# SOLUTION MANUAL ENGLISH UNIT PROBLEMS CHAPTER 11



# **CHAPTER 11**

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# **Correspondence List**

The correspondence between the new English unit problem set and the previous 5th edition chapter 11 problem set and the current SI problems.

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# Rankine cycles

## 11.167E

A steam power plant, as shown in Fig. 11.3, operating in a Rankine cycle has saturated vapor at 600 lbf/in.<sup>2</sup> leaving the boiler. The turbine exhausts to the condenser operating at 2.225 lbf/in.<sup>2</sup>. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

For the cycle as given:

1:  $h_1 = 97.97 \text{ Btu/lbm}, v_1 = 0.01625 \text{ ft}^3/\text{lbm},$ 

3: 
$$h_3 = h_g = 1204.06 \text{ Btu/lbm}, s_3 = s_g = 1.4464 \text{ Btu/lbm R}$$

C.V. Pump Reversible and adiabatic.

Energy: 
$$w_p = h_2 - h_1$$
; Entropy:  $s_2 = s_1$ 

since incompressible it is easier to find work (positive in) as

$$w_{p} = \int v dP = v_{1}(P_{2} - P_{1}) = 0.01625(600 - 2.2)\frac{144}{778} = 1.8 \text{ Btu/lbm}$$
  
 $h_{2} = h_{1} + w_{p} = 97.97 + 1.8 = 99.77 \text{ Btu/lbm}$ 

C.V. Boiler:  $q_H = h_3 - h_2 = 1204.06 - 99.77 = 1104.3$  Btu/lbm

C.V. Tubine:  $w_T = h_3 - h_4$ ,  $s_4 = s_3$   $s_4 = s_3 = 1.4464 = 0.1817 + x_4 \times 1.7292 \implies x_4 = 0.7314$ ,  $h_4 = 97.97 + 0.7314 \times 1019.78 = 843.84$  Btu/lbm  $w_T = 1204.06 - 843.84 = 360.22$  Btu/lbm  $\eta_{CYCLE} = (w_T - w_P)/q_H = (360.22 - 1.8)/1104.3 = 0.325$ 

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C.V. Condenser: q_L = h_4 - h_1 = 843.84 - 97.97 = 745.9 Btu/lbm
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## 11.168<mark>E</mark>

Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 350 F, and the condenser pressure is 0.95 lbf/in.<sup>2</sup>. Determine the thermal efficiency of this cycle.

H<sub>2</sub>O ideal Rankine cycle

## CV: turbine

State 3: Table F.7.1 
$$h_3 = 1193.1$$
 Btu/lbm,  $s_3 = 1.5793$  Btu/lbm R  
 $s_4 = s_3 = 1.5793 = 0.1296 + x_4 \times 1.8526 \implies x_4 = 0.7825$   
 $h_4 = 68.04 + 0.7825 \times 1036.98 = 879.5$  Btu/lbm  
 $w_T = h_3 - h_4 = 1193.1 - 879.5 = 313.6$  Btu/lbm  
 $w_P = \int vdP \approx v_1(P_2 - P_1) = 0.01613(134.54 - 0.95) \frac{144}{778} = 0.4$  Btu/lbm  
 $\Rightarrow w_{NET} = w_T - w_P = 313.6 - 0.4 = 313.2$  Btu/lbm  
 $h_2 = h_1 + w_P = 68.04 + 0.4 = 68.44$  Btu/lbm  
 $q_H = h_3 - h_2 = 1193.1 - 68.44 = 1124.7$  Btu/lbm  
 $\eta_{TH} = w_{NET}/q_H = 313.2/1124.7 = 0.278$ 



## 11.169E

A Rankine cycle uses ammonia as the working substance and powered by solar energy. It heats the ammonia to 320 F at 800 psia in the boiler/superheater. The condenser is water cooled, and the exit is kept at 70 F. Find (T, P, and x if applicable) for all four states in the cycle.

NH<sub>3</sub> ideal Rankine cycle

State 1: Table F.8.1, T = 70 F, x = 0,  $P_1 = 128.85$  psia,

$$h_1 = 120.21$$
 Btu/lbm,  $v_1 = 0.2631$  ft<sup>3</sup>/lbm

CV Pump:

$$w_{p} = h_{2} - h_{1} = \int v dP \approx v_{1}(P_{2} - P_{1}) = 0.02631(800 - 128.85)\frac{144}{778}$$
  
= 3.27 Btu/lbm  
$$h_{2} = h_{1} + w_{p} = 120.21 + 3.27 = 123.48 \text{ Btu/lbm} = h_{f} \implies T_{2} = 72.8 \text{ F}$$

[we need the computer software to do better  $(P_2, s_2 = s_1)$ ]

State 3: 320 F, 800 psia : superheated vapor,  $s_3 = 1.1915$  Btu/lbm CV: turbine

$$s_4 = s_3 = 1.1915 = 0.2529 + x_4 \times 0.9589 \implies x_4 = 0.9788$$
  
 $P_4 = P_1 = 128.85 \text{ psia}, \quad T_4 = T_1 = 70 \text{ F}$ 



## 11.170<mark>E</mark>

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 180 F, and the condenser temperature is 100 F. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-134a Table F.10)  $P_1 = 138.93 \text{ psia}, P_2 = P_3 = 400.4 \text{ psia}$   $h_3 = 184.36 \text{ Btu/lbm}, s_3 = 0.402 \text{ Btu/lbm R}$   $h_1 = 108.86 \text{ Btu/lbm}, v_1 = 0.01387 \text{ ft}^3/\text{lbm}$   $w_P = h_2 - h_1 = \int_1^2 \text{vdP} \approx v_1(P_2 - P_1)$   $= 0.01387(400.4 - 138.93) \frac{144}{778} = 0.671 \text{ Btu/lbm}$  $h_2 = h_1 + w_P = 108.86 + 0.671 = 109.53 \text{ Btu/lbm}$ 

CV: Boiler

$$q_{\rm H} = h_3 - h_2 = 184.36 - 109.53 = 74.83$$
 Btu/lbm

CV: Turbine

$$s_4 = s_3 = 0.402 \implies x_4 = (0.402 - 0.2819)/0.1272 = 0.9442$$
  
 $h_4 = 176.08$  Btu/lbm,

Energy Eq.:  $w_T = h_3 - h_4 = 8.276$  Btu/lbm  $w_{NET} = w_T - w_P = 8.276 - 0.671 = 7.605$  Btu/lbm

$$\eta_{TH} = w_{NET} / q_{H} = 7.605/74.83 = 0.102$$



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## 11.171E

Do Problem 11.170 with R-22 as the working fluid.

Standard Rankine cycle with properties from the R-22 tables,

$$h_{1} = 39.267 \text{ Btu/lbm}, v_{1} = 0.01404 \text{ ft}^{3}/\text{lbm}, P_{1} = 210.6 \text{ psia}, P_{2} = P_{3} = 554.8 \text{ psia}, h_{3} = 110.07 \text{ Btu/lbm}, s_{3} = 0.1913 \text{ Btu/lbm R}$$
CV: Pump  $w_{p} = v_{1}(P_{2}-P_{1}) = 0.01404 (554.8-210.6)\frac{144}{778} = 0.894 \text{ Btu/lbm}$ 
 $h_{2} = h_{1} + w_{p} = 39.267 + 0.894 = 40.16 \text{ Btu/lbm}$ 
CV: Turbine  $s_{4} = s_{3}$ 
 $\Rightarrow x_{4} = (0.1913 - 0.07942)/0.13014 = 0.9442$ 
 $h_{4} = 101.885 \text{ Btu/lbm}, w_{T} = h_{3} - h_{4} = 8.185 \text{ Btu/lbm}$ 
CV: Boiler
 $q_{H} = h_{3} - h_{2} = 110.07 - 40.16 = 69.91 \text{ Btu/lbm}$ 
 $\eta_{TH} = (w_{T} - w_{p})/q_{H} = (8.185 - 0.894)/157.21 = 0.104$ 

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## 11.172<mark>E</mark>

A smaller power plant produces 50 lbm/s steam at 400 psia, 1100 F, in the boiler. It cools the condenser with ocean water coming in at 55 F and returned at 60 F so that the condenser exit is at 110 F. Find the net power output and the required mass flow rate of the ocean water.

## Solution:

The states properties from Tables F.7.1 and F.7.2

- 1: 110 F, x = 0:  $h_1 = 78.01$  Btu/lbm,  $v_1 = 0.01617$  ft<sup>3</sup>/lbm,  $P_{sat} = 1.28$  psia
- 3: 400 psia, 1100 F:  $h_3 = 1577.44$  Btu/lbm,  $s_3 = 1.7989$  Btu/lbm R
- C.V. Pump Reversible and adiabatic.

Energy: 
$$w_p = h_2 - h_1$$
; Entropy:  $s_2 = s_1$ 

since incompressible it is easier to find work (positive in) as

$$w_p = \int v dP = v_1 (P_2 - P_1) = 0.01617 (400 - 1.3) \frac{144}{778} = 1.19 \text{ Btu/lbm}$$

C.V. Turbine :  $w_T = h_3 - h_4$ ;  $s_4 = s_3$ 

 $s_4 = s_3 = 1.7989 = 0.1473 + x_4 (1.8101) \implies x_4 = 0.9124$  $\implies h_4 = 78.01 + 0.9124 (1031.28) = 1018.95 \text{ Btu/lbm}$  $w_T = 1577.44 - 1018.95 = 558.5 \text{ Btu/lbm}$ 

$$\dot{W}_{NET} = \dot{m}(w_T - w_p) = 50 (558.5 - 1.19) = 27 866 \text{ Btu/s}$$
  
C.V. Condenser :  $q_L = h_4 - h_1 = 1018.95 - 78.01 = 940.94 \text{ Btu/lbm}$ 

$$\dot{Q}_L = \dot{m}q_L = 50 \times 940.94 = 47\ 047\ Btu/s = \dot{m}_{ocean}\ C_p\ \Delta T$$
  
 $\dot{m}_{ocean} = \dot{Q}_L / C_p\ \Delta T = 47\ 047 / (1.0 \times 5) = 9409\ lbm/s$ 



## 11.173E

The power plant in Problem 11.167 is modified to have a superheater section following the boiler so the steam leaves the super heater at 600  $lbf/in.^2$ , 700 F. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

