SOLUTION MANUAL
SI UNIT PROBLEMS
CHAPTER 11

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FUNDAMENTALS

of

Thermodynamics

Sixth Edition

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Problems re-solved with the Pr, vr functions from A.7.2:
79, 81, 93, 94, 100, 103, 110, 118

CORRESPONDANCE TABLE

The correspondence between the new problem set and the 5th edition chapter 11 problem set.

Problems 11.1-20 are all new

New	5th	New	5th	New	5th
21	1 mod	51	27 mod	81	61
22	2	52	new	82	57
23	3 mod	53	15	83	59 mod
24	new	54	31	84	56
25	4	55	32 mod	85	62
26	5	56	33	86	63
27	6	57	36 mod	87	64
28	7	58	37	88	new
29	10	59	34	89	65
30	11	60	35	90	67
31	8 mod	61	38	91	68
32	new	62	39	92	new
33	12 mod	63	41	93	69 mod
34	13	64	42 mod	94	70 mod
35	14 mod	65	44	95	71
36	new	66	45	96	new
37	16 mod	67	new	97	new
38	new	68	46	98	new
39	17 mod	69	47	99	73
40	18 mod	70	49	100	74
41	20	71	50	101	75
42	22	72	new	102	new
43	new	73	new	103	72
44	23	74	48	104	76
45	24 mod	75	54	105	77
46	40 mod	76	52 mod	106	new
47	26 mod	77	60	107	79
48	19	78	new	108	78
49	21	79	53	109	80
50	25 mod	80	51	110	new

For many of the cycle problems we recommend that the students be allowed to use the software for properties to reduce the time spent on interpolations.

New	5th	New	5th	New	5th
111	new	131	new	151	114
112	81	132	new	152	new
113	82	133	98	153	9 mod
114	83	134	99	154	29
115	84	135	100	155	15
116	85	136	new	156	28 mod
117	86 a	137	101	157	40
118	86 b	138	103 mod	158	43
119	90	139	102	159	55
120	87	140	104	160	58 mod
121	88	141	105	161	59
122	89	142	106	162	70
123	91	143	108 mod	163	111
124	92	144	109	164	113
125	93	145	107	165	115
126	94 mod	146	new	166	116
127	95	147	110		
128	new	148	new		
129	96	149	112		
130	new	150	new		

The correspondence between the new English unit problem set and the previous 5th edition chapter 11 problem set and the current SI problems.

New	5th	SI	New	5th	SI	New	5th	SI
167	117 mod	21	184	new	73	201	148b	118
168	118 mod	22	185	133	74	202	new	-
169	new	24	186	136	86	203	149	120
170	119	26	187	137	89	204	150	121
171	120	27	188	138	93	205	151	125
172	new	32	189	139	95	206	new	130
173	121 mod	33	190	new	97	207	new	137
174	122 mod	35	191	new	98	208	new	146
175	123 mod	37	192	141	104	209	155	147
176	125 mod	45	193	140	105	210	new	148
177	124	48	194	142	107	211	new	150
178	127 mod	55	195	143	109	212	154	144
179	128 mod	57	196	144	112	213	126	-
180	129	60	197	145	113	214	130	157
181	131 mod	66	198	146	114	215	134	160
182	132	71	199	147	116	216	135	160
183	new	72	200	148a	117	217	153	134

Concept-Study Guide Problems

11.1

Is a steam power plant running in a Carnot cycle? Name the four processes.

No. It runs in a Rankine cycle.

1-2:	An isentropic compression (constant s)	Pump
2-3:	An isobaric heating (constant P)	Boiler
3-4:	An isentropic expansion (constant s)	Turbine
4-1:	An isobaric cooling, heat rejection (constant P)	Condenser

11.2

Consider a Rankine cycle without superheat. How many single properties are needed to determine the cycle? Repeat the answer for a cycle with superheat.

a. No superheat. Two single properties.
High pressure (or temperature) and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid and the boiler output is saturated vapor. Physically the high pressure is determined by the pump and the low temperature is determined by the cooling medium.

b. Superheat. Three single properties.
High pressure and **temperature** and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid. Physically the high pressure is determined by the pump and the high temperature by the heat transfer from the hot source. The low temperature is determined by the cooling medium.

11.3

Which component determines the high pressure in a Rankine cycle? What determines the low pressure?

The high pressure in the Rankine cycle is determined by the pump. The low pressure is determined as the saturation pressure for the temperature you can cool to in the condenser.

11.4

Mention two benefits of a reheat cycle.

The reheat raises the average temperature at which you add heat.

The reheat process brings the states at the lower pressure further out in the superheated vapor region and thus raises the quality (if two-phase) in the last turbine section.

11.5

What is the difference between an open and a closed feedwater heater?

The open feedwater heater mixes the two flows at the extraction pressure and thus requires two feedwater pumps.

The closed feedwater heater does not mix the flows but let them exchange energy (it is a two fluid heat exchanger). The flows do not have to be at the same pressure. The condensing source flow is dumped into the next lower pressure feedwater heater or the condenser or it is pumped up to line pressure by a drip pump and added to the feedwater line.

11.6

Can the energy removed in a power plant condenser be useful?

Yes.

In some applications it can be used for heating buildings locally or as district heating. Other uses could be to heat green houses or as general process steam in a food process or paper mill. These applications are all based on economics and scale. The condenser then has to operate at a higher temperature than it otherwise would.

11.7

In a cogenerating power plant, what is cogenerated?

The electricity is cogenerated. The main product is a steam supply.

11.8

Why is the back work ratio in the Brayton cycle much higher than in the Rankine cycle?

Recall the expression for shaft work in a steady flow device

$$w = - \int v \, dP$$

The specific volume in the compressor is not so much smaller than the specific volume in the turbine of the Brayton cycle as it is in the pump (liquid) compared to turbine (superheated vapor) in the Rankine cycle.

11.9

The Brayton cycle has the same 4 processes as the Rankine cycle, but the T-s and P-v diagrams look very different; why is that?

The Brayton cycle have all processes in the superheated vapor (close to ideal gas) region. The Rankine cycle crosses in over the two-phase region.

11.10

Is it always possible to add a regenerator to the Brayton cycle? What happens when the pressure ratio is increased?

No. When the pressure ratio is high, the temperature after compression is higher than the temperature after expansion. The exhaust flow can then not heat the flow into the combustor.

11.11

Why would you use an intercooler between compressor stages?

The cooler provides two effects. It reduces the specific volume and thus reduces the work in the following compressor stage. It also reduces the temperature into the combustor and thus lowers the peak temperature. This makes the control of the combustion process easier (no autoignition or uncontrollable flame spread), it reduces the formation of NOx that takes place at high temperatures and lowers the cooling requirements for the chamber walls.

