.738 = 149.262$ kPa

$$w_1 = 0.622 \times \frac{0.7384}{149.262} = 0.003\ 08$$

$$w_2 = 0.622 \times \frac{6.6456}{149.262} = 0.0277$$

$m_a = 149.262 \times 0.3/(0.287 \times 313.2) = 0.5$ kg

$P_2 = 149.262 + 6.6456 = 155.9$ kPa

$m_{vi} = 0.5(0.0277 - 0.00308) =$ **0.0123 kg**

$u_{v1} = u_{v2} \approx u_G$ at 40 °C and $u_{a2} = u_{a1}$

Energy Eq.6.16:

$Q_{CV} = m_a(u_{a2} - u_{a1}) + m_{v2}u_{v2} - m_{v1}u_{v1} - m_{vi}h_i$

$= m_{vi}(u_{G \text{ at } T} - h_i) = 0.0123(2430.1 - 2850.1) =$ **-5.15 kJ**

12.88

A rigid container, 10 m^3 in volume, contains moist air at 45°C, 100 kPa, $\phi = 40\%$. The container is now cooled to 5°C. Neglect the volume of any liquid that might be present and find the final mass of water vapor, final total pressure and the heat transfer.

Solution:

CV container. $m_2 = m_1$; $m_2u_2 - m_1u_1 = {}_1Q_2$

State 1: 45°C, $\phi = 40\%$ => $w_1 = 0.0236$, $T_{dew} = 27.7°C$

Final state $T_2 < T_{dew}$ so condensation, $\phi_2 = 100\%$

$P_{v1} = 0.4\ P_g = 0.4 \times 9.593 = 3.837$ kPa, $P_{a1} = P_{tot} - P_{v1} = 97.51$ kPa

$m_a = P_{a1}V/RT_1 = 10.679$ kg, $m_{v1} = w_1\ m_a = 0.252$ kg

$P_{v2} = P_{g2} = 0.8721$ kPa, $P_{a2} = P_{a1}T_2/T_1 = 85.25$ kPa

$P_2 = P_{a2} + P_{v2} =$ **86.12 kPa**

$m_{v2} = P_{v2}V/R_vT_2 =$ **0.06794 kg** (= V/v_g = 0.06797 steam table)

$m_{f2} = m_{v1} - m_{v2} = 0.184$ kg

The heat transfer from the energy equation becomes

$${}_1Q_2 = m_a(u_2 - u_1)_a + m_{v2}u_{g2} + m_{f2}u_{f2} - m_{v1}u_{g1}$$

$$= m_a\ C_v(T_2 - T_1) + m_{v2}\ 2382.3 + m_{f2}\ 20.97 - m_{v1}\ 2436.8$$

$$= -306.06 + 161.853 + 3.858 - 614.07 = \mathbf{-\ 754.4\ kJ}$$

12.89

A water-filled reactor of 1 m^3 is at 20 MPa, 360°C and located inside an insulated containment room of 100 m^3 that contains air at 100 kPa and 25°C. Due to a failure the reactor ruptures and the water fills the containment room. Find the final pressure.

CV Total container.

$$m_v(u_2-u_1) + m_a(u_2-u_1) = {}_1Q_2 - {}_1W_2 = 0$$

Initial water: $v_1 = 0.0018226$, $u_1 = 1702.8$, $m_v = V/v = 548.67$ kg

Initial air: $m_a = PV/RT = 100\times99/0.287\times298.2 = 115.7$ kg

Substitute into energy equation

$$548.67\ (u_2 - 1702.8) + 115.7\times0.717\ (T_2 - 25) = 0$$

$u_2 + 0.1511\ T_2 = 1706.6$ kJ/kg & $v_2 = V_2/m_v = 0.18226\ m^3/kg$

Trial and error 2-phase (T_{guess}, v_2 => x_2 => u_2 => LHS)

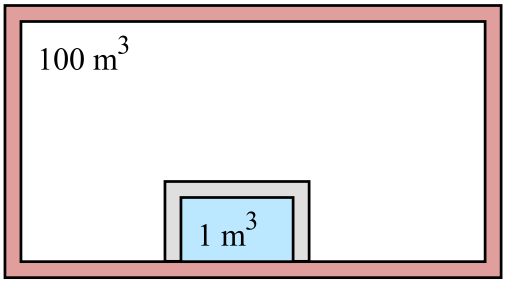
T = 150 LHS = 1546 T = 160 LHS = 1820.2

T = 155 LHS = 1678.1 => T = 156°C LHS = 1705.7 OK

$x_2 = 0.5372$, $P_{sat} = 557.5$ kPa

$P_{a2} = P_{a1}V_1T_2/V_2T_1 = 100\times 99 \times 429.15 / (100\times298.15) = 142.5$ kPa

=> $P_2 = P_{a2} + P_{sat} =$ **700 kPa.**



Tables and formulas or psychrometric chart

12.90

A flow moist air at 100 kPa, 40°C, 40% relative humidity is cooled to 15°C in a constant pressure device. Find the humidity ratio of the inlet and the exit flow, and the heat transfer in the device per kg dry air.

Solution:

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 7.384$ kPa , $P_{v1} = 2.954$ kPa, $\omega_1 = 0.0189$

$P_{v2} = 1.705 \text{ kPa} = P_{g2} \Rightarrow \omega_2 = 0.0108$

$h_{v1} = 2574.3$ kJ/kg, $h_{v2} = 2528.9$ kJ/kg, $h_f = 62.98$ kJ/kg

$$\bar{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2 h_{v2} - (\omega_1 - \omega_2) h_f$$

$$= 1.004(40 - 15) + 0.0189 \times 2574.3 - 0.0108 \times 2528.9 - 0.0073 \times 62.98$$

= **45.98 kJ/kg dry air**

Psychrometric chart: State 2: $T < T_{dew} = 23°C \Rightarrow \phi_2 = 100\%$

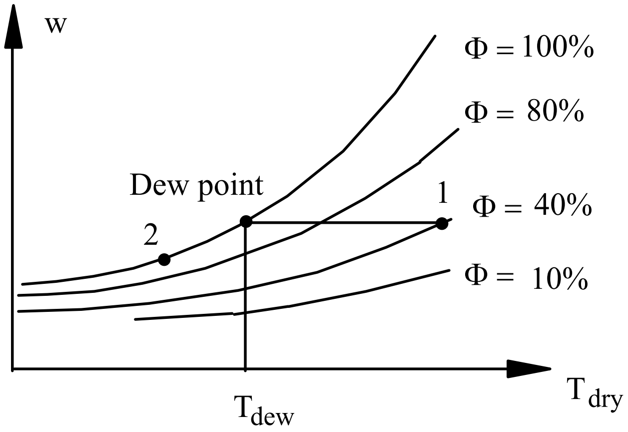
$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.018$, $\tilde{h}_1 = 106$; $\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0107$, $\tilde{h}_2 = 62$

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0073$, $h_f = 62.98$ kJ/kg

$$\dot{m}_a \bar{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq} h_f - \dot{m}_a \tilde{h}_2 \Rightarrow$$

$$\bar{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2) h_f - \tilde{h}_2 = 106 - 0.0073 \times 62.98 - 62$$

= **43.54 kJ/kg-dry air**



12.91

A flow, 0.2 kg/s dry air, of moist air at 40°C, 50% relative humidity flows from the outside state 1 down into a basement where it cools to 16°C, state 2. Then it flows up to the living room where it is heated to 25°C, state 3. Find the dew point for state 1, any amount of liquid that may appear, the heat transfer that takes place in the basement and the relative humidity in the living room at state 3.

Solve using psychrometric chart:

a) $T_{dew} = 27.2$ ($w = w_1$, $\phi = 100\%$) $\quad w_1 = 0.0232$, $\tilde{h}_1 = 118.2$ kJ/kg air

b) $T_2 < T_{dew}$ so we have $\phi_2 = 100\%$ liquid water appear in the basement.

$\Rightarrow w_2 = 0.0114 \quad \tilde{h}_2 = 64.4$ and from steam tbl. $h_f = 67.17$

$\dot{m}_{liq} = \dot{m}_{air}(w_1 - w_2) = 0.2(0.0232 - 0.0114) = 0.00236$ kg/s

c) Energy equation: $\dot{m}_{air}\,\tilde{h}_1 = \dot{m}_{liq}\,h_f + \dot{m}_{air}\,\tilde{h}_2 + Q_{out}$

$$Q_{out} = 0.2[118.2 - 64.4 - 0.0118 \times 67.17] = 10.6 \text{ kW}$$

d) $w_3 = w_2 = 0.0114$ & 25°C $\Rightarrow$ $\phi_3 = 58\%$.

If you solve by the formulas and the tables the numbers are:

$P_{g40} = 7.384$; $P_{v1} = 0.5 \times 7.384 = 3.692$ kPa

$w_1 = 0.622 \times 3.692 / (100 - 3.692) = 0.02384$

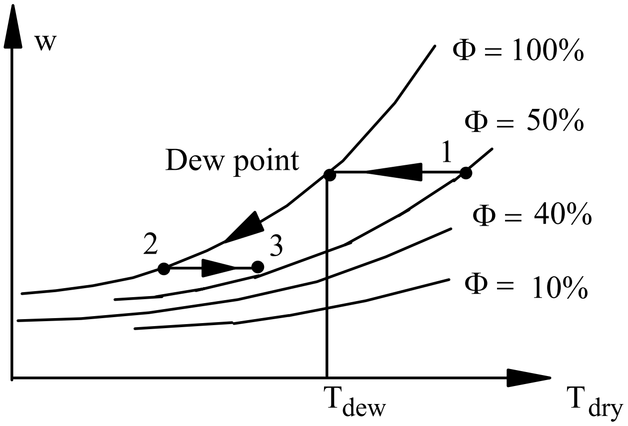
$P_{v1} = P_g(T_{dew}) \Rightarrow T_{dew\,1} = 27.5$ °C

2: $\phi = 100\%$, $P_{v2} = P_{g2} = 1.832$ kPa, $w_2 = 0.622 \times 1.832/98.168 = 0.0116$

$\dot{m}_{liq} = \dot{m}_{air}(w_1 - w_2) = 0.2 \times 0.01223 = 0.00245$ kg/s

3: $w_3 = w_2 \Rightarrow P_{v3} = P_{v2} = 1.832$ & $P_{g3} = 3.169$

$\phi_3 = P_v/P_g = 1.832/3.169 = 57.8\%$



12.92

Two moist air streams with 85% relative humidity, both flowing at a rate of 0.1 kg/s of dry air are mixed in a steady setup. One inlet flowstream is at 32.5°C and the other at 16°C. Find the exit relative humidity.

Solution:

CV mixing chamber.

Continuity Eq. water: $\dot{m}_{air}\, w_1 + \dot{m}_{air}\, w_2 = 2\dot{m}_{air}\, w_{ex}$;

Energy Eq.: $\dot{m}_{air}\, \tilde{h}_1 + \dot{m}_{air}\, \tilde{h}_2 = 2\dot{m}_{air}\, \tilde{h}_{ex}$

Properties from the tables and formulas

$P_{g32.5} = 4.937$; $P_{v1} = 0.85\times4.937 = 4.196$ kPa

$w_1 = 0.622 \times 4.196 / (100 - 4.196) = 0.0272$

$P_{g16} = 1.831$; $P_{v2} = 0.85\times1.831 = 1.556$ kPa

$w_2 = 0.622 \times 1.556 / (100 - 1.556) = 0.00983$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.0185$;

For the energy equation we have $\tilde{h} = h_a + wh_v$ so:

$2\, \tilde{h}_{ex} - \tilde{h}_1 - \tilde{h}_2 = 0 = 2h_{a\,ex} - h_{a\,1} - h_{a\,2} + 2w_{ex}h_{v\,ex} - w_1h_{v\,1} - wh_{v\,2}$

we will use constant heat capacity to avoid an iteration on T_{ex}.

$C_{p\,air}(2T_{ex} - T_1 - T_2) + C_{p\,H2O}(2w_{ex}T_{ex} - w_1T_1 - w_2T_2) = 0$

$T_{ex} = [\, C_{p\,air}(T_1 + T_2) + C_{p\,H2O}(w_1T_1 + w_2T_2)\,]/\, [2C_{p\,air} + 2w_{ex}C_{p\,H2O}]$

$= [\, 1.004\,(32.5 + 16) + 1.872(0.0272 \times 32.5 + 0.00983 \times 16]/2.0773$

$= 24.4°C$

$$P_{v\,ex} = \frac{w_{ex}}{0.622 + w_{ex}}\, P_{tot} = \frac{0.0185}{0.622 + 0.0185}\, 100 = 2.888 \text{ kPa},$$

$P_{g\,ex} = 3.069$ kPa => $\phi = 2.888 / 3.069 =$ **0.94 or 94%**

Properties taken from the psychrometric chart

State 1: $w_1 = 0.0266$, $\tilde{h}_1 = 120$ State 2: $w_2 = 0.0094$, $\tilde{h}_2 = 60$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.018$;

Energy Eq.: $\tilde{h}_{ex} = (\tilde{h}_1 + \tilde{h}_2)/2 = 90$ kJ/kg dry air

exit: w_{ex}, $\tilde{h}_{ex}$ => $T_{ex} =$ **24.5°C**, $\phi =$ **94%**

Notice how the energy in terms of temperature is close to the average of the two flows but the relative humidity is not.

12.93

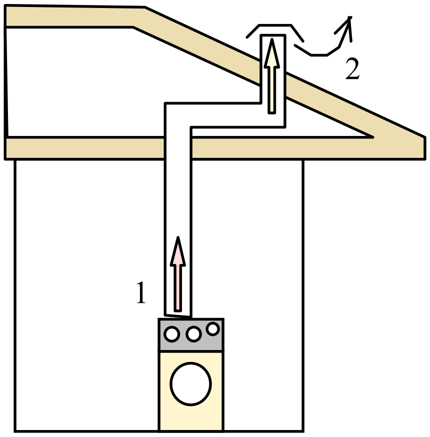
The discharge moist air from a clothes dryer is at 35°C, 80% relative humidity. The flow is guided through a pipe up through the roof and a vent to the atmosphere. Due to heat transfer in the pipe the flow is cooled to 24°C by the time it reaches the vent. Find the humidity ratio in the flow out of the clothes dryer and at the vent. Find the heat transfer and any amount of liquid that may be forming per kg dry air for the flow.

Solution:

State 1: $w = 0.0289$, $\tilde{h}_1 = 128$, $T_{dew} = 31^{o}C$

State 2: $24^{o}C < T_{dew}$ so it is saturated.