For this cycle from Table F.7 State 3: Superheated vapor  $h_3 = 1350.62$  Btu/lbm,  $s_3 = 1.5871$  Btu/lbm R,  $h_1 = 97.97 \text{ Btu/lbm}, v_1 = 0.01625 \text{ ft}^3/\text{lbm}$ State 1: Saturated liquid C.V. Pump: Adiabatic and reversible. Use incompressible fluid so  $w_{p} = \int v dP = v_{1}(P_{2} - P_{1}) = 0.01625(600 - 2.2)\frac{144}{778} = 1.8 \text{ Btu/lbm}$  $h_2 = h_1 + w_p = 95.81 \text{ Btu/lbm}$  $q_{\rm H} = h_3 - h_2 = 1350.62 - 97.97 = 1252.65$  Btu/lbm C.V. Boiler:  $w_{T} = h_{3} - h_{4},$ C.V. Tubine:  $s_4 = s_3$  $s_4 = s_3 = 1.5871$  Btu/lbm R = 0.1817 +  $x_4 1.7292 \implies x_4 = 0.8127$ ,  $h_a = 97.97 + 0.8127 \times 1019.78 = 926.75$  Btu/lbm  $w_{T} = 1350.62 - 926.75 = 423.87 \text{ Btu/lbm}$  $\eta_{\text{CYCLE}} = (w_{\text{T}} - w_{\text{P}})/q_{\text{H}} = (423.87 - 1.8)/1252.65 = 0.337$ C.V. Condenser:  $q_{I} = h_{A} - h_{1} = 926.75 - 97.97 = 828.8 \text{ Btu/lbm}$ 



## 11.174<mark>E</mark>

Consider a simple ideal Rankine cycle using water at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gasturbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 8000 lbf/in.<sup>2</sup>, 1300 F, and the condenser pressure is 0.95 lbf/in.<sup>2</sup>. What is the steam quality at the turbine exit?

Solution:

For the efficiency we need the net work and steam generator heat transfer.

State 1: 
$$s_1 = 0.1296$$
 Btu/lbm R,  $h_1 = 68.04$  Btu/lbm

State 3:  $h_3 = 1547.5$  Btu/lbm,  $s_3 = 1.4718$  Btu/lbm R

C.V. Pump. For this high exit pressure we use Table F.7.3 to get state 2.

Entropy Eq.: 
$$s_2 = s_1 \implies h_2 = 91.69$$
 Btu/lbm

 $w_p = h_2 - h_1 = 91.69 - 68.04 = 23.65$  Btu/lbm

C.V. Turbine. Assume reversible and adiabatic. Entropy Eq.:  $s_4 = s_3 = 1.4718 = 0.1296 + x_4 \times 1.8526$ 

 $x_4 = 0.7245$  Very low for a turbine exhaust

 $h_4 = 68.04 + x_4 \times 1036.98 = 751.29$  Btu/lbm,

$$w_T = h_3 - h_4 = 796.2$$
 Btu/lbm

Steam generator:  $q_H = h_3 - h_2 = 1547.5 - 91.69 = 1455.8$  Btu/lbm

 $w_{NET} = w_T - w_p = 796.2 - 23.65 = 772.6 \text{ Btu/lbm}$ 

$$\eta = w_{NET} / q_H = 772.6 / 1455.8 = 0.53$$



## 11.175<mark>E</mark>

Consider an ideal steam reheat cycle in which the steam enters the high-pressure turbine at 600 lbf/in.<sup>2</sup>, 700 F, and then expands to 150 lbf/in.<sup>2</sup>. It is then reheated to 700 F and expands to 2.225 lbf/in.<sup>2</sup> in the low-pressure turbine. Calculate the thermal efficiency of the cycle and the moisture content of the steam leaving the low-pressure turbine.

Solution:

Basic Rankine cycle with a reheat section. For this cycle from Table F.7 State 3: Superheated vapor  $h_3 = 1350.62$  Btu/lbm,  $s_3 = 1.5871$  Btu/lbm R, State 1: Saturated liquid  $h_1 = 97.97$  Btu/lbm,  $v_1 = 0.01625$  ft<sup>3</sup>/lbm C.V. Pump: Adiabatic and reversible. Use incompressible fluid so



## 11.176<mark>E</mark>

Consider an ideal steam regenerative cycle in which steam enters the turbine at 600 lbf/in.<sup>2</sup>, 700 F, and exhausts to the condenser at 2.225 lbf/in.<sup>2</sup>. Steam is extracted from the turbine at 150 lbf/in.<sup>2</sup> for an open feedwater heater. The feedwater leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and the feedwater heater. Calculate the thermal efficiency of the cycle and the net work per pound-mass of steam.



$$\Rightarrow$$
 h<sub>4</sub> = h<sub>3</sub> + w<sub>P34</sub> = 332.18 Btu/lbm

C.V. Turbine (high pressure section)

2nd law:  $s_6 = s_5 = 1.5871$  Btu/lbm R =>  $h_6 = 1208.93$  Btu/lbm

CV: feedwater heater, call the extraction fraction  $y = \dot{m}_6 / \dot{m}_3$ 

Continuity Eq.: 
$$\dot{m}_3 = \dot{m}_6 + \dot{m}_2$$
, Energy Eq.:  $\dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3$   
 $y_6 h_6 + (1 - y_6) h_2 = h_3 \implies y_6 = (h_3 - h_2)/(h_6 - h_2)$   
 $\Rightarrow y_6 = (330.67 - 98.41)/(1208.93 - 98.41) = 0.2091$ 

CV: Turbine from 5 to 7

$$s_7 = s_5 \implies x_7 = (1.5871 - 0.1817)/1.7292 = 0.8127$$
  
 $h_7 = 97.97 + 0.8127 \times 1019.78 = 926.75$  Btu/lbm

$$w_{T} = (h_{5} - h_{6}) + (1 - y_{6})(h_{6} - h_{7})$$
  
= (1350.62 - 1208.93) + 0.7909(1208.93 - 926.75) = 364.87 Btu/lbm

CV: pumps

 $w_{p} = (1 - y_{6})w_{P12} + w_{P34} = 0.7909 \times 0.44 + 1 \times 1.507 = 1.855 \text{ Btu/lbm}$  $w_{NET} = w_{T} - w_{p} = 364.87 - 1.855 = 363.0 \text{ Btu/lbm}$ 

CV: steam generator

$$\begin{split} \boldsymbol{q}_{H} &= \boldsymbol{h}_{5} \textbf{-} \boldsymbol{h}_{4} = 1350.62 - 332.18 = 1018.44 \text{ Btu/lbm} \\ \boldsymbol{\eta}_{TH} &= \boldsymbol{w}_{NET}^{}/\boldsymbol{q}_{H} = 363/1018.44 = \textbf{0.356} \end{split}$$

## 11**.**177E

A closed feedwater heater in a regenerative steam power cycle heats 40 lbm/s of water from 200 F, 2000 lbf/in.<sup>2</sup> to 450 F, 2000 lbf/in.<sup>2</sup>. The extraction steam from the turbine enters the heater at 600 lbf/in.<sup>2</sup>, 550 F and leaves as saturated liquid. What is the required mass flow rate of the extraction steam?



 From the steam tables F.7:

 F.7.3:
  $h_2 = 172.6$  Btu/lbm

 F.7.3:
  $h_4 = 431.13$  Btu/lbm

 F.7.2:
  $h_6 = 1255.36$  Btu/lbm

 Interpolate for this state

 F.7.1:
  $h_{6a} = 471.56$  Btu/lbm

C.V. Feedwater Heater

Energy Eq.:  $\dot{m}_2h_2 + \dot{m}_6h_6 = \dot{m}_2h_4 + \dot{m}_6h_{6a}$ 

Since all four state are known we can solve for the extraction flow rate

$$\dot{\mathbf{m}}_6 = \dot{\mathbf{m}}_2 \frac{\mathbf{h}_2 - \mathbf{h}_4}{\mathbf{h}_{6a} - \mathbf{h}_6} = 40 \frac{172.6 - 431.13}{471.56 - 1255.36} = \mathbf{13.2} \frac{\mathbf{lbm}}{\mathbf{s}}$$

## 11.178<mark>E</mark>

A steam power cycle has a high pressure of 600 lbf/in.<sup>2</sup> and a condenser exit temperature of 110 F. The turbine efficiency is 85%, and other cycle components are ideal. If the boiler superheats to 1400 F, find the cycle thermal efficiency.



## 11.179<mark>E</mark>

The steam power cycle in Problem 11.167 has an isentropic efficiency of the turbine of 85% and that for the pump it is 80%. Find the cycle efficiency and the specific work and heat transfer in the components.

States numbered as in fig 11.3 of text. CV Pump:  $w_{P,S} = v_1(P_2 - P_1) = 0.01625(600 - 2.2)144/778 = 1.8 Btu/lbm$   $\Rightarrow w_{P,AC} = 1.8/0.8 = 2.245 Btu/lbm$   $h_2 = h_1 + w_{P,AC} = 97.97 + 2.245 = 100.2 Btu/lbm$ CV Turbine:  $w_{T,S} = h_3 - h_{4s}$ ,  $s_4 = s_3 = 1.4464 Btu/lbm R$   $s_4 = s_3 = 1.4464 = 0.1817 + x_4 \times 1.7292 \implies x_4 = 0.7314$ ,  $h_4 = 97.97 + 0.7314 \times 1019.78 = 843.84 Btu/lbm$   $\Rightarrow w_{T,S} = 1204.06 - 843.84 = 360.22 Btu/lbm$   $w_{T,AC} = h_3 - h_{4AC} = 360.22 \times 0.85 = 306.2$   $\Rightarrow h_{4AC} = 897.86 Btu/lbm (still two-phase)$ CV Boiler:  $q_H = h_3 - h_2 = 1204.06 - 100.2 = 1103.9 Btu/lbm$   $q_L = h_{4AC} - h_1 = 897.86 - 97.97 = 799.9 Btu/lbm$  $\eta_{CYCLE} = (w_T - w_P)/q_H = (306.2 - 2.245)/1103.9 = 0.275$ 

Compared to (360.22-1.8)/1104.3 = 0.325 in the ideal case.





state 2s and 2ac nearly the same

## 11.180E

Steam leaves a power plant steam generator at 500 lbf/in.<sup>2</sup>, 650 F, and enters the turbine at 490 lbf/in.<sup>2</sup>, 625 F. The isentropic turbine efficiency is 88%, and the turbine exhaust pressure is 1.7 lbf/in.<sup>2</sup>. Condensate leaves the condenser and enters the pump at 110 F, 1.7 lbf/in.<sup>2</sup>. The isentropic pump efficiency is 80%, and the discharge pressure is 520 lbf/in.<sup>2</sup>. The feedwater enters the steam generator at 510 lbf/in.<sup>2</sup>, 100 F. Calculate the thermal efficiency of the cycle and the entropy generation of the flow in the line between the steam generator exit and the turbine inlet, assuming an ambient temperature of 77 F.