11.12

The jet engine does not produce shaft work; how is power produced?

The turbine produces just enough shaft work to drive the compressor and it makes a little electric power for the aircraft. The power is produced as thrust of the engine. In order to exhaust the gases at high speed they must be accelerated so the high pressure in the turbine exit provides that force (high P relative to ambient). The high P into the turbine is made by the compressor, that pushes the flow backwards, and thus has a net resulting force forwards on the blades transmitted to the shaft and the aircraft. The outer housing also has a higher pressure inside that gives a net component in the forward direction.

11.13

How is the compression in the Otto cycle different from the Brayton cycle?

The compression in an Otto cycle is a volume reduction dictated by the piston motion. The physical handles are the volumes V_1 and V_2.

The compression in a Brayton cycle is the compressor pushing on the flow so it determines the pressure. The physical control is the pressure P_2.

11.14

Does the inlet state (P_1, T_1) have any influence on the Otto cycle efficiency? How about the power produced by a real car engine?

Very little. The efficiency for the ideal cycle only depends on compression ratio when we assume cold air properties. The u's are slightly non-linear in T so there will be a small effect.

In a real engine there are several effects. The inlet state determines the density and thus the total mass in the chamber. The more mass the more energy is released when the fuel burns, the peak P and T will also change which affects the heat transfer loss to the walls and the formation of Nox (sensitive to T). The combustion process may become uncontrollable if T is too high (knocking). Some increase in P_1 like that done by a turbo-charger or super-charger increases the power output and if high, it must be followed by an intercooler to reduce T_1. If P_1 is too high the losses starts to be more than the gain so there is an optimum level.

11.15

How many parameters do you need to know to completely describe the Otto cycle? How about the Diesel cycle?

Otto cycle. State 1 (2 parameters) and the compression ratio CR and the energy release per unit mass in the combustion, a total of **4 parameters**. With that information you can draw the diagrams in Figure 11.28. Another way of looking at it is four states (8 properties) minus the four process equations ($s_2 = s_1$, $v_3 = v_2$, $s_4 = s_3$ and $v_4 = v_1$) gives 4 unknowns.

Diesel cycle. Same as for the Otto cycle namely **4 parameters**. The only difference is that one constant v process is changed to a constant P process.

11.16

The exhaust and inlet flow processes are not included in the Otto or Diesel cycles. How do these necessary processes affect the cycle performance?

Due to the pressure loss in the intake system and the dynamic flow process we will not have as much mass in the cylinder nor as high a P as in a reversible process. The exhaust flow requires a slightly higher pressure to push the flow out through the catalytic converter and the muffler (higher back pressure) and the pressure loss in the valve so again there is a loss relative to a reversible process. Both of these processes subtracts a pumping work from the net work out of the engine and a lower charge mass gives less power (not necessarily lower efficiency) than other wise could be obtained.

11.17

A refrigerator in my 20^{o}C kitchen uses R-12 and I want to make ice cubes at –5^o C. What is the minimum high P and the maximum low P it can use?

Since the R-12 must give heat transfer out to the kitchen air at 20^{o}C, it must at least be that hot at state 3.

From Table B.3.1: $P_3 = P_2 = P_{sat} =$ **567 kPa** is minimum high P.

Since the R-12 must absorb heat transfer at the freezers –5^{o}C, it must at least be that cold at state 4.

From Table B.3.1: $P_1 = P_4 = P_{sat} =$ **261 kPa** is maximum low P.

11.18

How many parameters are needed to completely determine a standard vapor compression refrigeration cycle?

Two parameters: The high pressure and the low pressure. This assumes the exit of the condenser is saturated liquid and the exit of the evaporator is saturated vapor.

11.19

Why would one consider a combined cycle system for a power plant? For a heat pump or refrigerator?

Dual cycle or combined cycle systems have the advantage of a smaller difference between the high and low ranges for P and T. The heat can be added at several different temperatures reducing the difference between the energy source T and the working substance T. The working substance vapor pressure at the desired T can be reduced from a high value by adding a topping cycle with a different substance or have a higher low pressure at very low temperatures.

11.20

Since any heat transfer is driven by a temperature difference, how does that affect all the real cycles relative to the ideal cycles?

Heat transfers are given as $\dot{Q} = CA\ \Delta T$ so to have a reasonable rate the area and the temperature difference must be large. The working substance then must have a different temperature than the ambient it exchanges energy with. This gives a smaller temperature difference for a heat engine with a lower efficiency as a result. The refrigerator or heat pump must have the working substance with a higher temperature difference than the reservoirs and thus a lower coefficient of performance (COP).

The smaller CA is the larger ΔT must be for a certain magnitude of the heat transfer rate. This can be a design problem, think about the front end air intake grill for a modern car which is very small compared to a car 20 years ago.

Simple Rankine cycles

11.21

A steam power plant as shown in Fig. 11.3 operating in a Rankine cycle has saturated vapor at 3.0 MPa leaving the boiler. The turbine exhausts to the condenser operating at 10 kPa. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 10) = 3.02$ kJ/kg

$\Rightarrow h_2 = h_1 + w_p = 191.81 + 3.02 = 194.83$ kJ/kg

C.V. Boiler : $q_H = h_3 - h_2 = 2804.14 - 194.83 =$ **2609.3 kJ/kg**

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

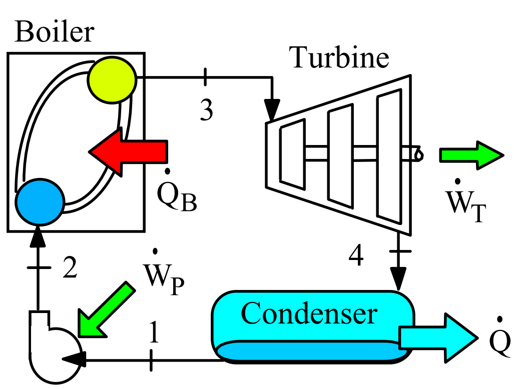
$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \Rightarrow x_4 = 0.7383$

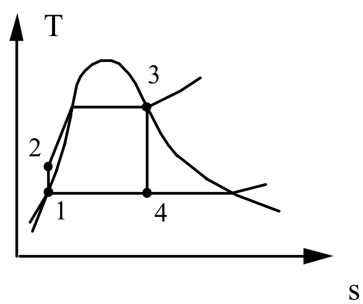
$\Rightarrow h_4 = 191.81 + 0.7383 (2392.82) = 1958.34$ kJ/kg

$w_T = 2804.14 - 1958.34 =$ **845.8 kJ/kg**

C.V. Condenser : $q_L = h_4 - h_1 = 1958.34 - 191.81 =$ **1766.5 kJ/kg**

$\eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (845.8 - 3.0) / 2609.3 =$ **0.323**





11.22

Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 175°C, and the condenser pressure is 10 kPa. Determine the thermal efficiency of this cycle.