$w = 0.019$, $\tilde{h}_2 = 92$ kJ/kg air



$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0099$ kg/kg dry air

Energy Eq.:

$$\dot{Q}/\dot{m}_a = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2)\, h_f$$
$$= 128 - 92 - 0.0099 \times 100.68$$
$$= 35 \text{ kJ/kg dry air}$$

12.94

A steady supply of 1.0 m^3/s air at 25°C, 100 kPa, 50% relative humidity is needed to heat a building in the winter. The outdoor ambient is at 10°C, 100 kPa, 50% relative humidity. What are the required liquid water input and heat transfer rates for this purpose?

Solution:

Air: $R_a = 0.287$ kJ/kg K, $C_p = 1.004$ kJ/kg-K

State 1: $T_1 = 10°C$, $\phi_1 = 50\%$, $P_1 = 100$ kPa

$P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.6138$ kPa,

$P_{a1} = P_1 - P_{v1} = 99.39$ kPa => $\omega_1 = 0.622\, P_{v1}/P_{a1} = 0.003841$

State 2: $T_2 = 25°C$, $P_2 = 100$ kPa, $\phi_2 = 50\%$, $\dot{V}_2 = 1\ m^3/s$

$P_{g2} = 3.169$ kPa, $P_{v2} = \phi_2 P_{g2} = 1.5845$ kPa,

$P_{a2} = P_2 - P_{v2} = 98.415$ kPa, $\omega_2 = 0.622\, P_{v2}/P_{a2} = 0.010014$

$\dot{m}_{a2} = P_{a2} \dot{V}_2/R_a T_2 = 98.415 \times 1/(0.287 \times 298.15) = 1.15$ kg/s

Steam tables B.1.1: $h_{v1} = 2519.7$ kJ/kg, $h_{v2} = 2547.2$ kJ/kg

State 3: Assume: Liq. Water at $T_3 = 25°C$, $h_{f3} = 104.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{f3} = \dot{m}_{v2} - \dot{m}_{v1}$

$\dot{m}_{f3} = \dot{m}_{a2}(\omega_2 - \omega_1) = 1.15 \times 0.006173 =$ **0.0071 kg/s**

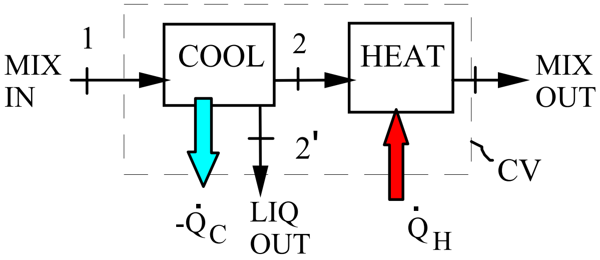
1^{st}Law: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f3}h_{f3} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}$

$$\frac{\dot{Q}}{\dot{m}_a} = C_p(T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} - \frac{\dot{m}_{f3}}{\dot{m}_a} h_{f3} \quad => \quad \dot{Q} = \mathbf{34.76\ kW}$$

12.95

A combination air cooler and dehumidification unit receives outside ambient air at 35°C, 100 kPa, 90% relative humidity. The moist air is first cooled to a low temperature T_2 to condense the proper amount of water, assume all the liquid leaves at T_2. The moist air is then heated and leaves the unit at 20°C, 100 kPa, relative humidity 30% with volume flow rate of 0.01 m^3/s. Find the temperature T_2, the mass of liquid per kilogram of dry air and the overall heat transfer rate.

Solution:



a) $P_{v1} = \phi_1 P_{G1} = 0.9 \times 5.628 = 5.0652$ kPa

$$w_1 = 0.622 \times \frac{5.0652}{100-5.0652} = 0.033\ 19$$

$P_{v3} = \phi_3 P_{G3} = 0.3 \times 2.339 = 0.7017$ kPa

$$w_2 = w_3 = 0.622 \times \frac{0.7017}{100-0.7017} = 0.0044$$

$\dot{m}_{LIQ\,2'}/\dot{m}_a = w_1 - w_2 = 0.033\ 19 - 0.0044 = \mathbf{0.028\ 79\ kg/kg\ air}$

$P_{G2} = P_{v3} = 0.7017 \text{ kPa} \quad \rightarrow \quad T_2 = \mathbf{1.7\ {}^oC}$

b) For a C.V. around the entire unit

$$\dot{Q}_{CV} = \dot{Q}_H + \dot{Q}_C$$

Net heat transfer, 1st law:

$\dot{Q}_{CV}/\dot{m}_a = (h_{a3}-h_{a1}) + w_3 h_{v3} - w_1 h_{v1} + \dot{m}_{L2'}\, h_{L2'}/\dot{m}_a$

$= 1.004(20-35) + 0.0044\times2538.1 - 0.033\ 19\times2565.3 + 0.028\ 79\times7.28$

$= -88.82$ kJ/kg air

$$\dot{m}_a = \frac{P_{a3}\dot{V}_3}{R_a T_3} = \frac{(100-0.7017)\times0.01}{0.287\times293.2} = 0.0118 \text{ kg/s}$$

$\dot{Q}_{CV} = 0.0118(-88.82) = \mathbf{-1.05\ kW}$

12.96

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{dry} = 25°C$, $T_{wet} = 21°C$

Solution:

a. From Eq.12.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\,\omega/(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582 \text{ kPa}$$

From Eq.12.25 $P_g = P_v/\phi = 1.582/0.5 = 3.165$ kPa => $\mathbf{T = 25°C}$

b. At 21°C:: $P_g = 2.505$ => $\omega_2 = 0.622 \times 2.505/(100 - 2.505) = 0.016$

From the steam tables B.1.1

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76$ kJ/kg, $h_{v1} = 2547.17$

From Eq.10.30:

$$\omega_1 = [C_p(T_2\text{-}T_1) + \omega_2\, h_{fg2}\,]/(h_{v1} - h_{f2}) = \mathbf{0.0143}$$

From Eq.12.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\,\omega/(0.622 + \omega) = 2.247,$$

From Eq.12.25: $\phi = 2.247/3.169 = \mathbf{0.71}$

Using the psychrometric chart E.4:

a: $\boldsymbol{T_{dry} = 25.3\ °C}$ b: $\boldsymbol{\omega = 0.0141}$, $\boldsymbol{\phi = 71\text{-}72\%}$

12.97

An insulated tank has an air inlet, $\omega_1 = 0.0084$, and an outlet, $T_2 = 22°C$, $\phi_2 = 90\%$ both at 100 kPa. A third line sprays 0.25 kg/s of water at 80°C, 100 kPa. For steady operation find the outlet specific humidity, the mass flow rate of air needed and the required air inlet temperature, T_1.

Solution:

Take CV tank in steady state. Continuity and energy equations are:

Continuity Eq. water: $\dot{m}_3 + \dot{m}_a w_1 = \dot{m}_a w_2$

Energy Eq.: $\dot{m}_3 h_f + \dot{m}_a \tilde{h}_1 = \dot{m}_a \tilde{h}_2$

All state properties are known except T_1.

From the psychrometric chart we get

State 2: $w_2 = \mathbf{0.015}$, $\tilde{h}_2 = 79.5$ State 3: $h_f = 334.91$ (steam tbl)

$$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.015-0.0084) = \mathbf{37.88\ kg/s}$$

$$\tilde{h}_1 = \tilde{h}_2 - (w_2 - w_1)h_f = 79.5 - 0.0066 \times 334.91 = 77.3$$

Chart $(w_1, \tilde{h}_1)$ => $T_1 = \mathbf{36.5°C}$

Using the tables and formulas we get

State 2: $P_{g22} = 2.671$; $P_{v2} = 0.9 \times 2.671 = 2.4039$ kPa

$$w_2 = 0.622 \times 2.4039 / (101.325 - 2.4039) = \mathbf{0.0151}$$

$$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.0151 - 0.0084) = \mathbf{37.31\ kg/s}$$

To avoid iterations on T_1 we use specific heat values also for water vapor by writing $h_{v1} = h_{v2} + C_{p\ h2o}(T_1 - T_2)$ so the energy equation is

$$C_{p\ a} T_1 + w_1 C_{p\ h2o}(T_1 - T_2) + w_1 h_{v2} = C_{p\ a} T_2 + w_2 h_{v2} - (w_2 - w_1) h_f$$

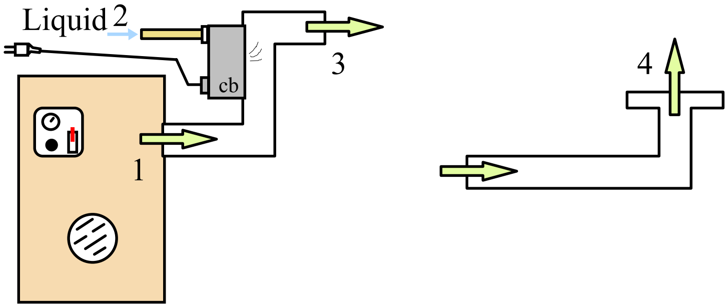
The equation now becomes

$$(1.004 + 0.0084 \times 1.872)T_1 = (0.0084 \times 1.872 + 1.004)\ 22 + (0.0151 - 0.0084)(2541.7 - 334.91) = 37.219$$

$$T_1 = \mathbf{36.5°C}$$

12.98

A flow of moist air from a domestic furnace, state 1, is at 45°C, 10% relative humidity with a flow rate of 0.05 kg/s dry air. A small electric heater adds steam at 100°C, 100 kPa generated from tap water at 15°C. Up in the living room the flow comes out at state 4: 30°C, 60% relative humidity. Find the power needed for the electric heater and the heat transfer to the flow from state 1 to state 4.



State 1: $w_1 = 0.0056$, $\tilde{h}_1 = 79$ kJ/kg dry air
State 4: $w_4 = 0.0160$, $\tilde{h}_4 = 90.5$ kJ/kg dry air

$\dot{m}_{liq} = \dot{m}_a (\omega_1 - \omega_2) = 0.05 (0.016 – 0.0056) = 0.00052$ kg/s

Energy Eq. for heater:

$$\dot{Q}_{heater} = \dot{m}_{liq} (h_{out} – h_{in}) = 0.00052 (2676.05 – 62.98) = \mathbf{1.36\ kW}$$

Energy Eq. for line:

$$\dot{Q}_{line} = \dot{m}_a (\tilde{h}_4 – \tilde{h}_1) – \dot{m}_{liq}\, h_{vap}$$
$$= 0.05(90.5 – 79) – 0.00052 \times 2676.05$$
$$= \mathbf{–0.816\ kW}$$

12.99

A water-cooling tower for a power plant cools 45°C liquid water by evaporation. The tower receives air at 19.5°C, $\phi = 30\%$, 100 kPa that is blown through/over the water such that it leaves the tower at 25°C, $\phi = 70\%$. The remaining liquid water flows back to the condenser at 30°C having given off 1 MW. Find the mass flow rate of air, and the amount of water that evaporates.

Solution:

CV Total cooling tower, steady state.

Continuity Eq. for water in air: $w_{in} + \dot{m}_{evap}/\dot{m}_a = w_{ex}$

Energy Eq.: $\dot{m}_a \tilde{h}_{in} + \dot{m}_1 h_{45} = \dot{m}_a \tilde{h}_{ex} + (\dot{m}_1 - \dot{m}_{evap}) h_{30}$

Inlet: 19.5°C, 30% rel hum => $w_{in} = 0.0041$, $\tilde{h}_{in} = 50$

Exit : 25°C, 70% rel hum => $w_{ex} = 0.0138$, $\tilde{h}_{ex} = 80$

Take the two water flow difference to mean the 1 MW

$$\dot{Q} = \dot{m}_1 h_{45} - (\dot{m}_1 - \dot{m}_{evap}) h_{30} = 1 \text{ MW}$$

$$\dot{m}_a(\tilde{h}_{ex} - \tilde{h}_{in}) = \dot{m}_a(80 - 50) = 1000 \text{ kW} \quad => \quad \dot{m}_a = \mathbf{33.33\ kg/s}$$

$$\dot{m}_{evap} = (w_{ex} - w_{in})\, \dot{m}_a = 0.0097 \times 33.33 = \mathbf{0.323\ kg/s}$$

The needed make-up water flow could be added to give a slightly different meaning to the 1 MW.

12.100

A flow of air at 5°C, $\phi = 90\%$, is brought into a house, where it is conditioned to 25°C, 60% relative humidity. This is done with a combined heater-evaporator where any liquid water is at 10°C. Find any flow of liquid, and the necessary heat transfer, both per kilogram dry air flowing. Find the dew point for the final mixture.

CV heater and evaporator. Use psychrometric chart.

Inlet: $w_1 = 0.0048$, $\tilde{h}_1 = 37.5$ kJ/kg dry air, $h_f = 42.01$ kJ/kg

Exit: $w_2 = 0.0118$, $\tilde{h}_2 = 75$ kJ/kg dry air, $\mathbf{T_{dew} = 16.5°C}$

From these numbers we see that water and heat must be added. Continuity eq. and energy equation give

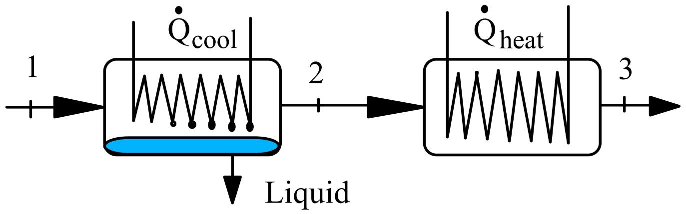
$$\dot{m}_{LIQ\ IN}/\dot{m}_A = w_2 - w_1 = \mathbf{0.007\ kg/kg\ dry\ air}$$

$$q = \tilde{h}_2 - \tilde{h}_1 - (w_2 - w_1)h_f = \mathbf{37.3\ kJ/kg\ dry\ air}$$

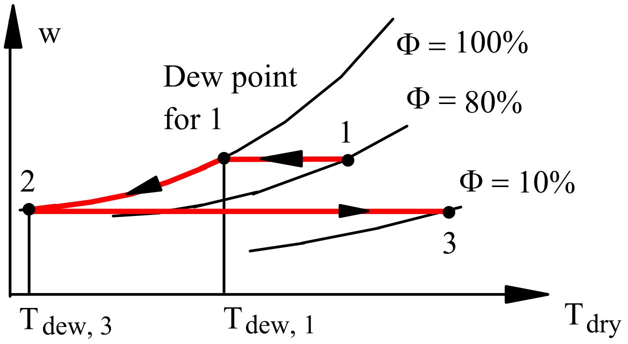
12.101

In a car's defrost/defog system atmospheric air, 21°C, relative humidity 80%, is taken in and cooled such that liquid water drips out. The now dryer air is heated to 41°C and then blown onto the windshield, where it should have a maximum of 10% relative humidity to remove water from the windshield. Find the dew point of the atmospheric air, specific humidity of air onto the windshield, the lowest temperature and the specific heat transfer in the cooler.