## 11.181E

A boiler delivers steam at 1500 lbf/in.<sup>2</sup>, 1000 F to a two-stage turbine as shown in Fig. 11.17. After the first stage, 25% of the steam is extracted at 200 lbf/in.<sup>2</sup> for a process application and returned at 150 lbf/in.<sup>2</sup>, 190 F to the feedwater line. The remainder of the steam continues through the low-pressure turbine stage, which exhausts to the condenser at 2.225 lbf/in.<sup>2</sup>. One pump brings the feedwater to 150 lbf/in.<sup>2</sup> and a second pump brings it to 1500 lbf/in.<sup>2</sup>. Assume the first and second stages in the steam turbine have isentropic efficiencies of 85% and 80% and that both pumps are ideal. If the process application requires 5000 Btu/s of power, how much power can then be cogenerated by the turbine?

3:  $h_3 = 1490.32$ ,  $s_3 = 1.6001$  Btu/lbmR C.V. Turbine T1 4s: Rev and adiabatic  $s_{4S} = s_3 \implies$ Table F.7.2 Sup. vapor  $h_{4S} = 1246.6$  Btu/lbm  $w_{T1,S} = h_3 - h_{4S} = 243.7$  Btu/lbm  $\Rightarrow w_{T1,AC} = 207.15$  Btu/lbm  $h_{4AC} = h_3 - w_{T1,AC} = 1283.16$ 4ac:  $P_4$ ,  $h_{4AC}$  $\Rightarrow s_{4AC} = 1.6384$  Btu/lbm R



5s: 
$$s_{5S} = s_{4AC} \implies x_{5S} = \frac{1.6384 - 0.1817}{1.7292} = 0.8424$$
  
 $h_{5S} = 97.97 + x_{5S} \ 1019.78 = 957.03 \ Btu/lbm$   
 $w_{T2,S} = h_{4AC} - h_{5S} = 326.13 \ Btu/lbm$   
 $w_{T2,AC} = 260.9 = h_{4AC} - h_{5AC} \implies h_{5AC} = 1022.3 \ Btu/lbm$   
7: Compressed liquid use sat. liq. same T:  $h_7 = 158.02 \ Btu/lbm$ ;

C.V. process unit. Assume no work only heat out.

 $q_{PROC} = h_{4AC} - h_7 = 1125.1 \text{ Btu/lbm}$ 

$$\dot{m}_4 = \dot{Q}/q_{PROC} = 5000/1125.1 = 4.444 \text{ lbm/s} = 0.25 \ \dot{m}_{TOT}$$
  
 $\Rightarrow \dot{m}_{TOT} = \dot{m}_3 = 17.776 \text{ lbm/s}, \qquad \dot{m}_5 = \dot{m}_3 - \dot{m}_4 = 13.332 \text{ lbm/s}$ 

C.V. Total turbine

$$\dot{W}_{T} = \dot{m}_{3}h_{3} - \dot{m}_{4}h_{4AC} - \dot{m}_{5}h_{5AC} = 7160 \text{ Btu/s}$$

## **Brayton Cycles**

#### 11.182E

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 000 hp to an electric generator. The minimum temperature in the cycle is 540 R, and the maximum temperature is 2900 R. The minimum pressure in the cycle is 1 atm, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine, the fraction of the turbine output required to drive the compressor and the thermal efficiency of the cycle?



Compression in compressor:  $s_2 = s_1 \implies$  Implemented in Eq.8.32

$$\rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540(14)^{0.286} = 1148.6 \text{ R}$$

 $w_{C} = h_{2} - h_{1} = C_{P0}(T_{2} - T_{1}) = 0.24 (1148.6 - 540) = 146.1 \text{ Btu/lbm}$ 

Expansion in turbine:  $s_4 = s_3 \implies$  Implemented in Eq.8.32

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 2900 \left(\frac{1}{14}\right)^{0.286} = 1363.3 \text{ R}$$

 $w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 0.24(2900 - 1363.3) = 368.8 \text{ Btu/lbm}$  $w_{NET} = w_T - w_C = 368.8 - 146.1 = 222.7 \text{ Btu/lbm}$ 

$$\dot{\mathbf{m}} = \dot{\mathbf{W}}_{\text{NET}} / \mathbf{w}_{\text{NET}} = 100\ 000 \times 2544 / 222.7 = 1\ 142\ 344\ \text{lbm/h}$$

$$\dot{W}_{T} = \dot{m}W_{T} = 165\ 600\ hp,$$
  $W_{C}/W_{T} = 0.396$ 

Energy input is from the combustor

.

$$q_H = C_{P0}(T_3 - T_2) = 0.24(2900 - 1148.6) = 420.3 \text{ Btu/lbm}$$
  
 $\eta_{TH} = w_{NET}/q_H = 222.7/420.3 = 0.530$ 

## 11.183E

A Brayton cycle produces 14 000 Btu/s with an inlet state of 60 F, 14.7 psia, and a compression ratio of 16:1. The heat added in the combustion is 400 Btu/lbm. What are the highest temperature and the mass flow rate of air, assuming cold air properties?

Solution:

Efficiency is from Eq.11.8

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{H}} = \frac{w_{net}}{q_{H}} = 1 - r_{p}^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

from the required power we can find the needed heat transfer

$$\dot{Q}_{H} = \dot{W}_{net} / \eta = \frac{14\ 000}{0.547} = 25\ 594\ Btu/s$$
  
 $\dot{m} = \dot{Q}_{H} / q_{H} = \frac{25\ 594\ Btu/s}{400\ Btu/lbm} = 63.99\ lbm/s$ 

Temperature after compression is

$$T_2 = T_1 r_p^{(k-1)/k} = 520 \times 16^{0.4/1.4} = 1148 R$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 1148 + \frac{400}{0.24} = 2815 R$$

## 11.184E

Do the previous problem with properties from table F.5 instead of cold air properties.

Solution:

With the variable specific heat we must go through the processes one by one to get net work and the highest temperature  $T_3$ .

From F.5:  $h_1 = 124.38 \text{ btu/lbm}, \quad s_{T1}^o = 1.63074 \text{ Btu/lbm R}$ 

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \implies s_{T2}^o = s_{T1}^o + R \ln(\frac{P_2}{P_1}) = 1.63074 + \frac{53.34}{778} \ln 16$$

## = 1.82083 Btu/lbm R

back interpolate in F.5  $\Rightarrow$  T<sub>2</sub> = 1133.5 R, h<sub>2</sub> = 274.58 Btu/lbm Energy equation with compressor work in

ergy equation with compressor work in

 $w_{C} = -1w_{2} = h_{2} - h_{1} = 274.58 - 124.383 = 150.2 \text{ Btu/lbm}$ Energy Eq. combustor:  $h_{3} = h_{2} + q_{H} = 274.58 + 400 = 674.6 \text{ Btu/lbm}$ 

Energy Eq. combustor.  $n_3 = n_2 + q_H = 2/4.58 + 400 = 0/4.0$  Btu/10

State 3: (P, h):  $T_3 = 2600 \text{ R}$ ,  $s_{T3}^o = 2.04523 \text{ Btu/lbm R}$ 

The expansion is reversible and adiabatic so constant s. From Eq.8.28

$$s_4 = s_3 \implies s_{T4}^o = s_{T3}^o + R\ln(P_4/P_3) = 2.04523 + \frac{53.34}{778}\ln(1/16) = 1.85514$$

 $\Rightarrow$  T<sub>4</sub> = 1297 R, h<sub>4</sub> = 316.21 Btu/lbm

Energy equation with turbine work out

 $w_T = h_3 - h_4 = 674.6 - 316.21 = 358.4$  Btu/lbm

Now the net work is

$$w_{net} = w_T - w_C = 358.4 - 150.2 = 208.2 \text{ Btu/lbm}$$

The total required power requires a mass flow rate as

$$\dot{\mathbf{m}} = \frac{W_{net}}{w_{net}} = \frac{14\ 000}{208.2} \frac{\text{Btu/s}}{\text{Btu/lbm}} = 67.2 \text{ lbm/s}$$

## 11.185E

An ideal regenerator is incorporated into the ideal air-standard Brayton cycle of Problem 11.182. Calculate the cycle thermal efficiency with this modification. Solution:



The compression is reversible and adiabatic so constant s. From Eq.8.32

$$\rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 540(14)^{0.286} = 1148.6 \text{ R}$$

 $w_{C} = h_{2} - h_{1} = C_{P0}(T_{2} - T_{1}) = 0.24 (1148.6 - 540) = 146.1 \text{ Btu/lbm}$ 

Expansion in turbine:  $s_4 = s_3 \implies$  Implemented in Eq.8.32

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 2900(\frac{1}{14})^{0.286} = 1363.3 \text{ R}$$

 $w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 0.24(2900 - 1363.3) = 368.8 Btu/lbm$  $w_{NET} = w_T - w_C = 368.8 - 146.1 = 222.7 Btu/lbm$ 

Ideal regenerator:  $T_X = T_4 = 1363.3 \text{ R}$ 

$$\begin{split} \boldsymbol{q}_{H} &= \boldsymbol{h}_{3} - \boldsymbol{h}_{X} = 0.24(2900 - 1363.3) = 368.8 ~Btu/lbm = \boldsymbol{w}_{T} \\ \boldsymbol{\eta}_{TH} &= \boldsymbol{w}_{NET}/\boldsymbol{q}_{H} = 222.7/368.8 = \boldsymbol{0.604} \end{split}$$

## 11.186<mark>E</mark>

An air-standard Ericsson cycle has an ideal regenerator as shown in Fig. P11.62. Heat is supplied at 1800 F and heat is rejected at 68 F. Pressure at the beginning of the isothermal compression process is 10 lbf/in.<sup>2</sup>. The heat added is 275 Btu/lbm. Find the compressor work, the turbine work, and the cycle efficiency.