Solution:

C.V. H_2O ideal Rankine cycle

State 3: $T_3 = 175°C \quad \Rightarrow \quad P_3 = P_{G\ 175°C} = 892$ kPa, $s_3 = 6.6256$

CV Turbine adiabatic and reversible so second law gives

$$s_4 = s_3 = 6.6256 = 0.6493 + x_4 \times 7.5009 \quad => \quad x_4 = 0.797$$

$$h_4 = 191.83 + 0.797 \times 2392.8 = 2098.3 \text{ kJ/kg}$$

The energy equation gives

$$w_T = h_3 - h_4 = 2773.6 - 2098.3 = 675.3 \text{ kJ/kg}$$

C.V. pump and incompressible liquid gives work into pump

$$w_P = v_1(P_2 - P_1) = 0.00101(892 - 10) = 0.89 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.83 + 0.89 = 192.72 \text{ kJ/kg}$$

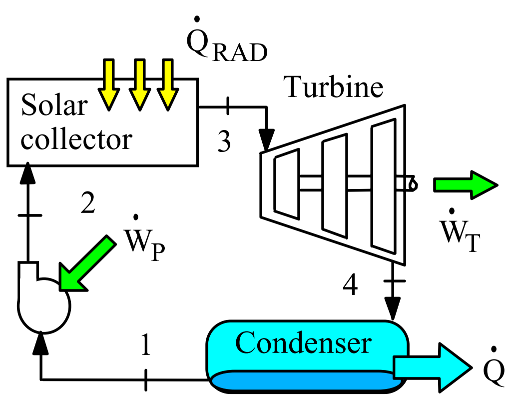
C.V. boiler gives the heat transfer from the energy equation as

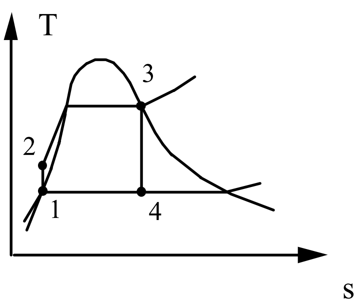
$$q_H = h_3 - h_2 = 2773.6 - 192.72 = 2580.9 \text{ kJ/kg}$$

The cycle net work and efficiency are found as

$$w_{NET} = w_T - w_P = 675.3 - 0.89 = 674.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 674.4/2580.9 = \mathbf{0.261}$$





11.23

A utility runs a Rankine cycle with a water boiler at 3.0 MPa and the cycle has the highest and lowest temperatures of 450°C and 45°C respectively. Find the plant efficiency and the efficiency of a Carnot cycle with the same temperatures.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45^oC , $x = 0$ => $h_1 = 188.42$, $v_1 = 0.00101$, $P_{sat} = 9.6$ kPa

3: 3.0 MPa , 450^oC => $h_3 = 3344$, $s_3 = 7.0833$

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101\ (3000 - 9.6) = 3.02$ kJ/kg

=> $h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. Boiler : $q_H = h_3 - h_2 = 3344 - 191 = 3152.56$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$s_4 = s_3 = 7.0833 = 0.6386 + x_4\ (7.5261)$ => $x_4 = 0.8563$

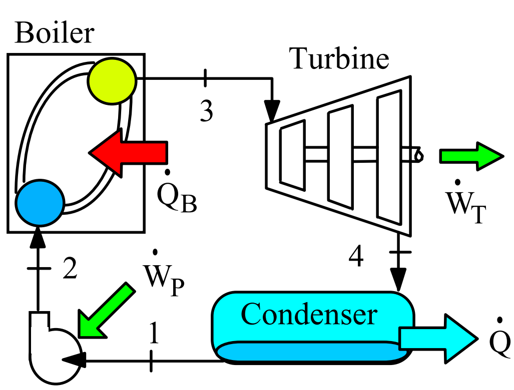
=> $h_4 = 188.42 + 0.8563\ (2394.77) = 2239.06$ kJ/kg

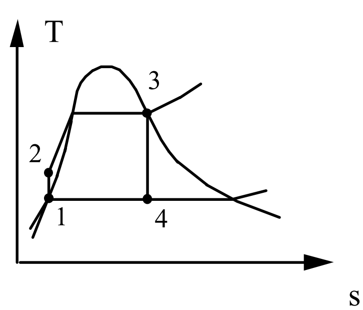
$w_T = 3344 – 2239.06 = 1105$ kJ/kg

C.V. Condenser : $q_L = h_4 - h_1 = 2239.06 - 188.42 = 2050.64$ kJ/kg

$\eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (1105 - 3.02) / 3152.56 = \mathbf{0.349}$

$$\eta_{carnot} = 1 - T_L / T_H = 1 - \frac{273.15 + 45}{273.15 + 450} = \mathbf{0.56}$$





11.24

A Rankine cycle uses ammonia as the working substance and powered by solar energy. It heats the ammonia to 140°C at 5000 kPa in the boiler/superheater. The condenser is water cooled and the exit kept at 25°C. Find (T, P and x if applicable) for all four states in the cycle.

Solution:

Based on the standard Rankine cycle and Table B.2 and Table A.4 for C_p.

State 1: Saturated liquid. $P_1 = P_{sat} = 1003$ kPa, $x_1 = 0$

State 2: $P_2 = 5000$ kPa, consider C.V. pump

Energy: $h_2 - h_1 = w_p = v_1 (P_2 - P_1) = 0.001658 (5000 – 1003) = 6.627$ kJ/kg

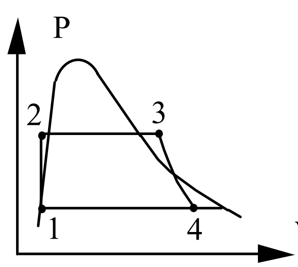
$$T_2 = T_1 + (h_2 - h_1)/C_p = 25 + 6.627/4.84 = 26.4^oC$$

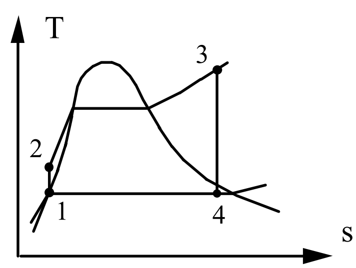
State 3: Table B.2.2 140°C at 5000 kPa, s = 4.9068 kJ/kg K

State 4: $P_4 = P_1 = 1003$ kPa. Consider the turbine for which $s_4 = s_3$.

$$s_3 < s_g = 5.0293 \text{ kJ/kg K at } 25^oC$$

$$x_4 = (s_3 – s_f)/s_{fg} = (4.9068 – 1.121)/3.9083 = 0.96866$$





11.25

A steam power plant operating in an ideal Rankine cycle has a high pressure of 5 MPa and a low pressure of 15 kPa. The turbine exhaust state should have a quality of at least 95% and the turbine power generated should be 7.5 MW. Find the necessary boiler exit temperature and the total mass flow rate.