Solution:



Solve using the psychrometric chart



Air inlet: 21°C, $\phi = 80\%$

=> $w_1 = 0.0124$,

$T_{dew} = 17.3°C$

$\tilde{h}_1 = 72$

Air exit: 41°C, $\phi = 10\%$

=> $w_3 = 0.0044$,

$T_{dew} = 1.9°C$

To remove enough water we must cool to the exit T_{dew}, followed by heating to T_{ex}. The enthalpy from chart $\tilde{h}_2 = 32.5$ and from B.1.1, $h_f(1.9°C) = 8$

CV cooler:

$$\dot{m}_{liq}/\dot{m}_{air} = w_1 - w_3 = 0.0124 - 0.0044 = \mathbf{0.008\ kg\ liq/kg\ air}$$

$$q = \dot{Q}_{CV}/\dot{m}_{air} = \tilde{h}_2 + (w_1 - w_3)\, h_f - \tilde{h}_1$$

$$= 32.5 + 0.008 \times 8 - 72 = \mathbf{-39.4\ kJ/kg\ dry\ air}$$

If the steam and air tables are used the numbers are

State 1: $P_{g1} = 2.505$, $P_{v1} = 2.004$ => $w_1 = 0.01259$

$h_{g1} = 2539.9$, $h_{a1} = 294.3$ => $\tilde{h}_1 = 326.3$

State 3: $P_{g3} = 7.826$, $P_{v3} = 0.783$ => $w_3 = 0.00486$

State 2: $w_{g3} = w_3$ => $T_2 = T_{3dew} = 3.3°C$, $h_{f2} = 13.77$

$h_{g2} = 2507.4$, $h_{a2} = 276.56$ => $\tilde{h}_2 = 288.75$

$\dot{m}_{liq}/\dot{m}_{air} = \mathbf{0.00773}$, $q = 288.75 + 0.00773 \times 13.77 - 326.3 = \mathbf{-37.45\ kJ/kg\ air}$

12.102

Atmospheric air at 35°C, relative humidity of 10%, is too warm and also too dry. An air conditioner should deliver air at 21°C and 50% relative humidity in the amount of 3600 m^3 per hour. Sketch a setup to accomplish this, find any amount of liquid (at 20°C) that is needed or discarded and any heat transfer.

Solution:

CV air conditioner. First we must check if water should be added or subtracted. We can know this from the absolute humidity ratio.

Properties from the tables and formulas

State 1: $P_{g35} = 5.628$; $P_{v1} = 0.10 \times 5.628 = 0.5628$ kPa

$w_1 = 0.622 \times 0.5628 / (101.325 - 0.5628) = 0.003474$

State 2: $P_{g21} = 2.505$; $P_{v2} = 0.5 \times 2.505 = 1.253$ kPa

$w_2 = 0.622 \times 1.253 / (101.325 - 1.253) = 0.007785$

As w goes up we must add liquid water. Now we get

Continuity Eq.: $\dot{m}_A(1 + w_1) + \dot{m}_{liq} = \dot{m}_A(1 + w_2)$

Energy Eq.: $\dot{m}_A\tilde{h}_{1mix} + \dot{m}_{liq}h_f + \dot{Q}_{CV} = \dot{m}_A\tilde{h}_{2mix}$

For the liquid flow we need the air mass flowrate out, 3600 $m^3/h = 1\ m^3/s$

$\dot{m}_A = P_{a\,2}\dot{V}/RT = (101.325 - 1.253)1/0.287 \times 294.15 = 1.185$ kg/s

$\dot{m}_{liq} = \dot{m}_A(w_2 - w_1) = 0.00511$ kg/s = **18.4 kg/h**

$\dot{Q}_{CV} = \dot{m}_A[C_{p\,a}(T_2 - T_1) + w_2h_{v2} - w_1h_{v1}] - \dot{m}_{liq}h_f$

$= 1.185 [1.004 (21 - 35) + 0.007785 \times 2539.9 - 0.003474 \times 2565.3]$

$- 0.00511 \times 83.96 =$ **- 4.21 kW**

If from psychrometric chart.

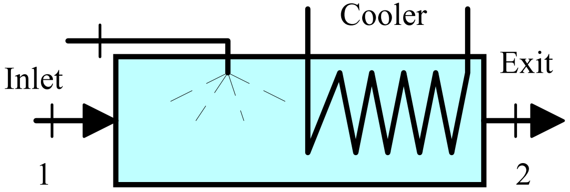
Inlet: $w_1 = 0.0030$, $\tilde{h}_{mix,1} = 63.0$, $h_{f,20} = 83.96$ kJ/kg

Exit: $w_2 = 0.0076$, $\tilde{h}_{mix,2} = 60.2$ kJ/kg dry air

P_{v2} and $\dot{m}_A = P_{a\,2}\dot{V}/RT$ same as above

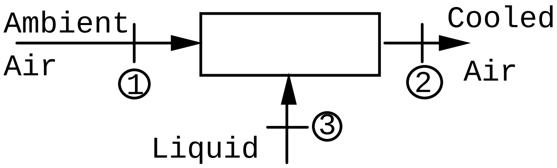
$\dot{Q}_{CV} = \dot{m}_A(\tilde{h}_{2mix} - \tilde{h}_{1mix}) - \dot{m}_{liq}h_f = 1.185(60.2 - 63) - 0.00511 \times 83.96$

$= - 3.74$ kW



12.103

One means of air-conditioning hot summer air is by evaporative cooling, which is a process similar to the adiabatic saturation process. Consider outdoor ambient air at 35°C, 100 kPa, 30% relative humidity. What is the maximum amount of cooling that can be achieved by such a technique? What disadvantage is there to this approach? Solve the problem using a first law analysis and repeat it using the psychrometric chart, Fig. E.4.



$P_1 = P_2 = 100$ kPa

$T_1 = 35\ ^oC,\ \phi_1 = 30\%$

$P_{v1} = \phi_1 P_{g1} = 0.30 \times 5.628 = 1.6884$

$\omega_1 = 0.622 \times 1.6884/98.31 = 0.01068$

For adiabatic saturation (Max. cooling is for $\phi_2 = 1$), 1st law, Eq.12.23

$$\omega_1\ (h_{v1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2\ h_{fg2}$$

$$\phi_2 = 1 \quad \& \quad \omega_2 = 0.622 \times P_{G2}/(P_2 - P_{G2})$$

Only one unknown: T_2 . Trial and error on energy equation:

$$C_pT_2 + \omega_2\ h_{fg2} + \omega_1\ h_{f2} = C_pT_1 + \omega_1 h_{v1}$$

$$= 1.004 \times 35 + 0.01068 \times 2565.3 = 62.537$$

$T_2 = 20\ ^oC$: $P_{G2} = 2.339$, $h_{f2} = 83.94$, $h_{fg2} = 2454.12$

=> $\omega_2 = 0.622 \times 2.339/\ 97.661 = 0.0149$

LHS $= 1.004 \times 20 + 0.0149 \times 2454.1 + 0.01068 \times 83.94 = 57.543$

$T_2 = 25\ ^oC$: $P_{G2} = 3.169$, $h_{f2} = 104.87$, $h_{fg2} = 2442.3$

=> $\omega_2 = 0.622 \times 3.169/\ 96.831 = 0.02036$

LHS $= 1.004 \times 25 + 0.02036 \times 2442.3 + 0.01068 \times 104.87 = 75.945$

linear interpolation: $T_2 = \mathbf{21.4\ ^oC}$

This method does lower the temperature but the relative and absolute humidity becomes very high and the slightest cooling like on a wall will result in condensation.

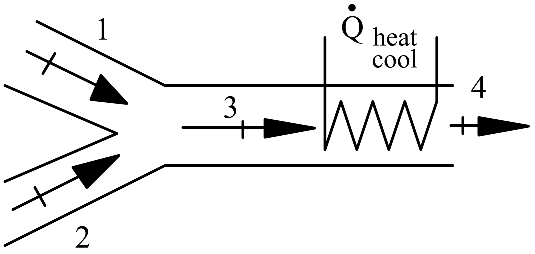
b) chart E.4 : Adiabatic saturation $T \approx$ WetBulbTemperature $\approx \mathbf{21.5\ ^oC}$

12.104

A flow of moist air at 45°C, 10% relative humidity with a flow rate of 0.2 kg/s dry air is mixed with a flow of moist air at 25°C, and absolute humidity of w = 0.018 with a rate of 0.3 kg/s dry air. The mixing takes place in an air duct at 100 kPa and there is no significant heat transfer. After the mixing there is heat transfer to a final temperature of 40°C. Find the temperature and relative humidity after mixing. Find the heat transfer and the final exit relative humidity.

Solution:

C.V : Total Setup
state 3 is internal to CV.



Continuity Eq.: $\dot{m}_{a1} w_1 + \dot{m}_{a2} w_2 = (\dot{m}_{a1} + \dot{m}_{a2}) w_3 = (\dot{m}_{a1} + \dot{m}_{a2}) w_4$

Energy Eq. $\dot{m}_{a1} \tilde{h}_1 + \dot{m}_{a2} \tilde{h}_2 = (\dot{m}_{a1} + \dot{m}_{a2}) \tilde{h}_3$

State 1: From Psychrometric chart $w_1 = 0.056$, $\tilde{h}_1 = 79$ kJ/kg dry air

State 2: From Psychrometric chart $\Phi_2 = 90\%$, $\tilde{h}_2 = 90.5$ kJ/kg dry air

$$w_3 = w_4 = \frac{\dot{m}_{a1}w_1 + \dot{m}_{a2}w_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5} 0.056 + \frac{0.3}{0.5} 0.018 = 0.01304$$

$$\tilde{h}_3 = \frac{\dot{m}_{a1}\tilde{h}_1 + \dot{m}_{a2}\tilde{h}_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5} 79 + \frac{0.3}{0.5} 90.5 = 85.9 \text{ kJ/kg dry air}$$

State 3: From Psychrometric chart $T_3 = 32.5°C$, $\Phi_3 = 45\%$

State 4: 40°C, $w_4 = 0.01304$ Read from Psychrometric chart

$\tilde{h}_4 = 94$, $\mathbf{\Phi_4 = 29\%}$

Now do the energy equation for the whole setup

Energy Eq. $\dot{m}_{a1} \tilde{h}_1 + \dot{m}_{a2} \tilde{h}_2 + \dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2}) \tilde{h}_4$

$$\dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2}) \tilde{h}_4 - \dot{m}_{a1} \tilde{h}_1 + \dot{m}_{a2} \tilde{h}_2$$

$$= 0.5\ 94 - 0.2\ 79 - 0.3\ 90.5 = \mathbf{4.05\ kW}$$

12.105

An indoor pool evaporates 1.512 kg/h of water, which is removed by a dehumidifier to maintain 21°C, $\phi = 70\%$ in the room. The dehumidifier, shown in Fig. P12.105, is a refrigeration cycle in which air flowing over the evaporator cools such that liquid water drops out, and the air continues flowing over the condenser. For an air flow rate of 0.1 kg/s the unit requires 1.4 kW input to a motor driving a fan and the compressor and it has a coefficient of performance, $\beta = Q_L/W_c = 2.0$. Find the state of the air as it returns to the room and the compressor work input.

Solution:

The unit must remove 1.512 kg/h liquid to keep steady state in the room. As water condenses out state 2 is saturated.

State 1: 21°C, 70% $\Rightarrow w_1 = 0.0108, \quad \tilde{h}_1 = 68.5$

CV 1 to 2: $\dot{m}_{liq} = \dot{m}_a(w_1 - w_2) \Rightarrow w_2 = w_1 - \dot{m}_{liq}/\dot{m}_a$

$q_L = \tilde{h}_1 - \tilde{h}_2 - (w_1 - w_2)\, h_{f2}$

$w_2 = 0.0108 - 1.512/3600\times0.1 = 0.0066$

State 2: w_2, 100% $\Rightarrow T_2 = 8°C, \quad \tilde{h}_2 = 45, \quad h_{f2} = 33.6$

$q_L = 68.5 - 45 - 0.0042\times33.6 = 23.36$ kJ/kg dry air

CV Total system : $\tilde{h}_3 = \tilde{h}_1 + \dot{W}_{el}/\dot{m}_a - (w_1 - w_2)\, h_f$

$= 68.5 + 14 - 0.14 = 82.36$ kJ/kg dry air

State 3: $w_3 = w_2, \quad \tilde{h}_3 \Rightarrow T_3 =$ **46°C**, $\phi_3 =$ **11-12%**

$\dot{W}_c = \dot{m}_a\, q_L/\beta =$ **1.165 kW**

Psychrometric chart only

12.106

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

a. T_{dry} = 25°C, ϕ =80%
b. T_{dry} =15°C, ϕ =100%
c. T_{dry} = 20°C, and ω = 0.008
d. T_{dry} = 25°C, T_{wet} = 23°C

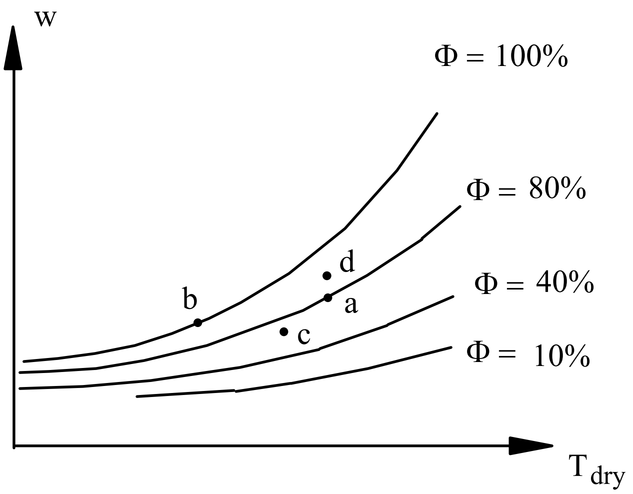
Solution:

a. 25°C, $\phi = 80\%$ => $\omega = 0.016$; $T_{wet} = 22.3°C$

b. 15°C, $\phi = 100\%$ => $\omega = 0.0106$; $T_{wet} = 15°C$

c. 20°C, $\omega = 0.008$ => $\phi = 57\%$; $T_{wet} = 14.4°C$

d. 25°C, $T_{wet} = 23°C$ => $\omega = 0.017$; $\phi = 86\%$



12.107

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

a. ϕ = 50%, ω = 0.012

b. T_{wet} =15°C, ϕ = 60%.