Identify the states

Heat supplied at high temperature  $T_4 = T_3 = 1800 \text{ F} = 2349.7 \text{ R}$ Heat rejected at low temperature  $T_1 = T_2 = 68 \text{ F} = 527.7 \text{ R}$ Beginning of the compression:  $P_1 = 10 \text{ lbf/in}^2$ Ideal regenerator:  $_2q_3 = -_4q_1 \implies q_H = _3q_4 \implies$   $w_T = q_H = 275 \text{ Btu/lbm}$   $\eta_{TH} = \eta_{CARNOT TH.} = 1 - T_L/T_H = 1 - 527.7/2349.7 = 0.775$   $w_{net} = \eta_{TH} q_H = 0.775 \times 275 = 213.13 \text{ Btu/lbm}$  $q_L = -w_C = 275 - 213.13 = 61.88 \text{ Btu/lbm}$ 



## 11.187E

The turbine in a jet engine receives air at 2200 R, 220 lbf/in.<sup>2</sup>. It exhausts to a nozzle at 35 lbf/in.<sup>2</sup>, which in turn exhausts to the atmosphere at 14.7 lbf/in.<sup>2</sup>. The isentropic efficiency of the turbine is 85% and the nozzle efficiency is 95%. Find the nozzle inlet temperature and the nozzle exit velocity. Assume negligible kinetic energy out of the turbine.

Solution:

C.V. Turbine:  $h_i = 560.588 \text{ Btu/lbm}, s_{Ti}^o = 1.99765 \text{ Btu/lbm R}, s_{es} = s_i$ 

Then from Eq.8.28

$$\Rightarrow s_{\text{Tes}}^{\text{o}} = s_{\text{Ti}}^{\text{o}} + R \ln(P_{\text{e}}/P_{\text{i}}) = 1.99765 + \frac{53.34}{778} \ln(35/220) = 1.8716 \frac{\text{Btu}}{\text{lbm R}}$$

Table F.5 
$$T_{es} = 1382 \text{ R}, h_{es} = 338.27 \text{ Btu/lbm},$$

Energy eq.:  $w_{T,s} = h_i - h_{es} = 560.588 - 338.27 = 222.3 \text{ Btu/lbm}$ 

Eq.9.27: 
$$w_{T,AC} = w_{T,s} \times \eta_T = 188.96 = h_i - h_{e,AC} \implies h_{e,AC} = 371.6$$

Table F.5  $\Rightarrow$  **T**<sub>e,AC</sub> = **1509 R**,  $s_{Te}^{o} = 1.8947$  Btu/lbm R

C.V. Nozzle:  $h_i = 371.6$  Btu/lbm,  $s_{Ti}^o = 1.8947$  Btu/lbm R,  $s_{es} = s_i$ Then from Eq.8.28

$$\Rightarrow s_{Tes}^{o} = s_{Ti}^{o} + R \ln(P_e/P_i) = 1.8947 + \frac{53.34}{778} \ln(\frac{14.7}{35}) = 1.8352 \frac{Btu}{lbm R}$$
  
Table F.5  $\Rightarrow T_{e,s} = 1199.6 R$ ,  $h_{e,s} = 291.3 Btu/lbm$   
Energy Eq.:  $(1/2)V_{e,s}^2 = h_i - h_{e,s} = 371.6 - 291.3 = 80.3 Btu/lbm$   
Eq.9.30:  $(1/2)V_{e,AC}^2 = (1/2)V_{e,s}^2 \times \eta_{NOZ} = 76.29 Btu/lbm$   
 $V_{e,AC} = \sqrt{2 \times 25037 \times 76.29} = 1954 \text{ ft/s}$ 

Recall 1 Btu/lbm = 25 037 ft<sup>2</sup>/s<sup>2</sup>

## **Otto, Diesel, Stirling and Carnot Cycles**

## 11.188E

Air flows into a gasoline engine at 14 lbf/in.<sup>2</sup>, 540 R. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 560 Btu/lbm of energy is released as the fuel burns. Find the temperature and pressure after combustion.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2:  $s_2 = s_1 \implies$  From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 540 \times 8^{0.4} = 1240.6 \text{ R}$$
  
 $P_2 = P_1 \times (v_1/v_2)^k = 14 \times 8^{1.4} = 257.3 \text{ lbf/in}^2$ 

Combustion 2 to 3 at constant volume:  $u_3 = u_2 + q_H$ 

$$T_3 = T_2 + q_H/C_v = 1240.6 + 560/0.171 = 4515 R$$

$$P_3 = P_2 \times (T_3/T_2) = 257.3 (4515 / 1240.6) = 936 lbf/in^2$$



## 11.189E

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 800 Btu/lbm of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 13 lbf/in.<sup>2</sup>, 50 F. Assuming constant specific heat, with the value from Table F.4, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:



State 1:  $v_1 = RT_1/P_1 = \frac{53.34 \times 510}{13 \times 144} = 14.532 \text{ ft}^3/\text{lbm}, \ v_2 = v_1/7 = 2.076 \text{ ft}^3/\text{lbm}$ 

The compression process, reversible adiabatic so then isentropic. The constant s is implemented with Eq.8.25 leading to Eqs.8.34 and 8.32

$$P_2 = P_1 (v_1/v_2)^k = 13(7)^{1.4} = 198.2 \text{ lbf/in}^2$$
  
 $T_2 = T_1 (v_1/v_2)^{k-1} = 510(7)^{0.4} = 1110.7 \text{ R}$ 

The combustion process with constant volume,  $q_{H} = 800$  Btu/lbm

$$T_3 = T_2 + q_H/C_{V0} = 1110.7 + 800/0.171 = 5789 R$$
  
 $P_3 = P_2T_3/T_2 = 198.2 \times 5789/1110.7 = 1033 lbf/in^2$ 

Cycle efficiency from the ideal cycle as in Eq.11.18

$$\eta_{TH} = 1 - (T_1/T_2) = 1 - 510/1110.7 = 0.541$$

To get the mean effective pressure we need the net work

$$w_{\text{NET}} = \eta_{\text{TH}} \times q_{\text{H}} = 0.541 \times 800 = 432.8 \text{ Btu/lbm}$$
$$P_{\text{m eff}} = \frac{w_{\text{NET}}}{v_1 - v_2} = \frac{432.8 \times 778}{(14.532 - 2.076) \times 144} = 188 \text{ lbf/in}^2$$

## 11.190<mark>E</mark>

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 520 R, 12.2 psia in the cylinder. The combustion peak pressure is 900 psia. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:

Compression. Isentropic so we use Eqs.8.33-8.34

$$P_2 = P_1(v_1/v_2)^k = 12.2 (10)^{1.4} = 306.45 \text{ psia}$$
  
 $T_2 = T_1(v_1/v_2)^{k-1} = 520 (10)^{0.4} = 1306.2 \text{ R}$ 

Combustion. Constant volume

 $T_3 = T_2 (P_3/P_2) = 1306.2 \times 900/306.45 = 3836 R$ 

Exhaust. Isentropic expansion so from Eq.8.33

$$T_4 = T_3 / (v_1/v_2)^{k-1} = T_3 / 10^{0.4} = 3836 / 2.5119 = 1527 R$$

Overall cycle efficiency is from Eq.11.18,  $r_v = v_1/v_2$ 

$$\eta = 1 - r_v^{1\text{-}k} = 1 - 10^{\text{-}0.4} = \textbf{0.602}$$

Comment: No actual gasoline engine has an efficiency that high, maybe 35%.

## 11.191<mark>E</mark>

A for stroke gasoline engine has a compression ratio of 10:1 with 4 cylinders of total displacement 75 in<sup>3</sup>. the inlet state is 500 R, 10 psia and the engine is running at 2100 RPM with the fuel adding 750 Btu/lbm in the combustion process. What is the net work in the cycle and how much power is produced?

Solution:

Overall cycle efficiency is from Eq.11.18,  $r_v = v_1/v_2$ 

$$\begin{split} \eta_{TH} &= 1 - r_v^{1\text{-}k} = 1 - 10^{-0.4} = \textbf{0.602} \\ w_{net} &= \eta_{TH} \times q_H = 0.602 \times 750 = 451.5 \text{ Btu/lbm} \end{split}$$

We also need specific volume to evaluate Eqs.11.15 to 11.17

$$v_1 = RT_1 / P_1 = 53.34 \times 500 / (10 \times 144) = 18.52 \text{ ft}^3/\text{lbm}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1 \left(1 - \frac{1}{r_v}\right)} = \frac{451.5}{18.52 \times 0.9} \frac{778}{144} = 146.3 \text{ psia}$$

Now we can find the power from Eq.11.17

$$\dot{W} = P_{\text{meff}} V_{\text{displ}} \frac{\text{RPM}}{60} \frac{1}{2} = 146.3 \times \frac{75}{12} \times \frac{2100}{60} \times \frac{1}{2} = 16\ 002\ \text{lbf-ft/s}$$

=**29 hp** Recall 1 hp = 550 lbf-ft/s.

## 11.192E

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent n somewhat larger than the specific heat ratio k. Repeat Problem 11.189 but assume the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with n equal to 1.50.

First find states 2 and 3. based on the inlet state we get

$$v_4 = v_1 = RT_1/P_1 = 53.34 \times 510/13 \times 144 = 14.532 \text{ ft}^3/\text{lbm}$$

$$v_3 = v_2 = v_1/7 = 2.076 \text{ ft}^3/\text{lbm}$$

After compression we have constant s leads to Eq.8.34 and Eq.8.32

$$P_2 = P_1 (v_1/v_2)^k = 13(7)^{1.4} = 198.2 \text{ lbf/in}^2$$
  
 $T_2 = T_1 (v_1/v_2)^{k-1} = 510(7)^{0.4} = 1110.7 \text{ R}$ 

Constant volume combustion

$$T_3 = T_2 + q_H/C_{V0} = 1110.7 + 800/0.171 = 5789 R$$
  
 $P_3 = P_2T_3/T_2 = 198.2 \times 5789/1110.7 = 1033 lbf/in^2$ 

Process 3 to 4:  $Pv^{1.5} = constant.$ 

$$P_4 = P_3 (v_3/v_4)^{1.5} = 1033(1/7)^{1.5} = 55.78 \text{ lbf/in}^2$$
  
 $T_4 = T_3 (v_3/v_4)^{0.5} = 5789(1/7)^{0.5} = 2188 \text{ R}$ 

For the mean effective pressure we need the net work and therefore the induvidual process work terms

$$\begin{split} {}_{1}w_{2} &= \int P \; dv = R(T_{2} - T_{1})/(1 - 1.4) \\ &= -53.34(1110.7 - 510)/(0.4 \times 778) = -102.96 \; Btu/lbm \\ {}_{3}w_{4} &= \int P \; dv = R(T_{4} - T_{3})/(1 - 1.5) \\ &= -53.34(2188 - 5789)/(0.5 \times 778) = 493.8 \; Btu/lbm \\ w_{NET} &= 493.8 - 102.96 = 390.84 \; Btu/lbm \\ \eta_{CYCLE} &= w_{NET}/q_{H} = 390.84/700 = \textbf{0.488} \end{split}$$

 $P_{meff} = w_{NET} / (v_1 - v_2) = 390.84 \times 778 / (14.532 - 2.076) = 169.5 \text{ lbf/in}^2$ Notice a smaller w<sub>NET</sub>,  $\eta_{CYCLE}$ ,  $P_{meff}$  compared to ideal cycle.