Solution:

C.V. Turbine assume adiabatic and reversible.

Energy: $w_T = h_3 - h_4$; Entropy: $s_4 = s_3$

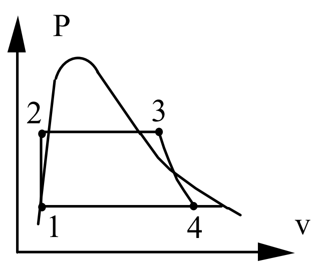
Since the exit state is given we can relate that to the inlet state from entropy.

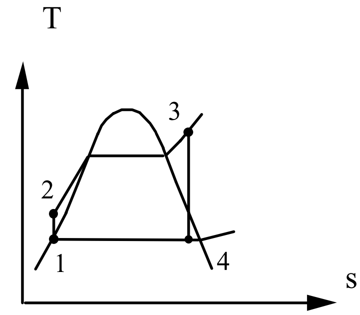
4: 15 kPa, $x_4 = 0.95$ => $s_4 = 7.6458$ kJ/kg K, $h_4 = 2480.4$ kJ/kg

3: $s_3 = s_4$, P_3 ⇒ $h_3 = 4036.7$ kJ/kg, $T_3 =$ **758°C**

$w_T = h_3 - h_4 = 4036.7 - 2480.4 = 1556.3$ kJ/kg

$\dot{m} = \dot{W}_T/w_T = 7.5 \times 1000/1556.3 =$ **4.82 kg/s**





11.26

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-134a Table B.5)

$$w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2 - P_1)$$

$$= 0.000873(2926.2 - 1017.0) = 1.67 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 256.54 + 1.67 = 258.21 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 428.10 - 258.21 = 169.89 \text{ kJ/kg}$$

CV: Turbine

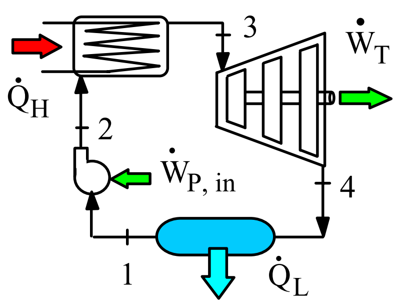
$$s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \;\;=>\;\; x_4 = 0.9346$$

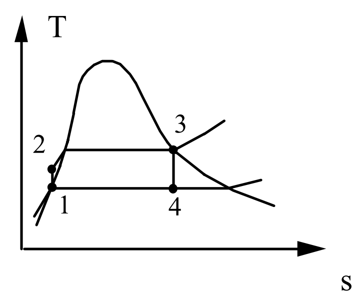
$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96 \text{ kJ/kg}$

$$w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29 \text{ kJ/kg}$$

$\eta_{TH} = w_{NET}/q_H = 17.29/169.89 =$ **0.102**





11.27

Do Problem 11.26 with R-22 as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-22 Table B.4)

$$w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2 - P_1) = 0.000884(4037 - 1534) = 2.21 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 94.27 + 2.21 = 96.48 \text{ kJ/kg}$$

CV: Boiler: $q_H = h_3 - h_2 = 253.69 - 96.48 = 157.21$ kJ/kg

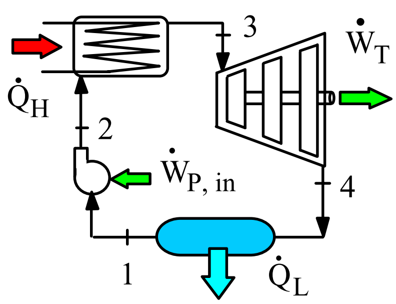
CV: Turbine

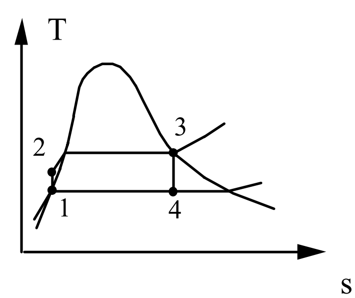
$$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329, \quad => \quad x_4 = 0.8446$$

$$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22$$

$$w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (18.47 - 2.21)/157.21 = \mathbf{0.1034}$$





11.28

Do Problem 11.26 with ammonia as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use Ammonia Table B.2)

$$w_P = h_2 - h_1 = \int_1^2 v dP = v_1(P_2 - P_1)$$

$$= 0.001725(4608.6 - 1554.9) = 5.27 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 371.43 + 5.27 = 376.7 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 1447.8 - 376.7 = 1071.1 \text{ kJ/kg}$$

CV: Turbine

$$s_4 = s_3 = 4.3901 = 1.3574 + x_4 \times 3.5088 \quad => \quad x_4 = 0.8643$$

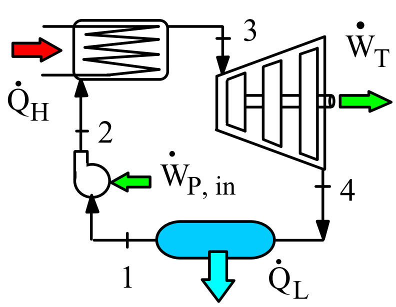
$$h_4 = 371.43 + 0.8643 \times 1098.8 = 1321.13 \text{ kJ/kg}$$

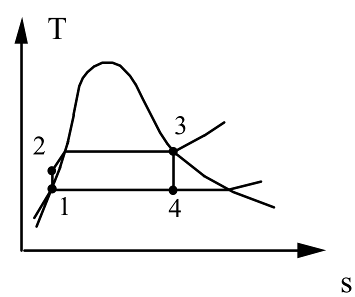
Energy Eq.:

$$w_T = h_3 - h_4 = 1447.8 - 1321.13 = 126.67 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_P = 126.67 - 5.27 = 121.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 121.4/1071.1 = \mathbf{0.113}$$

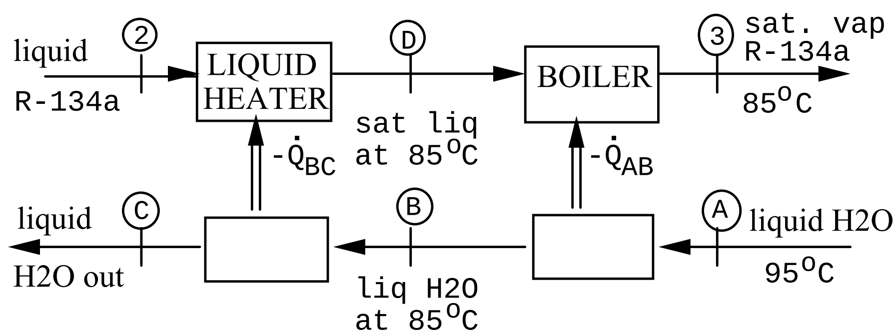




11.29

Consider the boiler in Problem 11.26 where the geothermal hot water brings the R-134a to saturated vapor. Assume a counter flowing heat exchanger arrangement. The geothermal water temperature should be equal to or greater than the R-134a temperature at any location inside the heat exchanger. The point with the smallest temperature difference between the source and the working fluid is called the pinch point. If 2 kg/s of geothermal water is available at 95°C, what is the maximum power output of this cycle for R-134a as the working fluid? (hint: split the heat exchanger C.V. into two so the pinch point with $\Delta T = 0$, T = 85°C appears).