c. ω = 0.008 and T_{wet} =17°C

d. T_{dry} = 10°C, ω = 0.006

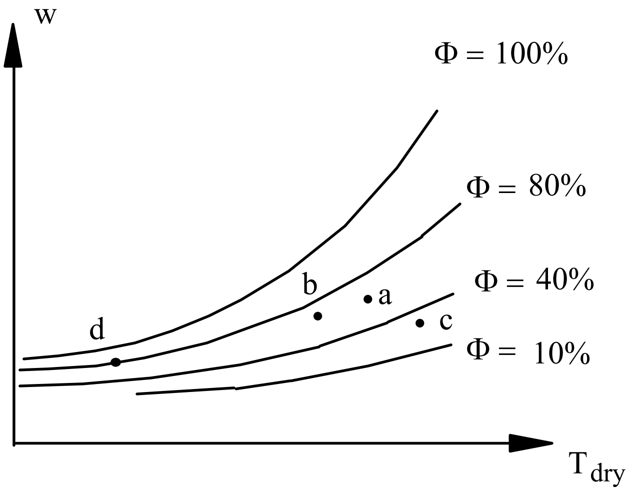
Solution:

a. ϕ = 50%, ω = 0.012 => T_{dry} = 23.5°C, T_{wet} = 20.6 °C

b. T_{wet} = 15°C, ϕ =60% => T_{dry} = 20.2°C, ω = 0.0086

c. ω = 0.008, T_{wet} =17°C => T_{dry} = 27.2°C, ϕ = 37%

d. T_{dry}= 10°C, ω = 0.006 => ϕ = 80%, T_{wet} = 8.2°C



12.108

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{wet} = 15°C$, $\phi = 50\%$ c. $T_{dry} = 25°C$, $T_{wet} = 21°C$

a. From Eq.12.21 $P_v = P\ \omega/(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582$ kPa

From Eq.12.18 $P_g = P_v/\phi = 1.582/0.5 = 3.165$ kPa => $T = 25°C$

b. Assume Twet is adiabatic saturation T and use energy Eq.12.23

At 15°C: $P_g = 1.705$ => $\omega = 0.622 \times 1.705/(100 - 1.705) = 0.01079$

$LHS = \omega_1 (h_{v1} - h_{f2}) + C_pT_1 = RHS = C_pT_2 + \omega_2 h_{fg2}$

$RHS = 1.004\times15 + 0.01079 \times 2465.93 = 41.667$ kJ/kg

$\omega_1 = 0.622\ \phi P_g/(100 - \phi P_g)$ where P_g is at T_1. Trial and error.

$LHS_{25C} = 49.98$, $LHS_{20C} = 38.3$ => $T = 21.4°C$, $\omega_1 = 0.008$

c. At 21°C:: $P_g = 2.505$ => $\omega_2 = 0.622 \times 2.505/(100 - 2.505) = 0.016$

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76$ kJ/kg, $h_{v1} = 2547.17$

From Eq.12.23: $\omega_1 = [C_p(T_2-T_1) + \omega_2\ h_{fg2}]/(h_{v1} - h_{f2}) = 0.0143$

$P_v = P\ \omega/(0.622 + \omega) = 2.247$, $\phi = 2.247/3.169 = 0.71$

Using the psychrometric chart E.4:

a: $T_{dry} = 25.3$ °C b. $T_{dry} = 21.6°C$, $\omega = 0.008$

c: $\omega = 0.0141$, $\phi = 71\text{-}72\%$

12.109

For each of the states in Problem 12.107 find the dew point temperature.

Solution:

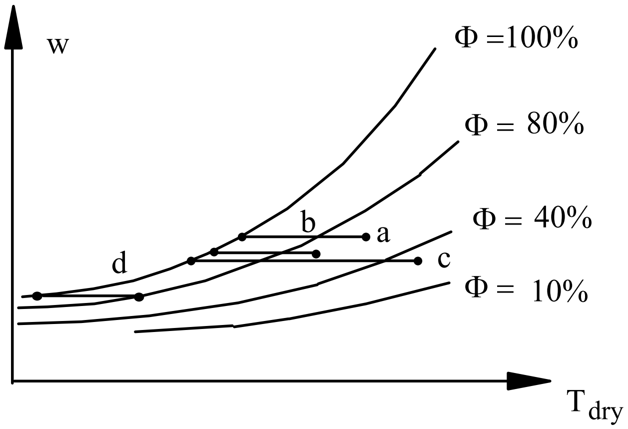
The dew point is the state with the same humidity ratio (abs humidity ω) and completely saturated $\phi = 100\%$. From psychrometric chart:

a.	$T_{\text{dew}} = 16.8°\text{C}$	c.	$T_{\text{dew}} = 10.9°\text{C}$
b.	$T_{\text{dew}} = 12°\text{C}$	d.	$T_{\text{dew}} = 6.5°\text{C}$

Finding the solution from the tables is done for cases a,c and d as

Eq.12.28 solve: $P_v = P_g = \omega P_{tot} /[\omega + 0.622] = Psat(T_{dew})$ in B.1.1

For case b use energy Eq. 10.30 to find ω_1 first from $T_{ad\ sat} = T_{wet}$.



12.110

Compare the weather two places where it is cloudy and breezy. At beach A it is 20°C, 103.5 kPa, relative humidity 90% and beach B has 25°C, 99 kPa, relative humidity 20%. Suppose you just took a swim and came out of the water. Where would you feel more comfortable and why?

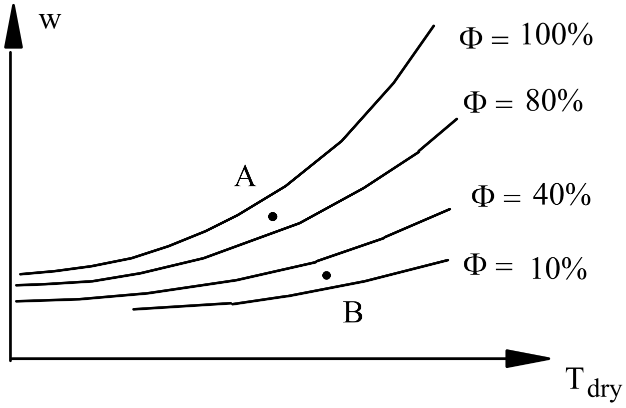
Solution:

Your skin being wet and air is flowing over it will feel T_{wet}. With the small difference in pressure from 100 kPa use the psychrometric chart.

A: 20°C, $\phi = 90\%$ => $T_{wet} = 18.7°C$

B: 25°C, $\phi = 20\%$ => $T_{wet} = 12.3°C$

At beach A it is comfortable, at B it feels chilly.



12.111

Ambient air at 100 kPa, 30°C, 40% relative humidity goes through a constant pressure heat exchanger as a steady flow. In one case it is heated to 45°C and in another case it is cooled until it reaches saturation. For both cases find the exit relative humidity and the amount of heat transfer per kilogram dry air.

Solution:

CV heat exchanger: $\dot{m}_{Ai} = \dot{m}_{Ae}$, $\dot{m}_{vi} = \dot{m}_{ve}$, $w_e = w_i$

$$(h_a + wh_v)_i + q = (h_a + wh_v)_e = \tilde{h}_e, \quad q = \tilde{h}_e - \tilde{h}_i$$

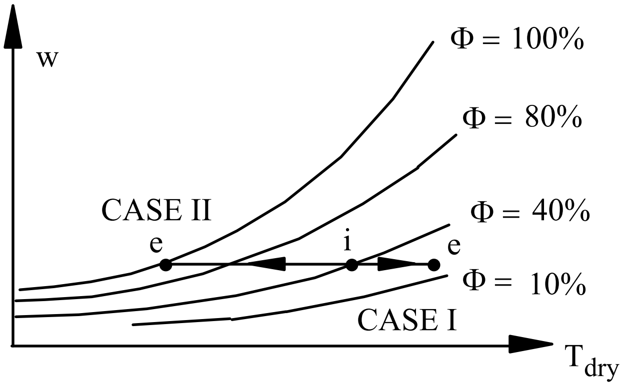
Using the psychrometric chart: i: $w_i = 0.0104$, $\tilde{h}_i = 76$

Case I) e: $T_e = 45\ ^oC$, $w_e = w_i \Rightarrow \tilde{h}_e = 92$,

$\boldsymbol{\phi_e = 17\%}$**,** $q = 92\text{-}76 =$ **16 kJ/kg dry air**

Case II) e: $w_e = w_i$, $\boldsymbol{\phi_e = 100\%} \Rightarrow \tilde{h}_e = 61$, $T_e = 14.5^oC$

$q = 61\text{-}76 =$ **-15 kJ/kg dry air**

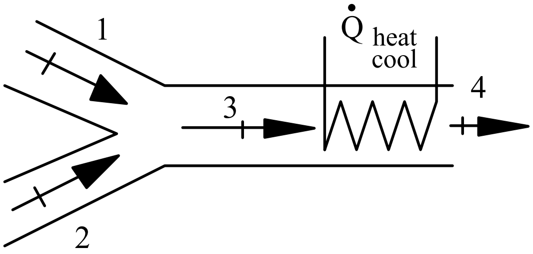


12.112

A flow of moist air at 21°C, 60% relative humidity should be produced from mixing of two different moist air flows. Flow 1 is at 10°C, relative humidity 80% and flow 2 is at 32°C and has T_{wet} = 27°C. The mixing chamber can be followed by a heater or a cooler. No liquid water is added and P = 100 kPa. Find the two controls one is the ratio of the two mass flow rates $\dot{m}_{a1}/\dot{m}_{a2}$ and the other is the heat transfer in the heater/cooler per kg dry air.

Solution:

C.V : Total Setup
state 3 is internal to CV.



Continuity Eq.: $\dot{m}_{a1}\, w_1 + \dot{m}_{a2}\, w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_4$

Energy Eq. $\dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 + \dot{Q}_{a1} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4$

Define $x = \dot{m}_{a1}/\dot{m}_{a2}$ and substitute into continuity equation

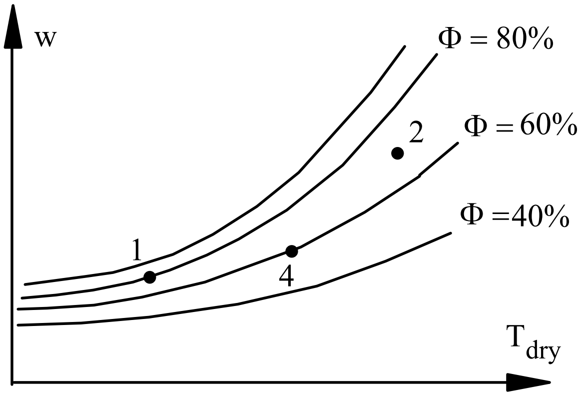
$$=> \quad x\, w_1 + w_2 = (1+x)\, w_4 \quad => \quad x = \frac{w_4 - w_2}{w_1 - w_4} = \mathbf{3.773}$$

Energy equation scaled to total flow of dry air

$$\tilde{q} = \dot{Q}_{a1}/(\dot{m}_{a1} + \dot{m}_{a2}) = \tilde{h}_4 - [x/(1+x)]\, \tilde{h}_1 - [1/(1+x)]\, \tilde{h}_2$$

$$= 64 - 0.7905 \times 45 - 0.2095 \times 105$$

$$= \mathbf{6.43\ kJ/kg\text{-}dry\ air}$$



State 1:
$w_1 = 0.006$, $\tilde{h}_1 = 45$

State 2:
$w_2 = 0.0208$, $\tilde{h}_2 = 105$

State 4:
$w_4 = 0.0091$, $\tilde{h}_4 = 64$,
$T_{dew\,4}$ = 12.5°C

12.113

In a hot and dry climate, air enters an air-conditioner unit at 100 kPa, 40°C, and 5% relative humidity, at the steady rate of 1.0 m^3/s. Liquid water at 20°C is sprayed into the air in the AC unit at the rate 20 kg/hour, and heat is rejected from the unit at at the rate 20 kW. The exit pressure is 100 kPa. What are the exit temperature and relative humidity?

State 1: $T_1 = 40°C$, $P_1 = 100$ kPa, $\phi_1 = 5\%$, $\dot{V}_{a1} = 1\ m^3/s$

$P_{g1} = 7.3837$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.369$ kPa, $P_{a1} = P - P_{v1} = 99.63$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.0023,\ \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.108 \text{ kg/s},\quad h_{v1} = 2574.3 \text{ kJ/kg}$$

State 2 : Liq. Water. 20°C, $\dot{m}_{f2} = 20$ kg/hr $= 0.00556$ kg/s, $h_{f2} = 83.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a3},\quad \dot{m}_{v1} + \dot{m}_{12} = \dot{m}_{v3}$

$\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.0023 = 0.0073$

State 3 : $P_3 = 100$ kPa and $P_{v3} = P_3\omega_3/(0.622 + \omega_3) = 1.16$ kPa

1st Law: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f2}h_{f2} = \dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3};\quad \dot{Q} = -20$ kW

$(h_{a3}-h_{a1}) + \omega_3 h_{v3} = \omega_1 h_{v1} + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1}$

$= 0.0023 * 2574.3 + (0.00556*83.9 - 20)/1.108 = -11.7$

Unknowns: h_{a3}, h_{v3} implicitly given be a single unknown: T_3

Trial and Error for T_3; $T_3 =$ **10 C**, $P_{g3} = 1.23$ kPa , $\phi_3 = \frac{P_{v3}}{P_{g3}} =$ **0.94**

If we solved with the psychrometric chart we would get:

State 1: $\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.002,\quad \tilde{h}_1 = 65;$

State 3: $\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.002 = 0.007$

Now the energy equation becomes

$\tilde{h}_3 = \tilde{h}_1 + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1} = 65 + (0.00556*83.9 - 20)/1.108 = 47.4$

Given ω_3 we find the state around 10 C and $\phi_3 = 90\%$

12.114

Consider two states of atmospheric air. (1) 35°C, $T_{\text{wet}} = 18°\text{C}$ and (2) 26.5°C, $\phi = 60\%$. Suggest a system of devices that will allow air in a steady flow process to change from (1) to (2) and from (2) to (1). Heaters, coolers (de)humidifiers, liquid traps etc. are available and any liquid/solid flowing is assumed to be at the lowest temperature seen in the process. Find the specific and relative humidity for state 1, dew point for state 2 and the heat transfer per kilogram dry air in each component in the systems.