## 11.193E

In the Otto cycle all the heat transfer  $q_{\rm H}$  occurs at constant volume. It is more realistic to assume that part of  $q_{\rm H}$  occurs after the piston has started its downwards motion in the expansion stroke. Therefore consider a cycle identical to the Otto cycle, except that the first two-thirds of the total  $q_{\rm H}$  occurs at constant volume and the last one-third occurs at constant pressure. Assume the total  $q_{\rm H}$  is 700 Btu/lbm, that the state at the beginning of the compression process is 13 lbf/in.<sup>2</sup>, 68 F, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.



## 11.194<mark>E</mark>

A diesel engine has a bore of 4 in., a stroke of 4.3 in. and a compression ratio of 19:1 running at 2000 RPM (revolutions per minute). Each cycle takes two revolutions and has a mean effective pressure of 200 lbf/in.<sup>2</sup>. With a total of 6 cylinders find the engine power in Btu/s and horsepower, hp.

Solution:

Work from mean effective pressure.

 $P_{meff} = w_{net} / (v_{max} - v_{min}) \rightarrow w_{net} = P_{meff} (v_{max} - v_{min})$ 

The displacement is

$$\Delta V = \pi Bore^2 \times 0.25 \times S = \pi \times 4^2 \times 0.25 \times 4.3 = 54.035 \text{ in}^3$$

Work per cylinder per power stroke

W =  $P_{meff}(V_{max} - V_{min}) = 200 \times 54.035 / (12 \times 778) = 1.1575 \text{ Btu/cycle}$ 

Only every second revolution has a power stroke so we can find the power

$$\dot{W} = W \times N_{cyl} \times RPM \times 0.5 \ (\frac{cycles}{min}) \times (\frac{min}{60 \text{ s}}) \times (\frac{Btu}{cycle})$$
$$= 1.1575 \times 6 \times 2000 \times 0.5 \times (1/60) = 115.75 \text{ Btu/s}$$
$$= 115.75 \times 3600/2544.43 \text{ hp} = 164 \text{ hp}$$

### 11.195<mark>E</mark>

At the beginning of compression in a diesel cycle T = 540 R, P = 30 lbf/in.<sup>2</sup> and the state after combustion (heat addition) is 2600 R and 1000 lbf/in.<sup>2</sup>. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Compression process (isentropic) from Eqs.8.33-8.34

$$P_2 = P_3 = 1000 \text{ lbf/in}^2 \implies v_1/v_2 = (P_2/P_1)^{1/k} = (1000/30)^{0.7143} = 12.24$$
$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 540(1000/30)^{0.2857} = 1470.6 \text{ R}$$

Expansion process (isentropic) first get the volume ratios

$$v_3/v_2 = T_3/T_2 = 2600/1470.6 = 1.768$$

$$v_4/v_3 = v_1/v_3 = (v_1/v_2)(v_2/v_3) = 12.24/1.768 = 6.923$$

The exhaust temperature follows from Eq.8.33

$$T_{4} = T_{3}(v_{3}/v_{4})^{k-1} = 2600*6.923^{-0.4} = 1199 \text{ R}$$

$$q_{L} = C_{V}(T_{4} - T_{1}) = 0.171(1199-540) = 112.7 \text{ Btu/lbm}$$

$$q_{H} = h_{3} - h_{2} = C_{P}(T_{3} - T_{2}) = 0.24(2600 - 1470.6) = 271.1 \text{ Btu/lbm}$$

$$\eta = 1 - q_{L}/q_{H} = 1 - 112.7 / 271.1 = 0.5843$$

$$w_{net} = q_{net} = 271.1 - 112.7 = 158.4 \text{ Btu/lbm}$$

$$v_{max} = v_{1} = RT_{1}/P_{1} = 53.34 \times 540/(30 \times 144) = 6.6675 \text{ ft}^{3}/\text{lbm}$$

$$v_{min} = v_{max}(v_{1}/v_{2}) = 6.6675 / 12.24 = 0.545 \text{ ft}^{3}/\text{lbm}$$

$$P_{meff} = [158.4/(6.6675 - 0.545)] \times (778/144) = 139.8 \text{ lbf/in}^{2}$$



Remark: This is a too low compression ratio for a practical diesel cycle.

## 11.196<mark>E</mark>

Consider an ideal air-standard diesel cycle where the state before the compression process is 14 lbf/in.<sup>2</sup>, 63 F and the compression ratio is 20. Find the maximum temperature(by iteration) in the cycle to have a thermal efficiency of 60%.

Diesel cycle:  $P_1 = 14$ ,  $T_1 = 522.67 \text{ R}$ ,  $v_1/v_2 = 20$ ,  $\eta_{TH} = 0.60$ 

From the inlet state and the compression we get

$$T_{2} = T_{1}(v_{1}/v_{2})^{k-1} = 522.67(20)^{0.4} = 1732.4 \text{ R}$$
$$v_{1} = \frac{53.34 \times 522.67}{14 \times 144} = 13.829 \text{ ft}^{3}/\text{lbm}, \quad v_{2} = \frac{13.829}{20} = 0.6915 \text{ ft}^{3}/\text{lbm}$$

Constant pressure combustion relates  $v_3$  and  $T_3$ 

$$v_3 = v_2 \times T_3 / T_2 = 0.6915 \times T_3 / 1732.4 = 0.000399 T_3$$

The expansion then gives  $T_4$  interms of  $T_3$ 

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} = \left(\frac{13.829}{0.000399 T_3}\right)^{0.4} \to T_4 = 0.0153 T_3^{1.4}$$

Now these T's relate to the given efficiency

$$\eta_{\rm TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.0153 T_3^{1.4} - 522.67}{1.4(T_3 - 1732.4)}$$
$$\Rightarrow \quad 0.0153 T_3^{1.4} - 0.56 T_3 + 447.5 = 0$$

Trial and error on this non-linear equation

5100 R: LHS = -35.54, 5500 R: LHS = 5.04, 5450 R: LHS = -0.5  
Linear interpolation, 
$$T_3 = 5455 R$$

## 11.197<mark>E</mark>

Consider an ideal Stirling-cycle engine in which the pressure and temperature at the beginning of the isothermal compression process are 14.7 lbf/in.<sup>2</sup>, 80 F, the compression ratio is 6, and the maximum temperature in the cycle is 2000 F. Calculate the maximum pressure in the cycle and the thermal efficiency of the cycle with and without regenerators.



## 11.198E

An ideal air-standard Stirling cycle uses helium as working fluid. The isothermal compression brings the helium from 15 lbf/in.<sup>2</sup>, 70 F to 90 lbf/in.<sup>2</sup>. The exspansion takes place at 2100 R and there is no regenerator. Find the work and heat transfer in all four processes per lbm helium and the cycle efficiency.

Substance helium F.4:  $R = 386 \text{ ft-lbf/lbmR}, C_v = 0.753 \text{ Btu/lbm R}$   $v_4/v_3 = v_1/v_2 = P_2/P_1 = 90/15 = 6$   $1 \rightarrow 2: -_1w_2 = -_1q_2 = \int P \, dV = RT \ln (v_1/v_2)$   $= 386 \times 530 \times \ln(6)/778 = 471.15 \text{ Btu/lbm}$   $2 \rightarrow 3: 2w_3 = 0; 2q_3 = C_P(T_3 - T_2) = 0.753(2100 - 530) = 1182.2$   $3 \rightarrow 4: 3w_4 = 3q_4 = RT_3 \ln(v_4/v_3) = 386 \times 2100 \times \ln(6)/778$  = 1866.8 Btu/lbm  $4 \rightarrow 1: 4w_1 = 0; 4q_1 = C_P(T_4 - T_1) = -1182.2 \text{ Btu/lbm}$  $\eta_{Cycle} = w_{net}/q_H = \frac{-471.15 + 1866.0}{1182.2 + 1866.8} = 0.458$
### 11.199E

The air-standard Carnot cycle was not shown in the text; show the T-s diagram for this cycle. In an air-standard Carnot cycle the low temperature is 500 R and the efficiency is 60%. If the pressure before compression and after heat rejection is 14.7 lbf/in.<sup>2</sup>, find the high temperature and the pressure just before heat addition.

Solution:

Carnot cycle efficiency from Eq.7.5

$$\eta = 0.6 = 1 - T_H/T_L$$
  
 $\Rightarrow T_H = T_L/0.4 = 500/0.4 = 1250 R$ 

Just before heat addition is state 2 and after heat rejection is state 1 so  $P_1 = 100$  kPa and the isentropic compression is from Eq.8.32

$$P_2 = P_1(T_H/T_L)^{\frac{1}{k-1}} = 14.7(\frac{1250}{500})^{3.5} = 363.2 \text{ lbf/in}^2$$

**OR** if we do not use constant specific heat, but use Table F.5 in Eq.8.28

$$P_2 = P_1 \exp[(s_{T2}^{\circ} - s_{T1}^{\circ})/R] = 14.7 \times \exp[\frac{1.84573 - 1.62115}{53.34 / 778}] = 389 \text{ lbf/in}^2$$



# 11.200E

Air in a piston/cylinder goes through a Carnot cycle in which  $T_L = 80.3$  F and the total cycle efficiency is  $\eta = 2/3$ . Find  $T_H$ , the specific work and volume ratio in the adiabatic expansion for constant  $C_p$ ,  $C_v$ .

Carnot cycle:

$$\eta = 1 - T_L/T_H = 2/3 \implies T_H = 3 \times T_L = 3 \times 540 = 1620 \text{ R}$$
  
Adiabatic expansion 3 to 4:  $Pv^k = \text{constant}$   
 ${}_{3}w_4 = (P_4v_4 - P_3v_3)/(1 - k) = [R/(1-k)](T_4 - T_3) = u_3 - u_4$   
 $= C_v(T_3 - T_4) = 0.171(1620 - 540) = 184.68 \text{ Btu/lbm}$   
 $v_4/v_3 = (T_3/T_4)^{1/(k-1)} = 3^{2.5} = 15.6$ 



## 11.201E

Do the previous problem 11.200E using Table F.5.