2 kg/s of water is available at 95 ^{o}C for the boiler. The restrictive factor is the boiling temperature of 85° C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.



Write the energy equation for the first section A-B and D-3:

$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{R134A}(428.1 - 332.65) \;\Rightarrow\; \dot{m}_{R134A} = 0.8813 \text{ kg/s}$$

To be sure that the boiling temp. is the restrictive factor, calculate T_C from the energy equation for the remaining section:

$$-\dot{Q}_{AC} = 0.8813(332.65 - 258.21) = 65.60 \text{ kW} = 2(355.88 - h_C)$$

$$\Rightarrow \; h_C = 323.1 \text{ kJ/kg}, \;\; T_C = 77.2°\text{C} > T_2 \quad \textbf{OK}$$

CV Pump: $w_P = v_1(P_2-P_1) = 0.000873(2926.2 - 1017.0) = 1.67$ kJ/kg

CV: Turbine: $s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \quad => \quad x_4 = 0.9346$

$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96$ kJ/kg

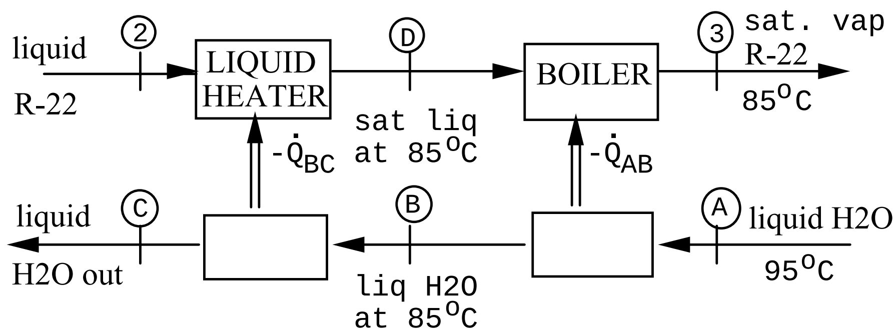
Cycle: $w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29$ kJ/kg

$$\dot{W}_{NET} = \dot{m}_{R134A} w_{NET} = 0.8813 \times 17.29 = \mathbf{15.24 \text{ kW}}$$

11.30

Do the previous problem with R-22 as the working fluid.

A flow with 2 kg/s of water is available at 95°C for the boiler. The restrictive factor is the boiling temperature of 85°C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.



$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{R\text{-}22}(253.69 - 165.09) \;\Rightarrow\; \dot{m}_{R\text{-}22} = 0.949 \text{ kg/s}$$

To verify that $T_D = T_3$ is the restrictive factor, find T_C.

$$-\dot{Q}_{AC} = 0.949(165.09 - 96.48) = 65.11 = 2.0(355.88 - h_C)$$

$$h_C = 323.32 \text{ kJ/kg} \;\Rightarrow\; T_C = 77.2^oC \quad \textbf{OK}$$

State 1: 40^oC, 1533.5 kPa, $v_1 = 0.000884 \text{ m}^3\text{/kg}$

CV Pump: $w_P = v_1(P_2 - P_1) = 0.000884(4036.8 - 1533.5) = 2.21 \text{ kJ/kg}$

CV: Turbine

$$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329 \;\Rightarrow\; x_4 = 0.8446$$

$$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47 \text{ kJ/kg}$

Cycle: $w_{NET} = w_T - w_P = 18.47 - 2.21 = 16.26 \text{ kJ/kg}$

$$\dot{W}_{NET} = \dot{m}_{R22} w_{NET} = 0.949 \times 16.26 = \mathbf{15.43 \text{ kW}}$$

11.31

Consider the ammonia Rankine-cycle power plant shown in Fig. P11.31. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

a. Determine the turbine power output and the pump power input for the cycle.
b. Determine the mass flow rate of water through each heat exchanger.
c. What is the thermal efficiency of this power plant?

Solution:

a) C.V. Turbine. Assume reversible and adiabatic.

$s_2 = s_1 = 5.0863 = 0.8779 + x_2 \times 4.3269 \quad => \quad x_2 = 0.9726$

$h_2 = 227.08 + 0.9726 \times 1225.09 = 1418.6 \text{ kJ/kg}$

$w_T = h_1 - h_2 = 1460.29 - 1418.6 = 41.69 \text{ kJ/kg}$

$\dot{W}_T = \dot{m}w_T = 1000 \times 41.69 = \mathbf{41\ 690\ kW}$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.0016(857 - 615) = 0.387 \text{ kJ/kg}$

$\dot{W}_P = \dot{m}w_P = 1000 \times 0.387 = \mathbf{387\ kW}$

b) Consider to condenser heat transfer to the low T water

$\dot{Q}_{\text{to low T H2O}} = 1000(1418.6 - 227.08) = 1.1915\times10^6 \text{ kW}$

$\dot{m}_{\text{low T H2O}} = \dfrac{1.1915\times10^6}{29.38 - 20.98} = \mathbf{141\ 850\ kg/s}$

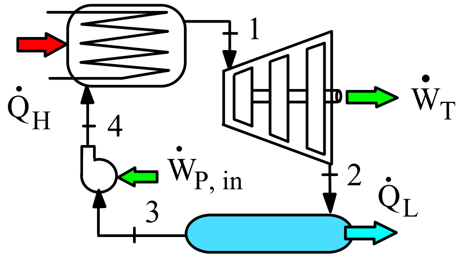
$h_4 = h_3 + w_P = 227.08 + 0.39 = 227.47 \text{ kJ/kg}$

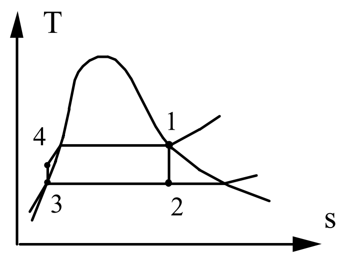
Now consider the boiler heat transfer from the high T water