Use the psychrometric chart E.4

1: $w_1 = 0.006$, $\tilde{h}_1 = 70.5$, $\phi_1 = 18\%$, $T_{\text{dew}} = 6.5°\text{C}$, $\tilde{h}_{\text{dew}} = 42$

2: $w_2 = 0.013$, $\tilde{h}_2 = 79.4$, $\phi_2 = 60\%$, $T_{\text{dew}} = 18°\text{C}$, $\tilde{h}_{\text{dew}} = 71$

Since $w_2 > w_1$ water must be added in process I to II and removed in the process II to I. Water can only be removed by cooling below dew point temperature so

I to II: Adiab. sat I to Dew,II, then heater from Dew,II to II

II to I: Cool to Dew,I then heat Dew,I to I

The first one can be done because $T_{\text{dew II}} = T_{\text{ad sat I}}$

I to II: $q = \tilde{h}_{II} - \tilde{h}_{\text{dewII}} = 79.4 - 71 = \mathbf{8.4\ kJ/kg\ air}$

II to I: $q_{\text{cool}} = \tilde{h}_{II} - \tilde{h}_{\text{dewI}} - (w_2 - w_1)h_f(\text{at } T_{\text{dewI}})$

$= 79.4 - 0.007 \times 27.29 = \mathbf{37.2\ kJ/kg\ air}$

$q_{\text{heat}} = \tilde{h}_I - \tilde{h}_{\text{dewI}} = 70.5 - 42 = \mathbf{28.5\ kJ/kg\ air}$

12.115

To refresh air in a room, a counterflow heat exchanger, see Fig. P12.72, is mounted in the wall, drawing in outside air at 0.5°C, 80% relative humidity and pushing out room air, 40°C, 50% relative humidity. Assume an exchange of 3 kg/min dry air in a steady flow device, and also that the room air exits the heat exchanger to the atmosphere at 23°C. Find the net amount of water removed from the room, any liquid flow in the heat exchanger and (*T*, ϕ) for the fresh air entering the room.

State 3: $w_3 = 0.0232$, $\tilde{h}_3 = 119.2$, $T_{dew\ 3} = 27°C$

The room air is cooled to 23°C $< T_{dew\ 3}$ so liquid will form in the exit flow channel and state 4 is saturated.

4: 23°C, $\phi = 100\% \Rightarrow$ $w_4 = 0.0178$, $\tilde{h}_4 = 88$, $h_{f4} = 96.52$ kJ/kg

1: 0.5°C, $\phi = 80\% \Rightarrow$ $w_1 = 0.0032$, $\tilde{h}_1 = 29.2$ kJ/kg dry air

CV 3 to 4:

$$\dot{m}_{liq\ 4} = \dot{m}_a (w_3 - w_4) = 3\ (0.0232 - 0.0178) = \mathbf{0.0162\ kg/min}$$

CV room: $\dot{m}_{v,out} = \dot{m}_a (w_3 - w_2) = \dot{m}_a (w_3 - w_1)$

$$= 3(0.0232-0.0032) = \mathbf{0.06\ kg/min}$$

CV Heat exchanger: $m_a(\tilde{h}_2 - \tilde{h}_1) = m_a(\tilde{h}_3 - \tilde{h}_4) - m_{liq}h_{f4}$

$$\tilde{h}_2 = \tilde{h}_1 + \tilde{h}_3 - \tilde{h}_4 - (w_3 - w_4)\, h_{f4} = 29.2 + 119.2 - 88 - 0.0054 \times 96.52$$

$$= 59.9 \text{ kJ/kg dry air}$$

2: $w_2 = w_3$, $\tilde{h}_2 \Rightarrow T_2 = \mathbf{32.5°C}$, $\phi = \mathbf{12\%}$

Availability (exergy) in mixtures

12.116

Consider the mixing of a steam flow with an oxygen flow in Problem 12.71. Find the rate of total inflowing availability and the rate of exergy destruction in the process.

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

Exergy Flow: $\dot{\Phi}_{in} = \dot{m}\, \psi_{in} = \dot{m}_{H2O}\, \psi_1 + \dot{m}_{O2}\, \psi_2$

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$

$$= C_{P\ H2O}(T_1 - T_o) - T_o\ [C_{P\ H2O} \ln(T_1/T_o) - R \ln(P_1/P_o)\ ]$$

$$= 1.872\ (400 - 25) - 298.15[\ 1.872 \ln \frac{673.15}{298.15} - 0.4615 \ln \frac{400}{100}\ ]$$

$$= 702 - 298.15\ (1.5245 - 0.63978) = 438.2 \text{ kJ/kg}$$

$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$

$$= C_{P\ O2}(T_2 - T_o) - T_o\ [C_{P\ O2} \ln(T_2/T_o) - R \ln(P_2/P_o)\ ]$$

$$= 0.922(126.85–25) - 298.15[0.922 \ln \frac{400}{298.15} - 0.2598 \ln \frac{400}{100}\ ]$$

$$= 93.906 - 298.15\ (0.27095 - 0.36016) = 120.5 \text{ kJ/kg}$$

$$\dot{\Phi}_{in} = \dot{m}_{H2O}\, \psi_1 + \dot{m}_{O2}\, \psi_2 = 1.8\ 438.2 + 3.2\ 120.5 = \mathbf{1174.4\ kW}$$

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{O2}\, h_2 = \dot{m}_{H2O}\, h_{3\ H2O} + \dot{m}_{O2}\, h_{3\ O2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{O2}\, s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\, s_{3\ H2O} + \dot{m}_{O2}\, s_{3\ O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\, (h_{3\ H2O} - h_1) + \dot{m}_{O2}\, (h_{3\ O2} - h_2) = 0$$

$$\dot{m}_{H2O}\, C_{P\ H2O}(T_3 - T_1) + \dot{m}_{O2}\, C_{P\ O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872\ (T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \mathbf{545.6\ K}$$

$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{3\ H2O} - s_1) + \dot{m}_{O2}\,(s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O}\,[\ C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O}\ ] + \dot{m}_{O2}\,[\ C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2}\ ]$$

$$= 1.8\ [\ 1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5\ ]$$

$$+ 3.2\ [\ 0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5\ ]$$

$$= -\,0.132 + 1.492 = \mathbf{1.36\ kW/K}$$

The exergy destruction is proportional to the entropy generation

$$\dot{\Phi}_{in} = T_o\,\dot{S}_{gen} = 298.15 \times 1.36 = \mathbf{405.5\ kW}$$

12.117

A mixture of 75% carbon dioxide and 25% water by mass is flowing at 1600 K, 100 kPa into a heat exchanger where it is used to deliver energy to a heat engine. The mixture leave the heat exchanger at 500 K with a mass flow rate of 2 kg/min. Find the rate of energy and the rate of exergy delivered to the heat engine.

C.V. Heat exchanger, steady flow and no work.

From Table A.8:

CO2: $h_{in} = 1748.12$ kJ/kg, $s^o_{T\,in} = 6.7254$ kJ/kg K

CO2: $h_{ex} = 401.52$ kJ/kg, $s^o_{T\,ex} = 5.3375$ kJ/kg K

H2O: $h_{in} = 3487.69$ kJ/kg, $s^o_{T\,in} = 14.0822$ kJ/kg K

H2O: $h_{ex} = 935.12$ kJ/kg, $s^o_{T\,ex} = 11.4644$ kJ/kg K

Energy Eq.: $\dot{Q} = \dot{m}\,(h_{in} - h_{ex}) = \dot{m} \sum y_i\,(h_{in} - h_{ex})_i$

$$= \frac{2}{60}\,[0.75\,(1748.12 - 401.52) + 0.25(3487.69 - 935.12)]$$

$$= \frac{1}{30}\,[\,1009.95 + 638.14\,] = \mathbf{54.94\ kW}$$

Entropy change:

$$s_{in} - s_{ex} = 0.75(6.7254 - 5.3375) + 0.25(14.0822 - 11.9644)$$

$$= 1.6954 \text{ kJ/kg K}$$

Exergy Flux:

$$\dot{\Phi} = \dot{m}\,(\psi_{in} - \psi_{ex}) = \dot{Q} - T_o\,\dot{m}\,(s_{in} - s_{ex})$$

$$= 54.94 - 298.15 \times \frac{1}{30} \times 1.6954$$

$$= \mathbf{38.09\ kW}$$

12.118

Find the second law efficiency of the heat exchanger in Problem 12.59.
A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process.

Solution:

The second law efficiency follows Eq.10.32 where the wanted term is the flow increase of exergy and the source is the radiation.

$$\dot{\Phi}_{flow} = \dot{m}(\psi_{ex} - \psi_{in}); \qquad \dot{\Phi}_{source} = \dot{Q}_{in}\,(1 - \frac{T_o}{T_{source}})$$

Heat exchanger Energy Eq.6.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2\,[\frac{1}{2} \times (971.67 - 303.76) + \frac{1}{2} \times (980.95 - 366.03)] = 1282.8 \text{ kW}$$

$$\dot{\Phi}_{source} = \dot{Q}_{in}\,(1 - \frac{T_o}{T_{source}}) = 1282.8\,(\,1 - \frac{298.15}{1400}) = 1009.6 \text{ kW}$$

Entropy Eq.9.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

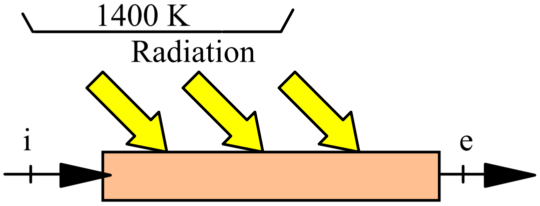
As P = C, the pressure correction in Eq.8.28 drops out to give generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$
$$= 2\,[0.5 \times (6.119 - 5.1196) + 0.5 \times (7.6121 - 6.6838)] - 1282.8/1400$$
$$= 1.01 \text{ kW/K}$$

$$\dot{\Phi}_{flow} = \dot{\Phi}_{source} - \dot{\Phi}_{destruction} = \dot{\Phi}_{source} - T\,\dot{S}_{gen}$$
$$= 1009.6 - 298.15 \times 1.01 = 708.5$$

$$\eta = \frac{\dot{\Phi}_{flow}}{\dot{\Phi}_{source}} = \frac{708.5}{1009.6} = \mathbf{0.70}$$

Remark: We could also explicitly have found the flow exergy increase.



Review Problems

12.119

A piston/cylinder contains helium at 110 kPa at ambient temperature 20°C, and initial volume of 20 L as shown in Fig. P12.119. The stops are mounted to give a maximum volume of 25 L and the nitrogen line conditions are 300 kPa, 30°C. The valve is now opened which allows nitrogen to flow in and mix with the helium. The valve is closed when the pressure inside reaches 200 kPa, at which point the temperature inside is 40°C. Is this process consistent with the second law of thermodynamics?

$P_1 = 110$ kPa, $T_1 = 20\ ^oC$, $V_1 = 20$ L, $V_{max} = 25\ L = V_2$

$P_2 = 200$ kPa, $T_2 = 40\ ^oC$, $P_i = 300$ kPa, $T_i = 30\ ^oC$

Constant P to stops, then constant $V = V_{max}$ => $W_{cv} = P_1(V_2 - V_1)$

$$Q_{cv} = U_2 - U_1 + W_{cv} - n_i\bar{h}_i,$$
$$= n_2\bar{h}_2 - n_1\bar{h}_1 - n_i\bar{h}_i - (P_2 - P_1)V_2$$
$$= n_A(\bar{h}_{A2} - \bar{h}_{Ai}) + n_B(\bar{h}_{B2} - \bar{h}_{B1}) - (P_2 - P_1)V_2$$

$n_B = n_1 = P_1V_1/\bar{R}T_1 = 110\times0.02/8.3145\times293.2 = 0.0009$ kmol

$n_2 = n_A + n_B = P_2V_2/\bar{R}T_2 = 200\times0.025/8.3145\times313.2 = 0.00192$ kmol,

$n_A = n_2 - n_B = 0.00102$ kmol

Mole fractions: $y_{A2} = 0.00102/0.00192 = 0.5313$, $y_{B2} = 0.4687$

$$Q_{cv} = 0.00102\times28.013\times1.042(40 - 30) + 0.0009\times4.003\times5.193(40 - 20) - (200 - 110)\ 0.025$$
$$= 0.298 + 0.374 - 2.25 = -\ 1.578 \text{ kJ}$$

$$S_{gen} = n_2\bar{s}_2 - n_1\bar{s}_1 - n_i\bar{s}_i - Q_{cv}/T_0$$
$$= n_A(\bar{s}_{A2} - \bar{s}_{Ai}) + n_B(\bar{s}_{B2} - \bar{s}_{B1}) - Q_{cv}/T_0$$

$$\bar{s}_{A2} - \bar{s}_{Ai} = 29.189 \ln\frac{313.2}{303.2} - \bar{R} \ln\frac{0.5313*200}{300} = 9.5763$$

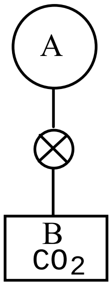
$$\bar{s}_{B2} - \bar{s}_{B1} = 20.7876 \ln\frac{313.2}{293.2} - \bar{R} \ln\frac{0.4687*200}{110} = 2.7015$$

$$S_{gen} = 0.00102\times9.5763 + 0.0009\times2.7015 + 1.578/293.2$$

= **0.0176 kJ/K > 0** **Satisfies 2nd law.**

12.120

A spherical balloon has an initial diameter of 1 m and contains argon gas at 200 kPa, 40°C. The balloon is connected by a valve to a 500-L rigid tank containing carbon dioxide at 100 kPa, 100°C. The valve is opened, and eventually the balloon and tank reach a uniform state in which the pressure is 185 kPa. The balloon pressure is directly proportional to its diameter. Take the balloon and tank as a control volume, and calculate the final temperature and the heat transfer for the process.



$$V_{A1} = \frac{\pi}{6} 1^3 = 0.5236,$$

$$m_{A1} = \frac{P_{A1}V_{A1}}{RT_{A1}} = \frac{200\times0.5236}{0.208\ 13\times313.2} = 1.606 \text{ kg}$$

$$m_{B1} = P_{B1}V_{B1}/RT_{B1} = 100\times0.50/0.18892\times373.2 = 0.709 \text{ kg}$$

$$P_2V_{A2}^{-1/3} = P_{A1}V_{A1}^{-1/3} \rightarrow$$

$$V_{A2} = V_{A1}\left(\frac{P_2}{P_{A1}}\right)^3 = 0.5236\left(\frac{185}{200}\right)^3 = 0.4144 \text{ m}^3$$

2: Uniform ideal gas mixture :

$$P_2(V_{A2}+V_B) = (m_AR_A+m_BR_B)T_2$$

$T_2 = 185(0.4144+0.50) / (1.606\times0.20813 + 0.709\times0.18892) =$ **361.3 K**

$$W_{12} = \frac{P_2V_{A2}\text{-}P_{A1}V_{A1}}{1\text{-}(-1/3)} = \frac{185\times0.4144\text{-}200\times0.5236}{(4/3)} = -21.0 \text{ kJ}$$

$$Q = m_AC_{V0A}(T_2\text{-}T_{A1}) + m_BC_{V0B}(T_2\text{-}T_{B1}) + W_{12}$$

$= 1.606\times0.312(361.3 - 313.2) + 0.709\times0.653(361.3 - 373.2) - 21.0$

$= 18.6 - 21.0 =$ **-2.4 kJ**

12.121

An insulated vertical cylinder is fitted with a frictionless constant loaded piston of cross sectional area 0.1 m^2 and the initial cylinder height of 1.0 m. The cylinder contains methane gas at 300 K, 150 kPa, and also inside is a 5-L capsule containing neon gas at 300 K, 500 kPa. The capsule now breaks, and the two gases mix together in a constant pressure process. What is the final temperature, final cylinder height and the net entropy change for the process.