Air in a piston/cylinder goes through a Carnot cycle in which  $T_L = 80.3$  F and the total cycle efficiency is  $\eta = 2/3$ . Find  $T_H$ , the specific work and volume ratio in the adiabatic expansion for constant  $C_p$ ,  $C_v$ .

Carnot cycle:

$$\eta = 1 - T_L/T_H = 2/3 \implies T_H = 3 \times T_L = 3 \times 540 = 1620 \text{ R}$$
  
 $_3w_4 = u_3 - u_4 = 290.13 - 92.16 = 197.97 \text{ Btu/lbm}$ 

Adiabatic expansion 3 to 4:  $s_4 = s_3 \implies Eq.8.28$ 

$$s_{T4}^{o} = s_{T3}^{o} + R \ln \frac{P_4}{P_3} \implies \text{Table F.5 for standard entropy}$$
  
 $\frac{P_4}{P_3} = \exp[(s_{T4}^{o} - s_{T3}^{o})/R] = \exp[\frac{1.63979 - 1.91362}{53.34/778}] = 0.018426$ 

Ideal gas law then gives

$$\frac{v_4}{v_3} = \frac{T_4}{T_3} \times \frac{P_3}{P_4} = \frac{540}{1620} \times \frac{1}{0.018426} = \mathbf{18.09}$$



# **Refrigeration Cycles**

## 11.202E

A car air-conditioner (refrigerator) in 70 F ambient uses R-134a and I want to have cold air at 20 F produced. What is the minimum high P and the maximum low P it can use?

Since the R-134a must give heat transfer out to the ambient at 70 F, it must at least be that hot at state 3.

From Table F.10.1:  $P_3 = P_2 = P_{sat} = 85.95$  psia is minimum high P. Since the R-134a must absorb heat transfer at the cold air 20 F, it must at least be that cold at state 4.

From Table F.10.1:  $P_1 = P_4 = P_{sat} = 33.29$  psia is maximum low P.

Ideal Ref. Cycle  $T_{cond} = 70 F = T_3$  $T_{evap} = 20 F$ 

Use Table F.10 for R-134a



# 11.203E

Consider an ideal refrigeration cycle that has a condenser temperature of 110 F and an evaporator temperature of 5 F. Determine the coefficient of performance of this refrigerator for the working fluids R-12 and R-22.

| Ideal Ref. Cycle<br>$T_{cond} = 110 F = T_3$<br>$T_{evap} = 5 F$<br>Use Table F.9 for R-22<br>Use computer table for R-12 |         |         |  |  |  |  |  |
|---|---------|---------|--|--|--|--|--|
|   | R-12    | R-22    |  |  |  |  |  |
| h <sub>1</sub> , Btu/lbm  | 77.803  | 104.954 |  |  |  |  |  |
| $s_2 = s_1$   | 0.16843 | 0.22705 |  |  |  |  |  |
| $P_2$ , lbf/in <sup>2</sup>   | 151.11  | 241.04  |  |  |  |  |  |
| $T_2, F$  | 127.29  | 161.87  |  |  |  |  |  |
| h <sub>2</sub> , Btu/lbm  | 91.107  | 123.904 |  |  |  |  |  |
| $h_3 = h_4$ , Btu/lbm   | 33.531  | 42.446  |  |  |  |  |  |
| $-w_{C} = h_{2} - h_{1}$  | 13.3    | 18.95   |  |  |  |  |  |
| $q_{L} = h_{1} - h_{4}$   | 44.27   | 62.51   |  |  |  |  |  |
| $\beta = q_L/(-w_C)$  | 3.33    | 3.30    |  |  |  |  |  |

## 11.204E

The environmentally safe refrigerant R-134a is one of the replacements for R-12 in refrigeration systems. Repeat Problem 11.203 using R-134a and compare the result with that for R-12.



C.V. Compressor: Adiabatic and reversible so constant s State 1: Table F.10.1  $h_1 = 167.32 \text{ Btu/lbm}, s_1 = 0.4145 \text{ Btu/lbm R}$ State 2:  $s_2 = s_1$  and  $P_2 = 161.1 \text{ psia} = P_3 = P_{\text{sat } 110 \text{ F}}$ Interpolate  $=> h_2 = 184.36 \text{ Btu/lbm}$  and  $T_2 = 121.8 \text{ F}$ Energy eq.:  $w_C = h_2 - h_1 = 184.36 - 167.32 = 17.04 \text{ Btu/lbm}$ Expansion valve:  $h_3 = h_4 = 112.46 \text{ Btu/lbm}$ Evaporator:  $q_L = h_1 - h_4 = 167.32 - 112.46 = 54.86 \text{ Btu/lbm}$ Overall performance, COP  $\beta = q_L/w_C = 54.86 / 17.04 = 3.22$ 

# 11.205E

Consider an ideal heat pump that has a condenser temperature of 120 F and an evaporator temperature of 30 F. Determine the coefficient of performance of this heat pump for the working fluids R-12, R-22, and ammonia.

| Ideal Heat Pump<br>$T_{cond} = 120 \text{ F}$<br>$T_{evap} = 30 \text{ F}$<br>Use Table F.8 for N<br>Use Table F.9 for R | 2        |        |                 |  |
|--|----------|--------|-----------------|--|
| Use computer table   | for R-12 | S      |                 |  |
|  | R-12     | R-22   | NH <sub>3</sub> |  |
| h <sub>1</sub> , Btu/lbm   | 80.42    | 107.28 | 619.58          |  |
| $\mathbf{s}_2 = \mathbf{s}_1$  | 0.1665   | 0.2218 | 1.2769          |  |
| $P_2$ , lbf/in <sup>2</sup>  | 172.3    | 274.6  | 286.5           |  |
| T <sub>2</sub> , F   | 132.2    | 160.4  | 239.4           |  |
| h <sub>2</sub> , Btu/lbm   | 91.0     | 122.17 | 719.5           |  |
| h <sub>3</sub> =h <sub>4</sub> , Btu/lbm   | 36.011   | 45.71  | 178.83          |  |
| $-w_{C} = h_{2} - h_{1}$   | 10.58    | 14.89  | 99.92           |  |
| $q_{\rm H} = h_2 - h_3$  | 54.995   | 76.46  | 540.67          |  |
| $\beta' = q_H'(-w_C)$  | 5.198    | 5.135  | 5.411           |  |

## 11.206E

The refrigerant R-22 is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 50 F; its exit temperature from the compressor is measured and found to be 185 F. If the compressor exit is 300 psia, what is the isentropic efficiency of the compressor and the coefficient of performance of the heat pump?

R-22 heat pump:  $T_2 = 185 \text{ F}$  $T_{EVAP} = 50 \text{ F}$ State 1: Table F.9.1

 $h_1 = 108.95 \text{ Btu/lbm},$  $s_1 = 0.2180 \text{ Btu/lbm R}$ 



State 2:  $h_2 = 126.525$  Btu/lbm Compressor work:  $w_C = h_2 - h_1 = 126.525 - 108.95 = 17.575$  Btu/lbm Isentropic compressor:  $s_{2S} = s_1 = 0.2180$  Btu/lbm R State 2s:  $(P_2, s)$   $T_{2S} = 160$  F,  $h_{2S} = 120.82$  Btu/lbm Ideal compressor work:  $w_{Cs} = h_{2S} - h_1 = 120.82 - 108.95 = 11.87$  Btu/lbm

The efficiency is the ratio of the two work terms

$$\eta_{\rm S \ COMP} = \frac{W_{\rm C \ s}}{W_{\rm C}} = \frac{11.87}{17.575} = 0.675$$

The condenser has heat transfer as  $(h_3 = h_f at 300 psia)$ 

$$q_{\rm H} = h_2 - h_3 = 126.525 - 48.02 = 78.505 \text{ Btu/lbm}$$

and a coefficient of performance of

$$\beta' = q_{H}^{}/w_{C}^{} = 4.47$$

#### 11.207E

Consider an air standard refrigeration cycle that has a heat exchanger included as shown in Fig. P11.137. The low pressure is 14.7 psia and the high pressure is 200 psia. The temperature into the compressor is 60 F which is  $T_1$  and  $T_3$  in

Fig.11.38, and  $T_4 = T_6 = -60$  F. Determine the coefficienct of performance of this cycle.

Solution:



Standard air refrigeration cycle with

 $T_1 = T_3 = 60 \text{ F} = 519.67 \text{ R}, P_1 = 14.7 \text{ psia}, P_2 = 200 \text{ psia}$  $T_4 = T_6 = -60 \text{ F} = 399.67 \text{ R}$ 

We will solve the problem with cold air properties.

Compressor, isentropic  $s_2 = s_1$  so from Eq.8.32

$$\Rightarrow T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 519.67 (200/14.7)^{0.2857} = 1095.5 \text{ R}$$
  
w<sub>C</sub> = -w<sub>12</sub> = C<sub>P0</sub>(T<sub>2</sub> - T<sub>1</sub>) = 0.24 (1095.5 - 519.67) = 138.2 Btu/lbm

Expansion in expander (turbine)

$$s_5 = s_4 \implies T_5 = T_4 (P_5/P_4)^{\frac{k-1}{k}} = 399.67 (14.7/200)^{0.2857} = 189.58 \text{ R}$$
  
 $w_E = C_{P0}(T_4 - T_5) = 0.24 (399.67 - 189.58) = 50.42 \text{ Btu/lbm}$ 

Net cycle work

 $w_{NET} = 50.42 - 138.2 = -87.78 \text{ kJ/kg}$ 

$$q_L = C_{P0}(T_6 - T_5) = w_E = 50.42 \text{ Btu/lbm}$$

Overall cycle performance, COP

 $\beta = q_L/(-w_{NET}) = 50.42 / 87.78 = 0.574$ 

## Availability and Combined Cycles

#### 11.208E

Find the flows and fluxes of exergy in the condenser of Problem 11.172E. Use those to determine the  $2^{nd}$  law efficiency.

A smaller power plant produces 50 lbm/s steam at 400 psia, 1100 F, in the boiler. It cools the condenser with ocean water coming in at 55 F and returned at 60 F so that the condenser exit is at 110 F. Find the net power output and the required mass flow rate of the ocean water.