$\dot{Q}_{\text{from high T H2O}} = 1000(1460.29 - 227.47) = 1.2328\times10^6 \text{ kW}$

$\dot{m}_{\text{high T H2O}} = \dfrac{1.2328\times10^6}{104.87 - 96.50} = \mathbf{147\ 290\ kg/s}$

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{41\ 690 - 387}{1.2328\times10^6} = \mathbf{0.033}$





11.32

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C in the boiler. It cools the condenser with ocean water coming in at 12°C and returned at 15°C so the condenser exit is at 45°C. Find the net power output and the required mass flow rate of ocean water.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45°C, x = 0: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600°C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 9.6) = 3.02$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$s_4 = s_3 = 7.5084 = 0.6386 + x_4 (7.5261)$ => $x_4 = 0.9128$

=> $h_4 = 188.42 + 0.9128 (2394.77) = 2374.4$ kJ/kg

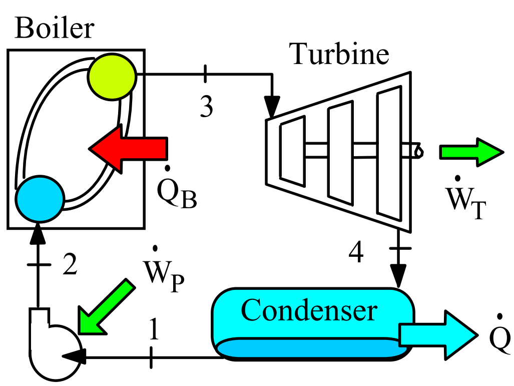
$w_T = 3682.34 – 2374.4 = 1307.94$ kJ/kg

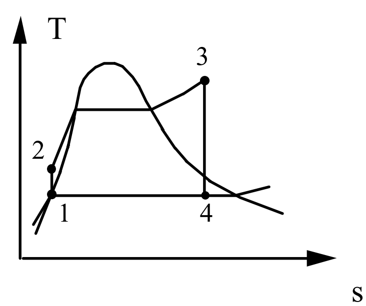
$\dot{W}_{NET} = \dot{m}(w_T – w_p) = 25 (1307.94 – 3.02) = \mathbf{32.6\ MW}$

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$\dot{Q}_L = \dot{m}q_L = 25 \times 2186 = 54.65\ \text{MW} = \dot{m}_{ocean}\, C_p\, \Delta T$

$\dot{m}_{ocean} = \dot{Q}_L / C_p\, \Delta T = 54\,650 / (4.18 \times 3) = \mathbf{4358\ kg/s}$





11.33

The power plant in Problem 11.21 is modified to have a super heater section following the boiler so the steam leaves the super heater at 3.0 MPa, 400°C. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4$;

Entropy: $s_4 = s_3 = 6.9211$ kJ/kg K

$$\Rightarrow \; x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.9211 - 0.6492}{7.501} = 0.83614 \;;$$

$$h_4 = 191.81 + 0.83614 \times 2392.82 = 2192.5 \text{ kJ/kg}$$

$$w_{T,s} = 3230.82 - 2192.5 = \mathbf{1038.3\ kJ/kg}$$

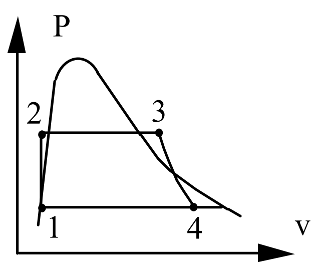
C.V. Pump: $w_P = \int v\, dP = v_1(P_2 - P_1) = 0.00101(3000 - 10) =$ **3.02 kJ/kg**

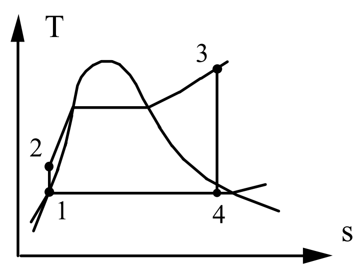
$$\Rightarrow \; h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83 \text{ kJ/kg}$$

C.V. Condenser: $q_C = h_4 - h_1 = 2192.5 - 191.81 =$ **2000.7 kJ/kg**

C.V. Boiler: $q_H = h_3 - h_2 = 3230.82 – 194.83 =$ **3036 kJ/kg**

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{1038.3 – 3.02}{3036} = \mathbf{0.341}$$





11.34

A steam power plant has a steam generator exit at 4 MPa, 500°C and a condenser exit temperature of 45°C. Assume all components are ideal and find the cycle efficiency and the specific work and heat transfer in the components.

Solution:

From the Rankine cycle we have the states:

1: 45°C $x = \emptyset$, $v_1 = 0.00101$ m^3/kg, $h_1 = 188.45$ kJ/kg

3: 4 MPa, 500°C, $h_3 = 3445.3$ kJ/kg, $s_3 = 7.0901$ kJ/kg K

C.V. Turbine: $s_4 = s_3 \Rightarrow x_4 = (7.0901 - 0.6386)/7.5261 = 0.8572$,

$$h_4 = 188.42 + 0.8572 \times 2394.77 = 2241.3$$

$$w_T = h_3 - h_4 = 3445.3 - 2241.3 = \textbf{1204 kJ/kg}$$

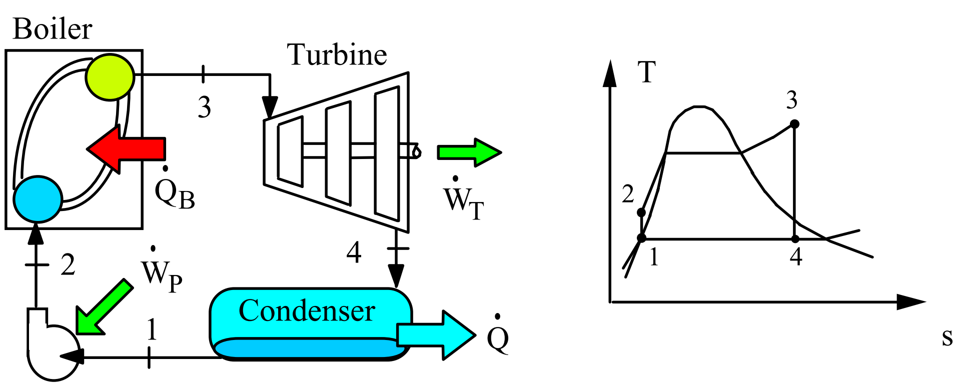
C.V. Pump: $w_P = v_1(P_2 - P_1) = 0.00101(4000 - 9.6) = \textbf{4.03 kJ/kg}$

$$w_P = h_2 - h_1 \Rightarrow h_2 = 188.42 + 4.03 = 192.45 \text{ kJ/kg}$$

C.V. Boiler: $q_H = h_3 - h_2 = 3445.3 - 192.45 = \textbf{3252.8 kJ/kg}$

C.V. Condenser: $q_{L,out} = h_4 - h_1 = 2241.3 - 188.42 = \textbf{2052.9 kJ/kg}$

$$\eta_{TH} = w_{net}/q_H = (w_T + w_P)/q_H = (1204 - 4.03)/3252.8 = \textbf{0.369}$$



11.35

Consider an ideal Rankine cycle using water with a high-pressure side of the cycle at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gas-turbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 30 MPa, 550°C, and the condenser pressure is 5 kPa. What is the steam quality at the turbine exit?