$A_p = 0.1\ m^2$, $h = 1.0$ m => $V_{tot} = V_{a1} + V_{b1} = 0.1\ m^3$

Methane: M = 16.04 kg/kmol, $C_p = 2.254$ kJ/kg-K, R = 0.51835 kJ/kg-K

Neon: M = 20.183 kg/kmol, $C_p = 1.03$ kJ/kg-K, R = 0.41195 kJ/kg-K

State 1: Methane, $T_{a1} = 300$ K, $P_{a1} = 150$ kPa, $V_{a1} = V_{tot} - V_{b1} = 0.095\ m^3$

Neon, $T_{b1} = 300$ K, $P_{b1} = 500$ kPa, $V_{b1} = 5$ L

$$m_a = \frac{P_{a1}V_{a1}}{R_aT_{a1}} = 0.0916 \text{ kg}, \quad n_a = \frac{m_a}{M_a} = 0.00571 \text{ kmol}$$

$$m_b = \frac{P_{b1}V_{b1}}{R_bT_{b1}} = 0.0202 \text{ kg}, \quad n_b = \frac{m_b}{M_b} = 0.001 \text{ kmol}$$

State 2: Mix, $P_{2mix} = P_{a1} = 150$ kPa

Energy Eq: ${}_1Q_2 = m_a(u_{a2}-u_{a1}) + m_b(u_{b2} - u_{b1}) + {}_1W_2$; ${}_1Q_2 = 0$

${}_1W_2 = \int P\,dV = P_2(V_2-V_1)_{tot}$; $P_2V_2 = m_{tot}R_{mix}T_2 = (m_aR_a + m_bR_b)T_2$

Assume Constant Specific Heat

$0 = m_aC_{pa}(T_2-T_{a1}) + m_bC_{pb}(T_2-T_{b1} + (m_aR_a + m_bR_b)T_2 - P_2V_{1\,tot}$

Solving for T_2 = **293.3 K**

$$V_2 = \frac{(m_aR_a + m_bR_b)T_2}{P_2} = 0.1087\ m^3, \qquad h_2 = \frac{V_2}{A_p} = \mathbf{1.087\ m}$$

Entropy Eq.: $\Delta S_{net} = m_a(s_{a2} - s_{a1}) + m_b(s_{b2}-s_{b1}) - \frac{{}_1Q_2}{T_0}$; ${}_1Q_2 = 0$

$$y_a = \frac{n_a}{n_a+n_b} = 0.851, \quad y_b = 1 - y_a = 0.149$$

$$s_{a2} - s_{a1} = C_{pa} \ln\frac{T_2}{T_{a1}} - R_a\ln\frac{y_aP_2}{P_{a1}} = 0.02503 \text{ kJ/kg-K}$$

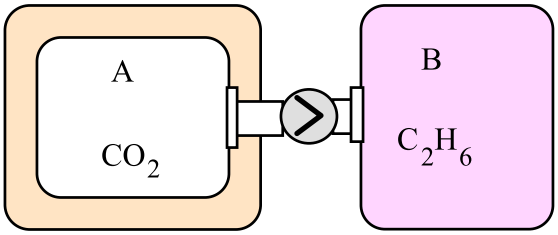
$$s_{b2} - s_{b1} = C_{pb} \ln\frac{T_2}{T_{b1}} - R_b \ln\frac{y_bP_2}{P_{b1}} = 1.2535 \text{ kJ/kg-K}$$

ΔS_{net} = **0.0276 kJ/K**

12.122

An insulated rigid 2 m^3 tank A contains CO_2 gas at 200°C, 1MPa. An uninsulated rigid 1 m^3 tank B contains ethane, C_2H_6, gas at 200 kPa, room temperature 20°C. The two are connected by a one-way check valve that will allow gas from A to B, but not from B to A. The valve is opened and gas flows from A to B until the pressure in B reaches 500 kPa and the valve is closed. The mixture in B is kept at room temperature due to heat transfer. Find the total number of moles and the ethane mole fraction at the final state in B. Find the final temperature and pressure in tank A and the heat transfer to/from tank B.

Solution:



Tank A: $V_A = 2\ m^3$, state A_1 : CO_2, $T_{A1} = 200°C = 473.2\ K$, $P_{A1} = 1\ MPa$

$\bar{C}_{v0\ CO_2} = 0.653 \times 44.01 = 28.74$, $\bar{C}_{P0\ CO_2} = 0.842 \times 44.01 = 37.06$ kJ/kmol K

Tank B: $V_B = 1\ m^3$, state B_1: C_2H_6, $T_{B1} = 20°C = 293.2\ K$, $P_{B1} = 200\ kPa$

Slow Flow A to B to $P_{B2} = 500$ kPa and assume $T_{B2} = T_{B1} = T_0$

Total moles scales to pressure, so with same V and T we have

$$P_{B1}V_B = n_{B1}\bar{R}T_{B1}\ , \qquad P_{B2}V_B = n_{B2\ mix}\ \bar{R}T_{B2}$$

Mole fraction: $$y_{C_2H_6\ B_2} = \frac{n_{B1}}{n_{B2}} = \frac{P_{B1}}{P_{B2}} = \frac{200}{500} = \mathbf{0.400}$$

$$n_{B1} = \frac{P_{B1}V_B}{RT_{B1}} = \frac{200 \times 1}{R \times 293.2} = 0.08204\ \text{kmol}$$

$$n_{B2\ mix} = \frac{P_{B2}V_B}{RT_{B2}} = \frac{500 \times 1}{R \times 293.2} = 0.2051\ \text{kmol}$$

$$n_{CO_2\ B2} = 0.2051 - 0.08201 = 0.12306\ \text{kmol}$$

Now we can work backwards to final state in A

$$n_{A1} = \frac{P_{A1}V_A}{RT_{A1}} = \frac{1000\times 2}{R\times 473.2} = 0.50833\ \text{kmol};\ \ n_{A2} = n_{A1} - n_{CO_2\ B2} = 0.38527\ \text{kmol}$$

C.V. A: All CO_2 Transient with flow out and adiabatic.

Energy Eq.: $Q_{CV\ A} = 0 = n_{A2}\ \bar{u}_{A2} - n_{A1}\ \bar{u}_{A1} + n_{ave}\ \bar{h}_{ave}$

$$0 = n_{A2}\,\bar{C}_{v0\,CO_2}T_{A2} - n_{A1}\,\bar{C}_{v0\,CO_2}T_{A1} + n_{ave}\,\bar{C}_{P0\,CO_2}\,(\,T_{A1} + T_{A2})/2$$

$$0 = 28.74\big(\,0.38527\times\,T_{A2} - 0.50833\times 473.2\big) + 0.12306 \times 37.06(473.2 + T_{A2})/2$$

$$=> \qquad T_{A2} = \mathbf{436.9\ K}$$

$$P_{A2} = \frac{n_{A2}\,RT_{A2}}{V_A} = \frac{0.38527 \times R \times 436.9}{2} = \mathbf{700\ kPa}$$

C.V. B: Transient with flow in and non-adiabatic.

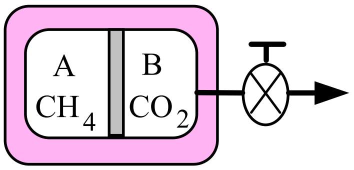
$$Q_{CV\,B} + n_{Bi}\,\bar{h}_{\,Bi\,ave} = (n\bar{u})_{\,B2} - (n\bar{u})_{\,B1} = (n\bar{u})_{CO_2\,B2} + (n\bar{u})_{C_2H_6\,B2} - (n\bar{u})_{C_2H_6\,B1}$$

$$Q_{CV\,B} = 0.12306 \times 28.74 \times 293.2 + 0\ \ - 0.12306 \times 37.06\,(473.2 + 436.9)/2$$

$$= \mathbf{-1038\ kJ}$$

12.123

A 0.2 m^3 insulated, rigid vessel is divided into two equal parts A and B by an insulated partition, as shown in Fig. P12.123. The partition will support a pressure difference of 400 kPa before breaking. Side A contains methane and side B contains carbon dioxide. Both sides are initially at 1 MPa, 30°C. A valve on side B is opened, and carbon dioxide flows out. The carbon dioxide that remains in B is assumed to undergo a reversible adiabatic expansion while there is flow out. Eventually the partition breaks, and the valve is closed. Calculate the net entropy change for the process that begins when the valve is closed.



$\Delta P_{MAX} = 400$ kPa, $\quad P_{A1} = P_{B1} = 1$ MPa

$V_{A1} = V_{B1} = 0.1$ m^3

$T_{A1} = T_{B1} = 30\ ^oC = 303.2$ K

CO_2 inside B: $s_{B2} = s_{B1}$ to $P_{B2} = 600$ kPa ($P_{A2} = 1000$ kPa)

$$\text{For } CO_2,\ k = 1.289 \;\Rightarrow\; T_{B2} = 303.2\left(\frac{600}{1000}\right)^{\frac{0.289}{1.289}} = 270.4 \text{ K}$$

$$n_{B2} = P_{B2}V_{B2}/\bar{R}T_{B2} = 600{\times}0.1/8.3145{\times}270.4 = 0.026\ 688$$

$$n_{A2} = n_{A1} = 1000{\times}0.1/8.3145{\times}303.2 = 0.039\ 668 \text{ kmol}$$

$$Q_{23} = 0 = n_3\bar{u}_3 - \sum_i n_{i2}\bar{u}_{i2} + 0 = n_{A2}\bar{C}_{V0\ A}(T_3 - T_{A2}) + n_{B2}\bar{C}_{V0\ B}(T_3 - T_{B2}) = 0$$

$$0.039\ 668{\times}16.04{\times}1.736(T_3 - 303.2) + 0.026\ 688{\times}44.01{\times}0.653(T_3 - 270.4) = 0$$

Solve $\quad T_3 = 289.8$ K

$$P_3 = 0.066356{\times}8.3145{\times}289.8/0.2 = 799.4 \text{ kPa}$$

$$P_{A3} = 0.5978{\times}799.4 = 477.9 \text{ kPa}, \qquad P_{B3} = P_3 - P_{A3} = 321.5 \text{ kPa}$$

$$\bar{s}_{A3} - \bar{s}_{A2} = 16.04{\times}2.254 \ln\left(\frac{289.8}{303.2}\right) - 8.3145 \ln\frac{477.9}{1000} = 4.505 \text{ kJ/kmol K}$$

$$\bar{s}_{B3} - \bar{s}_{B2} = 44.01{\times}0.842 \ln\left(\frac{289.8}{270.4}\right) - 8.3145 \ln\frac{321.5}{600} = 7.7546 \text{ kJ/kmol K}$$

$$\Delta S_{NET} = 0.039668{\times}4.505 + 0.026688{\times}7.7546 = \mathbf{+0.3857\ kJ/K}$$

12.124

An air-water vapor mixture enters a steady flow heater humidifier unit at state 1: 10°C, 10% relative humidity, at the rate of 1 m^3/s. A second air-vapor stream enters the unit at state 2: 20°C, 20% relative humidity, at the rate of 2 m^3/s. Liquid water enters at state 3: 10°C, at the rate of 400 kg per hour. A single air-vapor flow exits the unit at state 4: 40°C. Calculate the relative humidity of the exit flow and the rate of heat transfer to the unit.

Assume: P = 100 kPa

State 1: $T_1 = 10°C$, $\phi_1 = 10\%$, $\dot{V}_{a1} = 1\ m^3/s$

$P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.1228$ kPa,

$P_{a1} = P - P_{v1} = 99.877$ kPa

$$\omega_1 = 0.622\,\frac{P_{v1}}{P_{a1}} = 0.000765, \quad \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.2288 \text{ kg/s}$$

$\dot{m}_{v1} = \omega_1 \dot{m}_{a1} = 0.00094$ kg/s, $h_{v1} = h_{g1} = 2519.7$ kJ/kg

State 2: $T_2 = 20°C$, $\phi_2 = 20\%$, $\dot{V}_{a2} = 2\ m^3/s$

$P_{g2} = 2.3385$ kPa, $P_{v2} = \phi\, P_{g2} = 0.4677$ kPa, $P_{a2} = P - P_{v2} = 99.532$ kPa

$$\omega_2 = 0.622\,\frac{P_{v2}}{P_{a2}} = 0.002923, \quad \dot{m}_{a2} = \frac{P_{a2}\,\dot{V}_{a2}}{RT_{a2}} = 2.3656 \text{ kg/s}$$

$\dot{m}_{v2} = \omega_2 \dot{m}_{a2} = 0.00691$ kg/s, $h_{v2} = h_{g2} = 2538.1$ kJ/kg

State 3: Liquid. $T_3 = 10°C$, $\dot{m}_{f3} = 400$ kg/hr $= 0.1111$ kg/s, $h_{f3} = 42$ kJ/kg

State 4: $T_4 = 40°C$

Continuity Eq. air: $\dot{m}_{a4} = \dot{m}_{a2} + \dot{m}_{a1} = 3.5944$ kg/s,

Continuity Eq. water: $\dot{m}_{v4} = \dot{m}_{v1} + \dot{m}_{v2} + \dot{m}_{f3} = 0.11896$ kg/s

$$\omega_4 = \frac{\dot{m}_{v4}}{\dot{m}_{a4}} = 0.0331 = 0.622\,\frac{P_{v4}}{P - P_{v4}} \quad \rightarrow P_{v4} = 5.052 \text{ kPa}$$

$$P_{g4} = 7.384 \text{ kPa}, \quad \phi_4 = \frac{P_{v4}}{P_{g4}} = \mathbf{0.684}, \quad h_{v4} = h_{g4} = 2574.3 \text{ kJ/kg}$$

1st Law: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{f3}h_{f3} = \dot{m}_{a4}h_{a4} + \dot{m}_{v4}h_{v4}$

$$\dot{Q} = 1.004(3.5944 \times 40 - 1.2288 \times 10 - 2.3656 \times 20) + 0.11896 \times 2574.3$$
$$- 0.00094 \times 2519.7 - 0.00691 \times 2538.1 - 0.1111 \times 42.0$$
$$= \mathbf{366\ kW}$$

12.125

You have just washed your hair and now blow dry it in a room with 23°C, ϕ = 60%, (1). The dryer, 500 W, heats the air to 49°C, (2), blows it through your hair where the air becomes saturated (3), and then flows on to hit a window where it cools to 15°C (4). Find the relative humidity at state 2, the heat transfer per kilogram of dry air in the dryer, the air flow rate, and the amount of water condensed on the window, if any.