Solution:

Take the reference state at the ocean temperature 55 F = 514.7 R

The states properties from Tables F.7.1 and F.7.2. Ref. state 14.7 lbf/in<sup>2</sup>, 55 F,  $h_0 = 23.06$  Btu/lbm,  $s_0 = 0.0458$  Btu/lbm R



State 1: 110 F, x = 0:  $h_1 = 78.01$  Btu/lbm,  $s_1 = 0.1473$  Btu/lbm R, State 3: 400 psia, 1100 F:  $h_3 = 1577.44$  Btu/lbm,  $s_3 = 1.7989$  Btu/lbm R C.V. Turbine :  $w_T = h_3 - h_4$ ;  $s_4 = s_3$ 

$$s_4 = s_3 = 1.7989 = 0.1473 + x_4 (1.8101) \implies x_4 = 0.9124$$
  
 $\implies h_4 = 78.01 + 0.9124 (1031.28) = 1018.95 \text{ Btu/lbm}$ 

C.V. Condenser :  $q_L = h_4 - h_1 = 1018.95 - 78.01 = 940.94$  Btu/lbm

$$\dot{\mathbf{Q}}_{\mathrm{L}} = \dot{\mathbf{m}}\mathbf{q}_{\mathrm{L}} = 50 \times 940.94 = 47\ 047\ \mathrm{Btu/s} = \dot{\mathbf{m}}_{\mathrm{ocean}}\ \mathrm{C}_{\mathrm{p}}\ \Delta\mathrm{T}$$

$$\dot{m}_{ocean} = Q_L / C_p \Delta T = 47.047 / (1.0 \times 5) = 9409 \text{ lbm/s}$$

The specific flow exergy for the two states are from Eq.10.24 neglecting kinetic and potential energy

$$\psi_4 = h_4 - h_0 - T_0(s_4 - s_0), \quad \psi_1 = h_1 - h_0 - T_0(s_1 - s_0)$$

The net drop in exergy of the water is

$$\dot{\Phi}_{water} = \dot{m}_{water} [h_4 - h_1 - T_0(s_4 - s_1)]$$
  
= 50 [ 1018.95 - 78.01 - 514.7 (1.7989 - 0.1473)]  
= 47 047 - 42 504 = **4543 Btu/s**

The net gain in exergy of the ocean water is

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean} [h_6 - h_5 - T_0 (s_6 - s_5)]$$

$$= \dot{m}_{ocean} [C_p (T_6 - T_5) - T_0 C_p \ln(\frac{T_6}{T_5})]$$

$$= 9409 [1.0 (60 - 55) - 514.7 \times 1.0 \ln \frac{459.7 + 60}{459.7 + 55}]$$

$$= 47 047 - 46 818 = 229 \text{ Btu/s}$$
The second law efficiency is

 $\eta_{\rm II} = \dot{\Phi}_{\rm ocean} / \dot{\Phi}_{\rm water} = \frac{229}{4543} = 0.05$ 

In reality all the exergy in the ocean water is destroyed as the 60 F water mixes with the ocean water at 55 F after it flows back out into the ocean and the efficiency does not have any significance. Notice the small rate of exergy relative to the large rates of energy being transferred.

### 11.209E

(Adv.) Find the availability of the water at all four states in the Rankine cycle described in Problem 11.173. Assume the high-temperature source is 900 F and the low-temperature reservoir is at 65 F. Determine the flow of availability in or out of the reservoirs per pound-mass of steam flowing in the cycle. What is the overall cycle second law efficiency?

Ref. state 14.7 lbf/in<sup>2</sup>, 77°F,  $h_0 = 45.08$  Btu/lbm,  $s_0 = 0.08774$  Btu/lbm R For this cycle from Table F.7 State 3: Superheated vapor  $h_3 = 1350.62$  Btu/lbm,  $s_3 = 1.5871$  Btu/lbm R,  $h_1 = 97.97$  Btu/lbm,  $v_1 = 0.01625$  ft<sup>3</sup>/lbm State 1: Saturated liquid C.V. Pump: Adiabatic and reversible. Use incompressible fluid so  $w_{p} = \int v dP = v_{1}(P_{2} - P_{1}) = 0.01625(600 - 2.2)\frac{144}{778} = 1.8 \text{ Btu/lbm}$  $h_2 = h_1 + w_p = 95.81 \text{ Btu/lbm}$  $q_{\rm H} = h_3 - h_2 = 1350.62 - 97.97 = 1252.65$  Btu/lbm C.V. Boiler:  $w_{T} = h_{3} - h_{4}, \qquad s_{4} = s_{3}$ C.V. Tubine:  $s_4 = s_3 = 1.5871$  Btu/lbm R = 0.1817 +  $x_4 1.7292 \implies x_4 = 0.8127$ ,  $h_{A} = 97.97 + 0.8127 \times 1019.78 = 926.75 \text{ Btu/lbm}$  $w_{T} = 1350.62 - 926.75 = 423.87 \text{ Btu/lbm}$  $\eta_{\text{CYCLE}} = (w_{\text{T}} - w_{\text{P}})/q_{\text{H}} = (423.87 - 1.8)/1252.65 = 0.337$ C.V. Condenser:  $q_{I} = h_{A} - h_{1} = 926.75 - 97.97 = 828.8 \text{ Btu/lbm}$ 



From solution to 11.121:

 $s_1 = 0.17497$ ,  $s_2 = 0.175 = s_1$ ,  $s_4 = s_3 = 1.5871$  Btu/lbm R  $h_1 = 94.01$ ,  $h_2 = 95.81$ ,  $h_3 = 1350.6$ ,  $h_4 = 921.23$  Btu/lbm

$$\psi = h - h_0 - T_0(s - s_0)$$
  

$$\psi_1 = 94.01 - 45.08 - 536.67(0.17497 - 0.08774) = 2.116 \text{ Btu/lbm}$$
  

$$\psi_2 = 3.92, \quad \psi_3 = 500.86, \quad \psi_4 = 71.49 \text{ Btu/lbm}$$
  

$$\Delta \psi_H = (1 - T_0/T_H)q_H = 0.6054 \times 1254.79 = 759.65 \text{ Btu/lbm}$$
  

$$\Delta \psi_L = (1 - T_0/T_0)q_C = \emptyset$$
  

$$\eta_{II} = w_{NET}/\Delta \psi_H = (429.37 - 1.8)/759.65 = 0.563$$

Notice  $T_H > T_3$ ,  $T_L < T_4 = T_1$ , so cycle is externally irreversible. Both  $q_H$  and  $q_C$  over finite  $\Delta T$ .

#### 11.210E

Find the flows of exergy into and out of the feedwater heater in Problem 11.176E.

State 1:  $x_1 = 0$ ,  $h_1 = 97.97$  Btu/lbm,  $v_1 = 0.01625$  ft<sup>3</sup>/lbm, s = 0.17497State 3:  $x_3 = 0$ ,  $h_3 = 330.67$  Btu/lbm,  $s_3 = 0.49199$  Btu/lbm R State 5:  $h_5 = 1350.52$  Btu/lbm,  $s_5 = 1.5871$  Btu/lbm R State 6:  $s_6 = s_5 = 1.5871$  Btu/lbm R  $=> h_6 = 1208.93$  Btu/lbm C.V Pump P1  $w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.01625(150 - 2.225)\frac{144}{778} = 0.44$  Btu/lbm  $=> h_2 = h_1 + w_{P1} = 97.97 + 0.4439 = 98.41$  Btu/lbm  $s_2 = s_1 = 0.17497$  Btu/lbm R

C.V. Feedwater heater: Call  $\dot{m}_6 / \dot{m}_{tot} = x$  (the extraction fraction) Energy Eq.:  $(1 - x) h_2 + x h_6 = 1 h_3$ 



Ref. State: 14.7 psia, 77 F,  $s_0 = 0.08774$  Btu/lbm R,  $h_0 = 45.08$  Btu/lbm  $\psi_2 = h_2 - h_0 - T_0(s_2 - s_0)$  = 98.41 - 45.08 - 536.67(0.17497 - 0.08774) = 6.52 Btu/lbm  $\psi_6 = 1208.93 - 45.08 - 536.67(1.5871 - 0.08774) = 359.2$  Btu/lbm  $\psi_3 = 330.67 - 45.08 - 536.67(0.49199 - 0.08774) = 68.64$  Btu/lbm

The rate of exergy flow scaled with maximum flow rate is then

 $\dot{\Phi}_2/\dot{m}_3 = (1 - x) \psi_2 = 0.7909 \times 6.52 = 5.157 \text{ Btu/lbm}$  $\dot{\Phi}_6/\dot{m}_3 = x\psi_6 = 0.2091 \times 359.2 = 75.109 \text{ Btu/lbm}$  $\dot{\Phi}_3/\dot{m}_3 = \psi_3 = 68.64 \text{ Btu/lbm}$ 

The mixing is destroying 5.157 + 75.109 - 68.64 = 11.6 Btu/lbm of exergy

## 11.211**E**

Consider the Brayton cycle in problem 11.183E. Find all the flows and fluxes of exergy and find the overall cycle second-law efficiency. Assume the heat transfers are internally reversible processes and we then neglect any external irreversibility.

Solution:

Efficiency is from Eq.11.8

$$\eta = \dot{W}_{NET} / \dot{Q}_{H} = \frac{W_{net}}{q_{H}} = 1 - r_{p}^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

from the required power we can find the needed heat transfer

$$\dot{Q}_{H} = \dot{W}_{net} / \eta = 14\ 000 / 0.547 = 25\ 594\ Btu/s$$
  
 $\dot{m} = \dot{Q}_{H} / q_{H} = 25\ 594\ (Btu/s) / 400\ Btu/lbm = 63.99\ lbm/s$ 

Temperature after compression is

$$T_2 = T_1 r_p^{(k-1)/k} = 519.67 \times 16^{0.4/1.4} = 1148 R$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 1148 + \frac{400}{0.24} = 2815 R$$

For the exit flow I need the exhaust temperature

$$T_4 = T_3 r_p^{-(k-1)/k} = 2815 \times 16^{-0.2857} = 1274.8 R$$

The high T exergy input from combustion is

$$\dot{\Phi}_{\rm H} = \dot{m}(\psi_3 - \psi_2) = \dot{m}[h_3 - h_2 - T(s_3 - s_2)]$$
  