Solution:

For the efficiency we need the net work and steam generator heat transfer.

C.V. Pump. For this high exit pressure we use Table B.1.4

State 1: $s_1 = 0.4764$ kJ/kg K, $h_1 = 137.82$ kJ/kg

Entropy Eq.: $s_2 = s_1 \;\; => \;\; h_2 = 168.36$ kJ/kg

$$w_p = h_2 - h_1 = 30.54 \text{ kJ/kg}$$

C.V. Turbine. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 6.0342 = 0.4764 + x_4 \times 7.9187$

$x_4 = \mathbf{0.70186}$ Very low for a turbine exhaust

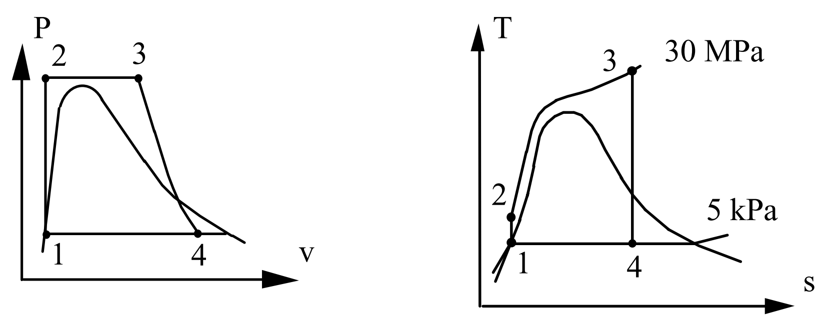
$$h_4 = 137.79 + x_4 \times 2423.66 = 1838.86, \quad h_3 = 3275.36 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 1436.5 \text{ kJ/kg}$$

Steam generator: $q_H = h_3 - h_2 = 3107$ kJ/kg

$$w_{NET} = w_T - w_p = 1436.5 - 30.54 = 1406 \text{ kJ/kg}$$

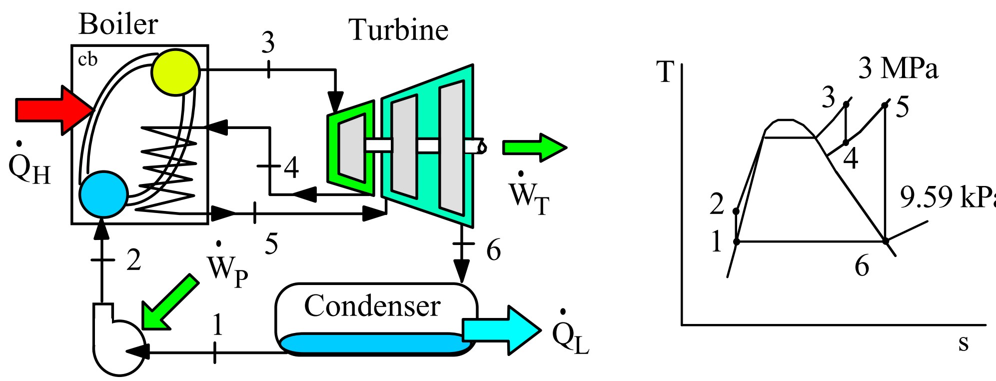
$$\eta = w_{NET}/q_H = 1406 / 3107 = \mathbf{0.45}$$



Reheat Cycles

11.36

A smaller power plant produces steam at 3 MPa, 600°C in the boiler. It keeps the condenser at 45°C by transfer of 10 MW out as heat transfer. The first turbine section expands to 500 kPa and then flow is reheated followed by the expansion in the low pressure turbine. Find the reheat temperature so the turbine output is saturated vapor. For this reheat find the total turbine power output and the boiler heat transfer.



The states properties from Tables B.1.1 and B.1.3

1: 45°C, x = 0: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600°C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

6: 45°C, x = 1: $h_6 = 2583.19$ kJ/kg, $s_6 = 8.1647$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101\ (3000 - 9.59) = 3.02$ kJ/kg

$h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. HP Turbine section

Entropy Eq.: $s_4 = s_3$ => $h_4 = 3093.26$ kJ/kg; $T_4 = 314°C$

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5 = 8.1647$ kJ/kg K => state 5

State 5: 500 kPa, s_5 => $h_5 = 3547.55$ kJ/kg, $T_5 =$ **529°C**

C.V. Condenser.

Energy Eq.: $q_L = h_6 – h_1 = h_{fg} = 2394.77$ kJ/kg

$\dot{m} = \dot{Q}_L / q_L = 10\ 000 / 2394.77 = 4.176$ kg/s

Both turbine sections

$\dot{W}_{T,tot} = \dot{m}w_{T,tot} = \dot{m}(h_3 - h_4 + h_5 - h_6)$

$= 4.176\ (3682.34 - 3093.26 + 3547.55 – 2583.19) =$ **6487 kW**

Both boiler sections

$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$

$= 4.176\ (3682.34 – 191.44 + 3547.55 - 3093.26) =$ **16 475 kW**

11.37

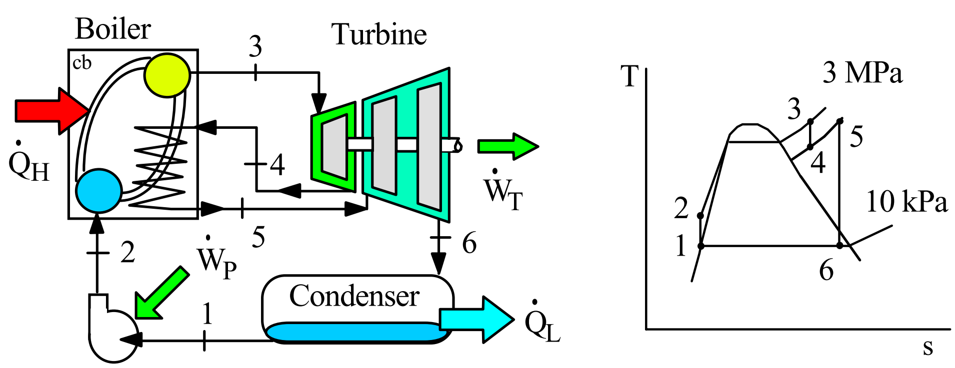
Consider an ideal steam reheat cycle where steam enters the high-pressure turbine at 3.0 MPa, 400°C, and then expands to 0.8 MPa. It is then reheated to 400°C and expands to 10 kPa in the low-pressure turbine. Calculate the cycle thermal efficiency and the moisture content of the steam leaving the low-pressure turbine.