The blowdryer heats the air at constant specific humidity to 2 and it then goes through an adiabatic saturation process to state 3, finally cooling to 4.

1: 23°C, 60% rel hum => $w_1 = 0.0104$, $\tilde{h}_1 = 69$ kJ/kg dry air

2: $w_2 = w_1$, T_2 => ϕ_2 **= 15%**, $\tilde{h}_2 = 95$ kJ/kg dry air

CV. 1 to 2:

$w_2 = w_1$; $q = \tilde{h}_2 - \tilde{h}_1 = 95 - 69 =$ **26 kJ/kg dry air**

$\dot{m}_a = Q/q = 0.5/26 =$ **0.01923 kg/s**

CV. 2 to 3: $w_3 - w_2 = \dot{m}_{liq}/\dot{m}_a$; $\dot{m}_a \tilde{h}_2 + \dot{m}_{liq} h_f = \dot{m}_a \tilde{h}_3$

3: ϕ = 100% => $T_3 = \mathbf{T_{wet,2} = 24.8°C}$, $w_3 = \mathbf{0.0198}$

4: ϕ = 100%, T_4 => $w_4 = 0.01065$

$\dot{m}_{liq} = (w_3 - w_4)\dot{m}_a = (0.0198-0.01065)\times0.01923 =$ **0.176 g/s**

If the steam tables and formula's are used then we get

$h_{g1} = 2543.5$, $h_{g2} = 2590.3$, $P_{g1} = 2.837$, $P_{v1} = 1.7022$,

$P_{g2} = 11.8$, $w_1 = 0.01077$, $w_2 = w_1$, $P_{v2} = P_{v1}$

$\phi_2 = P_{v2}/P_{g2} = 14.4\%$, $h_{f3} = 114$,

Trial and error for adiabatic saturation temperature.

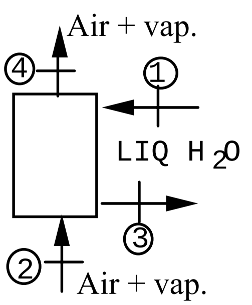
$T_3 = 25°C$, $w_3 = 0.02$, $P_{v4} = P_{g4} = 1.705$ kPa,

$w_4 = 0.622\times1.705/(100-1.705) = 0.0108$

12.126

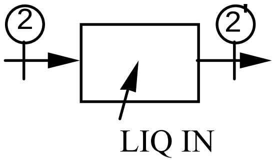
Steam power plants often utilize large cooling towers to cool the condenser cooling water so it can be recirculated; see Fig. P12.126. The process is essentially evaporative adiabatic cooling, in which part of the water is lost and must therefore be replenished. Consider the setup shown in Fig. P12.126, in which 1000 kg/s of warm water at 32°C from the condenser enters the top of the cooling tower and the cooled water leaves the bottom at 20°C. The moist ambient air enters the bottom at 100 kPa, dry bulb temperature of 18°C and a wet bulb temperature of 10°C. The moist air leaves the tower at 95 kPa, 30°C, and relative humidity of 85%. Determine the required mass flow rate of dry air, and the fraction of the incoming water that evaporates and is lost.

$P_4 = 95$ kPa
$T_4 = 30$ °C
$\phi_4 = 0.85$
$P_2 = 100$ kPa
$T_2 = 18$ °C
$WBT_2 = 10$ °C



$T_1 = 32$ °C
$\dot{m}_1 = 1000$ kg/s
$T_3 = 20$ °C

18 °C



$T_{2'} = 10$ °C, $\phi_{2'} = 1.0$

$$w_{2'} = 0.622 \times \frac{1.2276}{100 - 1.2276} = 0.00773$$

$$w_2 = \frac{(h_{a2'} - h_{a2}) + w_{2'}\, h_{FG2'}}{h_{v2} - h_{F2'}} = \frac{1.0035(10-18) + 0.00773 \times 2477.7}{2534.4 - 42.0} = 0.00446$$

$$P_{v4} = 0.85 \times 4.246 = 3.609, \quad w_4 = 0.622 \times \frac{3.609}{95 - 3.609} = 0.02456$$

Cons. mass: $\dot{m}_{a2} = \dot{m}_{a4} = \dot{m}_a, \quad \dot{m}_1 + \dot{m}_{v2} = \dot{m}_3 + \dot{m}_{v4}$

or $\dot{m}_3 = \dot{m}_1 + \dot{m}_a(w_2 - w_4)$ and set $r = \dot{m}_1/\dot{m}_a$

1st law: $\dot{m}_1 h_1 + \dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2} = \dot{m}_3 h_3 + \dot{m}_a h_{a4} + \dot{m}_{v4} h_{v4}$

or $r\, h_1 + (h_{a2} - h_{a4}) + w_2 h_{v2} = (r + w_2 - w_4) h_3 + w_4 h_{v4}$

$r(h_1 - h_3) = h_{a4} - h_{a2} + w_4 h_{v4} - w_2 h_{v2} - (w_4 - w_2) h_3$

$r(134.15 - 83.96) = 1.004(30-18) + 0.024\,56 \times 2556.3$
$- 0.004\,46 \times 2534.4 - 0.0201 \times 83.96$

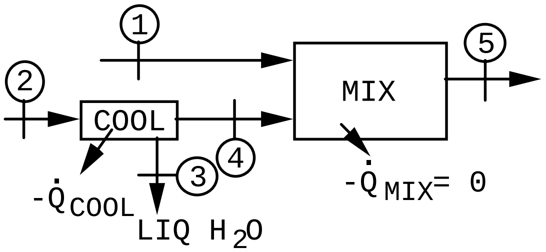
$r = \dot{m}_1/\dot{m}_a = 1.232 \quad \rightarrow \quad \dot{m}_a =$ **811.7 kg/s**

$\dot{m}_3 = \dot{m}_1 + \dot{m}_a(w_2 - w_4) = 1000 - 811.7 \times 0.0201 = 983.7$

$\Delta\dot{m}/\dot{m}_1 =$ **0.0163**

12.127

Ambient air is at a condition of 100 kPa, 35°C, 50% relative humidity. A steady stream of air at 100 kPa, 23°C, 70% relative humidity, is to be produced by first cooling one stream to an appropriate temperature to condense out the proper amount of water and then mix this stream adiabatically with the second one at ambient conditions. What is the ratio of the two flow rates? To what temperature must the first stream be cooled?



$P_1 = P_2 = 100$ kPa

$T_1 = T_2 = 35\ ^oC$

$\phi_1 = \phi_2 = 0.50,\ \phi_4 = 1.0$

$P_5 = 100,\ T_5 = 23\ ^oC$

$\phi_5 = 0.70$

$$P_{v1} = P_{v2} = 0.5\times5.628 = 2.814 \text{ kPa} \quad => \quad w_1 = w_2 = 0.622\times\frac{2.814}{100-2.814} = 0.0180$$

$$P_{v5} = 0.7\times2.837 = 1.9859 \text{ kPa} \quad => \quad w_5 = 0.622\times\frac{1.9859}{100-1.9859} = 0.0126$$

C.V.: Mixing chamber: Call the mass flow ratio $r = m_{a2}/m_{a1}$

cons. mass: $w_1 + r\,w_4 = (1+ r)w_5$

Energy Eq.: $h_{a1} + w_1h_{v1} + rh_{a4} + rw_4h_{v4} = (1+r)h_{a5} + (1+r)w_5h_{v5}$

$\rightarrow 0.018 + rw_4 = (1+r)\ 0.0126$

$$\text{or} \qquad r = \frac{m_{a2}}{m_{a1}} = \frac{0.018-0.0126}{0.0126-w_4}, \qquad \text{with} \qquad w_4 = 0.622 \times \frac{P_{G4}}{100-P_{G4}}$$

$$1.004\times308.2 + 0.018\times2565.3 + r\times1.004\times T_4 + r\,w_4h_{v4}$$

$$= (1+r)\times1.004 \times 296.2 + (1+r)\times0.0126\times2543.6$$

$$\text{or} \qquad r\Big[1.004\times T_4 + w_4h_{G4} - 329.3\Big] + 26.2 = 0$$

Assume $T_4 =$ **5 °C** $\rightarrow P_{G4} = 0.8721,\ \ h_{G4} = 2510.5$

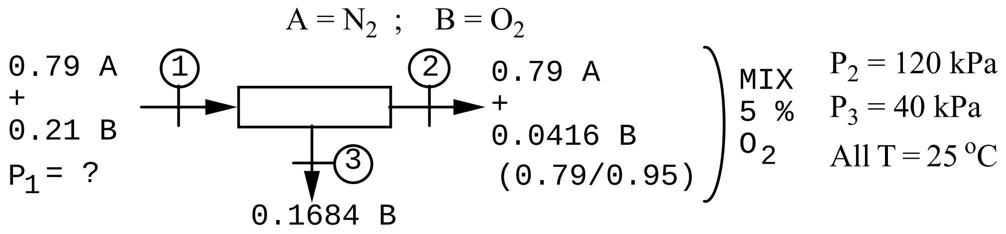
$w_4 = 0.622\times0.8721/(100-0.8721) = 0.0055$

$$r = m_{a2}/m_{a1} = \frac{0.018-0.0126}{0.0126-0.0055} = \mathbf{0.7606}$$

$0.7606[1.004\times278.2 + 0.0055\times2510.5 - 329.6] + 26.2 = -1.42 \approx 0$ OK

12.128

A semipermeable membrane is used for the partial removal of oxygen from air that is blown through a grain elevator storage facility. Ambient air (79% nitrogen, 21% oxygen on a mole basis) is compressed to an appropriate pressure, cooled to ambient temperature 25°C, and then fed through a bundle of hollow polymer fibers that selectively absorb oxygen, so the mixture leaving at 120 kPa, 25°C, contains only 5% oxygen. The absorbed oxygen is bled off through the fiber walls at 40 kPa, 25°C, to a vacuum pump. Assume the process to be reversible and adiabatic and determine the minimum inlet air pressure to the fiber bundle.



Let $s_{A1} = s_{B1} = 0$ at T = 25 °C & P_1

$$\bar{s}_{MIX\,1} = 0 + 0 - y_{A1}\bar{R}\ln y_{A1} - y_{B1}\bar{R}\ln y_{B1}$$

$$\bar{s}_{MIX\,2} = 0 + 0 - \bar{R}\ln(P_2/P_1) - y_{A2}\bar{R}\ln y_{A2} - y_{B2}\bar{R}\ln y_{B2}$$

Pure B: $\bar{s}_3 = 0 - \bar{R}\ln(P_3/P_1)$

For $\dot{n}_1\bar{s}_1 = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3$

$$\bar{R}\Big[-0.8316\ln(P_2/P_1) - 0.79\ln 0.95 - 0.0416\ln 0.05$$

$$- 0.1684\ln(P_3/P_1) + 0.79\ln 0.79 + 0.21\ln 0.21\Big] = 0$$

$$0.8316\ln(P_2/P_1) + 0.1684\ln(P_3/P_1) = -0.3488 + 4.6025 - \ln P_1 = -0.3488$$

$P_{1\,min}$ = **141 kPa**

For $P_1 > P_{1\,min}$ we would have entropy generation $\Delta\dot{S} > 0$

12.129

A dehumidifier receives a flow of 0.25 kg/s dry air at 28°C, 80% relative humidity as shown in figure P12.105. It is cooled down to 20°C as it flows over the evaporator and then heated up again as it flows over the condenser. The standard refrigeration cycle uses R-22 with an evaporator temperature of 5°C and a condensation pressure of 1600 kPa. Find the amount of liquid water removed and the heat transfer in the cooling process. How much compressor work is needed? What is the final air exit temperature and relative humidity?

Solution:

This set-up has a standard refrigeration cycle with R-22. This cycle and the air flow interacts through the two heat transfer processes. The cooling of the air is provided by the refrigeration cycle and thus requires an amount of work that depends on the cycle COP.

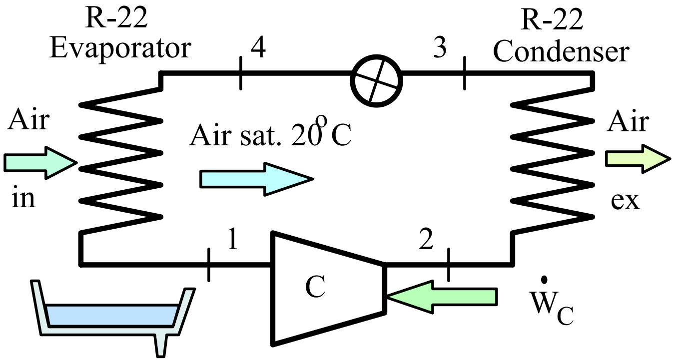
Refrigeration cycle:

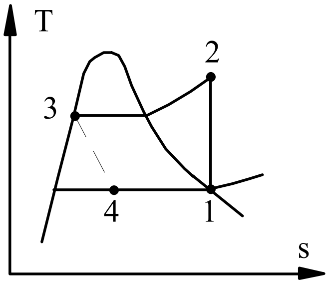
State 1: $x = 1$ $h_1 = 251.73$ kJ/kg, $s_1 = 0.9197$ kJ/kg K

State 2: $s_2 = s_1$, $h_2 = 276.75$ kJ/kg, $T_2 = 58^oC$

State 3: $x_3 = 0.0$, $h_3 = h_f = 96.55$ kJ/kg, $(T_3 = 41.7°C)$

State 4: $h_4 = h_3$ and $P_4 = P_1$





$T_1 = 5^oC$, $P_2 = 1600$ kPa

Now we get

$$w_C = h_2 - h_1 = 276.75 - 251.73 = 25.02 \text{ kJ/kg}$$

$$q_H = h_2 - h_3 = 276.75 - 96.55 = 180.2 \text{ kJ/kg}$$

$$q_L = h_1 - h_4 = 251.73 - 96.55 = 155.18 \text{ kJ/kg}$$

For the air processes let us use the psychrometric chart.