= 63.99 [400 - 536.67 × 0.24 ln ( $\frac{2815}{1148}$ )] = **17 895 Btu/s**

Since the low T exergy flow out is lost the second law efficiency is

$$\eta_{\rm II} = \dot{W}_{\rm NET} / \dot{\Phi}_{\rm H} = 14\ 000\ /\ 17\ 895 = 0.782$$

$$\dot{\Phi}_{\rm flow\ out} = \dot{m}(\psi_4 - \psi_0) = \dot{m}[h_4 - h_0 - T(s_4 - s_0)]$$

$$= 63.99\ [\ 0.24(1274.8 - 536.7) - 536.7 \times 0.24\ \ln\left(\frac{1274.8}{536.7}\right)\ ] = 4205\ \text{Btu/s}$$

$$\dot{\Phi}_{\rm flow\ in} = \dot{m}(\psi_1 - \psi_0) = \dot{m}[h_1 - h_0 - T(s_1 - s_0)]$$

$$= 63.99\ [\ 0.24(60 - 77) - 536.7 \times 0.24\ \ln\left(\frac{519.7}{536.7}\right)\ ] = 4.2\ \text{Btu/s}$$

## 11.212E

Consider an ideal dual-loop heat-powered refrigeration cycle using R-12 as the working fluid, as shown in Fig. P11.144. Saturated vapor at 220 F leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at 0 F leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the condenser at 110 F is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to that through the refrigeration loop. Find also the performance of the cycle, in terms of the ratio  $Q_I/Q_{H'}$ .



|   | Т   | Р                   | h       | S         | Computer tables for            |
|---|-----|---------------------|---------|-----------|--------------------------------|
|   | F   | lbf/in <sup>2</sup> | Btu/lbm | Btu/lbm R | properties.                    |
| 1 | 0   | 23.849              | 77.271  | 168.88    | $P_2 = P_3 = P_{SAT}$ at 110 F |
| 2 | -   | 151.11              |         | 168.88    | $P_5 = P_6 = P_{SAT}$ at 220 F |
| 3 | 110 | 151.11              | 33.531  | 0.067 45  | $s_2 = s_1 = 0.168\ 88$        |
| 4 | 0   | 23.849              | 33.531  |           | h <sub>2</sub> =91.277         |
| 5 | -   | 524.43              |         | 0.067 45  | Pump work:                     |
| 6 | 220 | 524.43              | 89.036  | 0.151 49  | $-w_{p} = h_{5} - h_{3}$       |
| 7 | 110 | 151.11              |         | 0.151 49  | $\approx v_5(P_5 - P_3)$       |

 $-w_{p} = 0.0129(524.4 - 151.1)\frac{144}{778} = 0.894$   $h_{5} = 33.531 + 0.894 = 34.425 \text{ Btu/lbm}$   $(1-x_{7}) = \frac{0.162}{0.095} \frac{79}{34} - 0.151}{0.095} \frac{49}{34} = \frac{0.011}{0.095} \frac{30}{34} = 0.1187$   $h_{7} = 87.844 - 0.1187(54.313) = 81.397 \text{ Btu/lbm}$ 

CV: turbine + compressor

Continuity Eq.:  $\dot{m}_1 = \dot{m}_2$ ,  $\dot{m}_6 = \dot{m}_7$ Energy Eq.:  $\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_7 h_7$   $\dot{m}_1 / \dot{m}_6 = \frac{89.036 - 81.397}{91.277 - 77.271} = \frac{7.639}{14.006} = 0.545$ ,  $\dot{m}_6 / \dot{m}_1 = 1.833$ CV: pump:  $-w_p = v_3 (P_5 - P_3)$ ,  $h_5 = h_3 - w_p$ 

CV evaporator:  $\dot{Q}_L = \dot{m}_1(h_1 - h_4)$ , CV boiler:  $\dot{Q}_H = \dot{m}_6(h_6 - h_5)$ 

$$\Rightarrow \beta = \dot{Q}_{L} / \dot{Q}_{H} = \frac{\dot{m}_{1} (h_{1} - h_{4})}{\dot{m}_{6} (h_{6} - h_{5})} = \frac{77.271 - 33.531}{1.833(89.036 - 34.425)} = 0.436$$

### 11.213E

Consider an ideal combined reheat and regenerative cycle in which steam enters the high-pressure turbine at 500 lbf/in.<sup>2</sup>, 700 F, and is extracted to an open feedwater heater at 120 lbf/in.<sup>2</sup> with exit as saturated liquid. The remainder of the steam is reheated to 700 F at this pressure, 120 lbf/in.<sup>2</sup>, and is fed to the low-pressure turbine. The condenser pressure is 2 lbf/in.<sup>2</sup>. Calculate the thermal efficiency of the cycle and the net work per pound-mass of steam.



 $\eta_{cvcle} = w_{net} / q_H = 421.15 / 1178.2 = 0.357$ 

# 11.214<mark>E</mark>

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 700 lbf/in.<sup>2</sup> exits this heat exchanger and is then superheated to 1100 F in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 60 lbf/in.<sup>2</sup>. The isentropic turbine efficiency is 87%, and the condenser pressure is 1 lbf/in.<sup>2</sup>. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1000 Btu/s.



$$\begin{split} -w_{P12} &= 0.016136(60 - 1)144/778 = 0.18 \text{ Btu/lbm} \\ h_2 &= h_1 - w_{P12} = 69.73 + 0.18 = 69.91 \text{ Btu/lbm} \\ -w_{P34} &= 0.017378(700 - 60)144/778 = 2.06 \text{ Btu/lbm} \\ h_4 &= h_3 - w_{P34} = 262.24 + 2.06 = 264.3 \text{ Btu/lbm} \\ s_{7S} &= s_6 = 1.7682, P_7 \implies T_{7S} = 500.8 \text{ F}, \quad h_{7S} = 1283.4 \\ h_7 &= h_6 - \eta_{ST}(h_6 - h_{7S}) = 1625.8 - 0.87(1625.8 - 1283.4) = 1327.9 \\ s_{8S} &= s_6 = 1.7682 = 0.13264 + x_{8S} \times 1.8453 \implies x_{8S} = 0.8863 \\ h_{8S} &= 69.73 + 0.8863 \times 1036 = 987.9 \text{ Btu/lbm} \\ h_8 &= h_6 - \eta_{ST}(h_6 - h_{8S}) = 1625.8 - 0.87(1625.8 - 987.9) = 1070.8 \end{split}$$

CV: heater: cont:  $m_2 + m_7 = m_3 = 1.0$  lbm, 1st law:  $m_2h_2 + m_7h_7 = m_3h_3$  $m_7 = (262.24-69.91) / (1327.9-69.91) = 0.1529$ 

CV: turbine:  $w_T = (h_6 - h_7) + (1 - m_7)(h_7 - h_8)$ 

= 1625.8-1327.9 + 0.8471(1327.9-1070.8) = 515.7 Btu/lbm

CV pumps:  $w_p = m_1 w_{P12} + m_3 w_{P34} = -(0.8471 \times 0.18 + 1 \times 2.06) = -2.2 \text{ Btu/lbm}$ 

$$w_{NET} = 515.7 - 2.2 = 513.5 \text{ Btu/lbm} \implies \dot{m} = 1000/513.5 = 1.947 \text{ lbm/s}$$

CV: reactor  $\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 1.947(1202 - 264.3) = 1825.7 \text{ Btu/s}$ CV: superheater  $\dot{Q}_{SUP} = \dot{m}(h_6 - h_5) = 1.947(1625.8 - 1202) = 825 \text{ Btu/s}$ 

## 11.215<mark>E</mark>

Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance to the first compressor is 14 lbf/in.<sup>2</sup>, the temperature entering each compressor is 70 F, and the temperature entering each turbine is 2000 F. An ideal regenerator is also incorporated into the cycle. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.



Assume const. specific heat

$$s_2 = s_1 \text{ and } s_4 = s_3$$
  
 $T_4 = T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 529.67(8)^{0.2857} = 959.4$ 

7/9

s

Total compressor work

 $-w_{C} = 2 \times (-w_{12}) = 2C_{P0}(T_{2} - T_{1}) = 2 \times 0.24(959.4 - 529.67) = 206.3 \text{ Btu/lbm}$ Also  $s_{6} = s_{7}$  and  $s_{8} = s_{9}$ 

R

$$\Rightarrow T_7 = T_9 = T_6 \left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 2459.67 \left(\frac{1}{8}\right)^{0.2857} = 1357.9 \text{ R}$$

Total turbine work

 $w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 0.24(2459.67 - 1357.9) = 528.85 \text{ Btu/lbm}$  $w_{NET} = 528.85 - 206.3 = 322.55 \text{ Btu/lbm}$ 

Ideal regenerator:  $T_5 = T_9$ ,  $T_{10} = T_4$ 

$$\Rightarrow q_{\rm H} = (h_6 - h_5) + (h_8 - h_7) = 2C_{\rm P0}(T_6 - T_5)$$
$$= 2 \times 0.24(2459.67 - 1357.9) = w_{\rm T} = 528.85 \text{ Btu/lbm}$$
$$\eta_{\rm TH} = w_{\rm NET}/q_{\rm H} = 322.55/528.85 = 0.61$$

# 11.216<mark>E</mark>

Repeat Problem 11.215, but assume that each compressor stage and each turbine stage has an isentropic efficiency of 85%. Also assume that the regenerator has an efficiency of 70%.





$$\begin{split} & w_{T} = \eta_{T} w_{TS} = 449.5 \text{ Btu/lbm} \\ & T_{7} = T_{9} = T_{6} - (+w_{67}/C_{P0}) = 2459.67 - 449.5/2 \times 0.24 = 1523 \text{ R} \\ & \eta_{REG} = \frac{h_{5} - h_{4}}{h_{9} - h_{4}} = \frac{T_{5} - T_{4}}{T_{9} - T_{4}} = \frac{T_{5} - 1035.3}{1523 - 1035.3} = 0.7 \implies T_{5} = 1376.7 \text{ R} \\ & q_{H} = C_{P0}(T_{6} - T_{5}) + C_{P0}(T_{8} - T_{7}) \\ & = 0.24(2459.67 - 1376.7) + 0.24(2459.67 - 1523) = 484.7 \text{ Btu/lbm} \\ & w_{NET} = w_{T} + w_{C} = 449.5 - 242.7 = 206.8 \text{ Btu/lbm} \\ & \eta_{TH} = w_{NET}/q_{H} = 206.8/484.7 = 0.427 \end{split}$$

## 11.217E

Consider a small ammonia absorption refrigeration cycle that is powered by solar energy and is to be used as an air conditioner. Saturated vapor ammonia leaves the generator at 120 F, and saturated vapor leaves the evaporator at 50 F. If 3000 Btu of heat is required in the generator (solar collector) per pound-mass of ammonia vapor generated, determine the overall performance of this system.



= 445.49 Btu/lbm 
$$\Rightarrow$$
 q<sub>L</sub>/q<sub>H</sub> = 445.49/3000 = **0.1485**