Air inlet: $w_{in} = 0.019$, $\tilde{h}_{in} = 96$ kJ/kg dry air, $T_{dew} = 24^oC > 20^oC$

Air 20: $\phi = 100\%$, $w_{20} = 0.0148$, $\tilde{h}_{20} = 77.5$, $h_f = 83.94$ (B.1.1)

Now do the continuity (for water) and energy equations for the cooling process

$\dot{m}_{liq} = \dot{m}_{air} (w_{in} - w_{20}) = 0.25 (0.019 – 0.0148) = \mathbf{0.00105\ kg/s}$

$\dot{Q}_{cool} = \dot{m}_{air}(\tilde{h}_{in} - \tilde{h}_{20}) - \dot{m}_{liq}h_f = 0.25(96 – 77.5) – 0.00105\ 83.94$

$= \mathbf{4.537\ kW}$

Now the heater from the R-22 cycle has

$\dot{Q}_{heat} = \dot{Q}_{cool} (q_H / q_L) = 4.537 (180.2 / 155.18) = 5.267$ kW

so the compressor work is the balance of the two

$\dot{W}_C = \dot{Q}_{heat} - \dot{Q}_{cool} = 5.267 – 4.537 = \mathbf{0.73\ kW}$

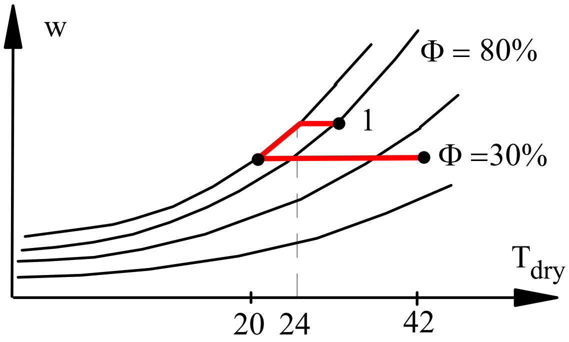
Energy eq. for the air flow being heated

$\dot{Q}_{heat} = \dot{m}_{air}(\tilde{h}_{ex} - \tilde{h}_{20}) \Rightarrow \tilde{h}_{ex} = \tilde{h}_{20} + \dot{Q}_{heat} / \dot{m}_{air}$

$\tilde{h}_{ex} = 77.5 + 5.267 / 0.25 = 98.6$ kJ/kg dry air and $w_{ex} = w_{20}$

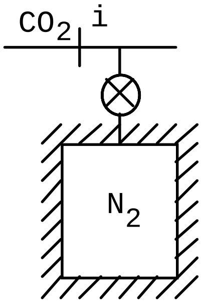
Locate state in the psychrometric chart

$T = 42^oC$ and $\phi = 30\%$



12.130

A 100-L insulated tank contains N_2 gas at 200 kPa and ambient temperature 25°C. The tank is connected by a valve to a supply line flowing CO_2 at 1.2 MPa, 90°C. A mixture of 50% N_2, 50% CO_2 by mole should be obtained by opening the valve and allowing CO_2 flow in to an appropriate pressure is reached and close the valve. What is the pressure? The tank eventually cools to ambient temperature. Find the net entropy change for the overall process.



$V = 100$ L, $P_1 = 200$ kPa, $T_1 = T_0 = 25$ °C

$P_i = 1.2$ MPa, $T_i = 90$ °C

At state 2: $y_{N_2} = y_{CO_2} = 0.50$

$n_{2\,CO_2} = n_{2\,N_2} = n_{1\,N_2} = P_1V/\bar{R}T_1$

$= 200*0.1/8.3145*298.2 = 0.00807$ kmol

$n_2 = 0.01614$ kmol

1st law: $n_i\bar{h}_i = n_2\bar{u}_2 - n_1\bar{u}_1$, for const specific heats

$n_i\bar{C}_{Poi}T_i = (n_i\bar{C}_{Voi}+n_1\bar{C}_{Vo1})T_2 - n_1\bar{C}_{Vo1}T_1$

But $n_i = n_1$ $\rightarrow \bar{C}_{Poi}T_i = \bar{C}_{Voi}T_2 + \bar{C}_{Vo1}(T_2-T_1)$

$44.01\times0.842\times363.2 = 44.01\times0.653\ T_2 + 28.013\times0.745(T_2-298.2)$

$T_2 = 396.7$ K

$P_2 = n_2\bar{R}T_2/V = 0.01614\times8.3145\times396.7/0.1 =$ **532 kPa**

Cool to $T_3 = T_0 = 298.2$ K

$P_3 = P_2\times T_3/T_2 = 532\times 298.2/396.7 = 400$ kPa

$Q_{23} = n_2\bar{C}_{Vo2}(T_3-T_2) = 0.016\,14(0.5\times28.013\times0.745$

$+ 0.5\times44.01\times0.653)(298.2-396.7) = -39.4$ kJ

$\Delta S_{NET} = n_3\bar{s}_3 - n_1\bar{s}_1 - n_i\bar{s}_i - Q_{23}/T_0$

$= n_i[(\bar{s}_{CO_2})_3 - \bar{s}_i] + n_1[(\bar{s}_{N_2})_3 - \bar{s}_1] - Q_{23}/T_0$

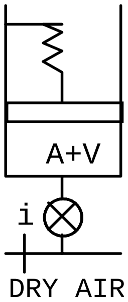
$$= 0.00807(44.01\times0.8418 \ln\frac{298.2}{363.2} - 8.3145 \ln\frac{0.5\times400}{1200})$$

$$+ 0.00807\times(- 8.3145 \ln\frac{0.5\times400}{200}) - \frac{-39.4}{298.2}$$

$= +0.0613 + 0 + 0.1321 =$ **+0.1934 kJ/K**

12.131

A cylinder/piston loaded with a linear spring contains saturated moist air at 120 kPa, 0.1 m^3 volume and also 0.01 kg of liquid water, all at ambient temperature 20°C. The piston area is 0.2 m^2, and the spring constant is 20 kN/m. This cylinder is attached by a valve to a line flowing dry air at 800 kPa, 80°C. The valve is opened, and air flows into the cylinder until the pressure reaches 200 kPa, at which point the temperature is 40°C. Determine the relative humidity at the final state, the mass of air entering the cylinder and the work done during the process.



$P_1 = 120$ kPa, $T_1 = 20\ ^oC = T_0$, $V_1 = 0.1$ m^3,

$m_{LIQ\ 1} = 0.01$ kg, $A_P = 0.2$ m^2, $k_s = 20$ kN/m

$P_i = 800$ kPa $\quad P_2 = 200$ kPa

$T_i = 80\ ^oC \quad T_2 = 40\ ^oC$

$P_2 = P_1 + (k_s/A_p^2)(V_2\text{-}V_1)$

$200 = 120 + (20/0.2^2)(V_2\text{-}0.1) \rightarrow V_2 = 0.26$ m^3

$\phi_1 = 1.0$ (or $w_1 = 0.622 \times 2.339/117.66 = 0.012\ 36$)

$$m_{v1} = \frac{P_{v1}V_1}{R_vT_1} = \frac{2.339 \times 0.1}{0.461\ 52 \times 293.2} = 0.001\ 73\ (= w_1 m_{A1})$$

Assume no liquid at state 2

$m_{v2} = m_{v1} + m_{L1} = 0.01173$ kg

$$\rightarrow P_{v2} = \frac{m_vR_vT_2}{V_2} = \frac{0.0011\ 73 \times 0.461\ 52 \times 313.2}{0.26} = 6.521 \text{ kPa}$$

a) $\phi_2 = \frac{6.521}{7.384} = \mathbf{0.883}$

b) $m_{A1} = \frac{P_{A1}V_1}{R_AT_1} = \frac{117.66 \times 0.1}{0.287 \times 293.2} = 0.1398$

$m_{A2} = \frac{193.479 \times 0.26}{0.287 \times 313.2} = 0.5596$

$m_{Ai} = m_{A2} - m_{A1} =$ **0.4198 kg**

c) $W_{CV} = \int PdV = \frac{1}{2}(P_1+P_2)(V_2\text{-}V_1) = \frac{1}{2}(120+200)(0.26\text{-}0.1) =$ **25.6 kJ**

12.132

Consider the previous problem and additionally determine the heat transfer. Show that the process does not violate the second law.

a) $Q_{CV} = m_{A2}h_{A2} - m_{A1}h_{A1} - m_{Ai}h_{Ai} + m_{v2}h_{v2} - m_{v1}h_{v1} - m_{L1}h_{L1} - P_2V_2 + P_1V_1 + W_{CV}$

$= 0.5596\times1.004\times313.2 - 0.1398\times1.004\times293.2$

$- 0.4198\times1.004\times353.2 + 0.001173\times2574.3$

$- 0.00173\times2538.4 – 0.01 \times83.9 - 200\times0.26 + 120\times0.1 + 25.6$

$=$ **-3.48 kJ**

b) $\Delta S_{CV} = S_2 - S_1 = m_{A2}s_{A2} - m_{A1}s_{A1} + m_vs_{v2} - m_{v1}s_{v1} - m_{L1}s_{L1}$

$\Delta S_{SURR} = - Q_{CV}/T_0 - m_{Ai}s_{Ai}$

$\Delta S_{NET} = m_{A2}s_{A2} - m_{A1}s_{A1} - m_{Ai}s_{Ai} + m_vs_{v2} - m_{v1}s_{v1} - m_{L1}s_{L1} - Q_{CV}/T_0$

$= m_{A2}(s_{A2}-s_{Ai}) + m_{A1}(s_{Ai}-s_{A1}) + m_vs_{v2} - m_{v1}s_{v1} - m_{L1}s_{L1} - Q_{CV}/T_0$

$$s_{A2} - s_{Ai} = 1.004 \ln\frac{313.2}{353.2} - 0.287 \ln\frac{193.479}{800} = +0.2866$$

$$s_{Ai} - s_{A1} = 1.004 \ln\frac{313.2}{293.2} - 0.287 \ln\frac{800}{117.66} = -0.3633$$

$s_{v2} = s_{G2} - R_v \ln \phi_2 = 8.2569 - 0.46152 \ln 0.883 = 8.3143$

$s_{v1} = s_{G1} - R_v \ln \phi_1 = 8.6671 – 0 = 8.6671$

$s_{L1} = s_{F1} = 0.2966$

$\rightarrow \Delta S_{NET} = 0.5596\ (+0.2868) + 0.1398\ (-0.3633) + 0.01173\times8.3143 –$

$0.00173\times8.6671 – 0.01\times0.2966 + 3.48/293.2$

$=$ **$+0.201$ kJ/K**

12.133

The air-conditioning by evaporative cooling in Problem 12.103 is modified by adding a dehumidification process before the water spray cooling process. This dehumidification is achieved as shown in Fig. P12.133 by using a desiccant material, which absorbs water on one side of a rotating drum heat exchanger. The desiccant is regenerated by heating on the other side of the drum to drive the water out. The pressure is 100 kPa everywhere and other properties are on the diagram. Calculate the relative humidity of the cool air supplied to the room at state 4, and the heat transfer per unit mass of air that needs to be supplied to the heater unit.

States as noted on Fig. P12.133, text page 506.

At state 1, 35 °C: $P_{V1} = \phi_1 P_{G1} = 0.30 \times 5.628 = 1.6884$

$$w_1 = 0.622 \times 1.6884/98.31 = 0.010\ 68$$

At $T_3 = 25\ ^oC$: $w_3 = w_2 = w_1/2 = 0.00534$

Evaporative cooling process to state 4, where $T_4 = 20^oC$

As in Eq. 12.30: $w_3(h_{v3} - h_{f4}) = C_{P0A}(T_4 - T_3) + w_4\, h_{fg4}$

$$0.005\ 34\ (2547.2 - 83.9) = 1.004(20 - 25) + w_4 \times 2454.2$$

$$w_4 = 0.0074 = 0.622 \times P_{v4} / (100 - P_{v4})$$

$P_{V4} = 1.176$ kPa, $\phi_4 = 1.176 / 2.339 = \mathbf{0.503}$

Following now the flow back we have

At $T_5 = 25\ ^oC$, $w_5 = w_4 = 0.0074$

Evaporative cooling process to state 6, where $T_6 = 20^oC$

$$w_5(h_{v5} - h_{f6}) = C_{P0A}(T_6 - T_5) + w_6\, h_{fg6}$$

$$0.0074(2547.2 - 83.9) = 1.004(20 - 25) + w_6 \times 2454.2$$

$$\Rightarrow \quad w_6 = 0.009\ 47$$

For adiabatic heat exchanger,

$\dot m_{A2} = \dot m_{A3} = \dot m_{A6} = \dot m_{A7} = \dot m_A$, Also $w_2 = w_3$, $w_6 = w_7$

So now only T_7 is unknown in the energy equation

$$h_{A2} + w_2 h_{v2} + h_{A6} + w_6 h_{v6} = h_{A3} + w_3 h_{v3} + h_{A7} + w_7 h_{v7}$$

or

$$C_{P0A}T_7 + w_6(h_{v7} - h_{v6}) = C_{P0A}(T_2 + T_6 - T_3) + w_2(h_{v2} - h_{v3})$$

$$1.004\ T_7 + 0.009\ 47(h_{v7} - 2538.1) = 1.004(60 + 20 - 25)$$

$$+ 0.005\ 34(2609.6 - 2547.2) = 55.526$$

By trial and error, $T_7 = 54.7\ ^oC$, $h_{v7} = 2600.3$ kJ/kg

For the heater 7-8, $w_8 = w_7$,

$\dot{Q}/\dot{m}_A = C_{P0A}(T_8 - T_7) + w_7(h_{v8} - h_{v7})$

$= 1.004(80 - 54.7) + 0.009\ 47(2643.7 - 2600.3)$ = **25.8 kJ/kg dry air**