Solution:

C.V. Pump reversible, adiabatic and assume incompressible flow

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = 191.81 + 3.02 = 194.83$ kJ/kg



C.V. HP Turbine section

$P_3 = 3$ MPa, $T_3 = 400^oC$ => $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

$s_4 = s_3$ => $h_4 = 2891.6$ kJ/kg;

C.V. LP Turbine section

State 5: 400^oC, 0.8 MPa => $h_5 = 3267.1$ kJ/kg, $s_5 = 7.5715$ kJ/kg K

Entropy Eq.: $s_6 = s_5 = 7.5715$ kJ/kg K => two-phase state

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{7.5715 - 0.6492}{7.501} = 0.92285 = \mathbf{0.923}$$

$h_6 = 191.81 + 0.92285 \times 2392.82 = 2400$ kJ/kg

$w_{T,tot} = h_3 - h_4 + h_5 - h_6 = 3230.82 - 2891.6+3267.1 - 2400 = 1237.8$ kJ/kg

$q_{H1} = h_3 - h_2 = 3230.82 - 194.83 = 3036$ kJ/kg

$q_H = q_{H1} + h_5 - h_4 = 3036 + 3267.1 - 2891.6 = 3411.5$ kJ/kg

$\eta_{CYCLE} = (1237.8 - 3.02)/3411.5 = \mathbf{0.362}$

11.38

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C in the boiler. It cools the condenser with ocean water so the condenser exit is at 45°C. There is a reheat done at 500 kPa up to 400°C and then expansion in the low pressure turbine. Find the net power output and the total heat transfer in the boiler.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45°C, x = 0: h_1 = 188.42 kJ/kg, v_1 = 0.00101 m^3/kg, P_{sat} = 9.59 kPa

3: 3.0 MPa, 600°C: h_3 = 3682.34 kJ/kg, s_3 = 7.5084 kJ/kg K

5: 500 kPa, 400°C: h_5 = 3271.83 kJ/kg, s_5 = 7.7937 kJ/kg K

C.V. Pump Reversible and adiabatic. Incompressible flow so

Energy: $w_p = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101\ (3000 - 9.6) = 3.02$ kJ/kg

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5 = 7.7937$ kJ/kg K => two-phase state

$$x_6 = (s_6 - s_f)/s_{fg} = \frac{7.7937 - 0.6386}{7.5261} = 0.9507$$

$$h_6 = 188.42 + 0.9507 \times 2394.77 = 2465.1 \text{ kJ/kg}$$

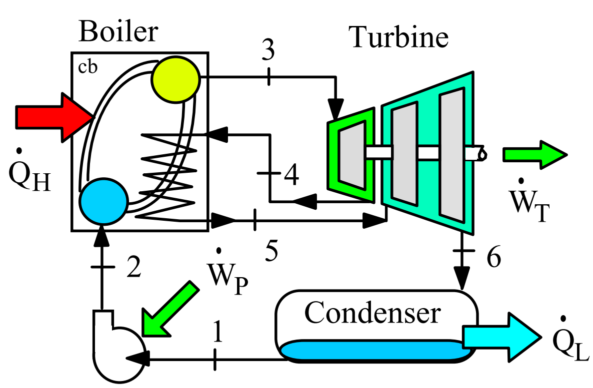
Both turbine sections

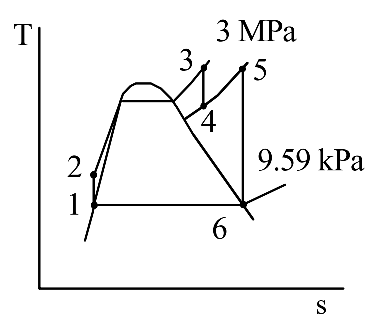
$$w_{T,tot} = h_3 - h_4 + h_5 - h_6$$
$$= 3682.34 - 3093.26 + 3271.83 - 2465.1 = 1395.81 \text{ kJ/kg}$$

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_p = \dot{m}(w_{T,tot} - w_p) = 25\ (1395.81 - 3.02) = \mathbf{34\ 820\ kW}$$

Both boiler sections

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$
$$= 25\ (3682.34 - 191.44 + 3271.83 - 3093.26) = \mathbf{91\ 737\ kW}$$

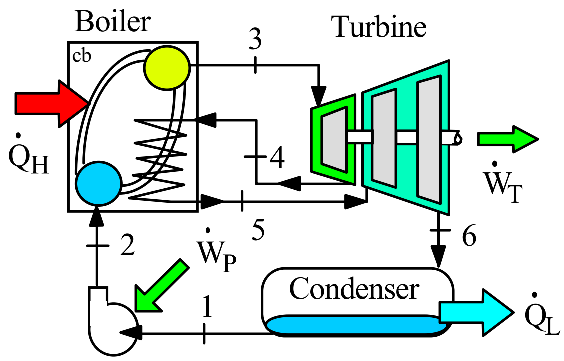


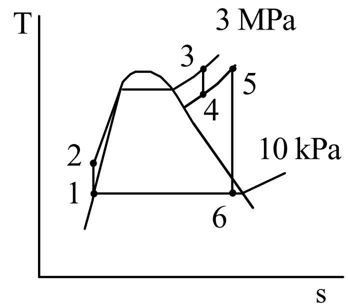


11.39

The reheat pressure effect the operating variables and thus turbine performance. Repeat Problem 11.37 twice, using 0.6 and 1.0 MPa for the reheat pressure.

Solution





C.V. Pump reversible, adiabatic and assume incompressible flow

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

State 3: 3 MPa, 400°C => $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

Low T boiler section: $q_{H1} = h_3 - h_2 = 3230.82 - 194.83 = 3035.99$ kJ/kg

State 4: P_4, $s_4 = s_3$

For $P_4 = 1$ MPa: $h_4 = 2940.85$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3263.9$ kJ/kg, $s_5 = 7.465$ kJ/kg K,

For $P_4 = 0.6$ MPa: $h_4 = 2793.2$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3270.3$ kJ/kg, $s_5 = 7.7078$ kJ/kg K,

State 6: 10 kPa, $s_6 = s_5$ => $x_6 = (s_6 - s_f)/s_{fg}$

Total turbine work: $w_{T,tot} = h_3 - h_4 + h_5 - h_6$

Total boiler H.Tr.: $q_H = q_{H1} + h_5 - h_4$

Cycle efficiency: $\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H$

P_4=P_5	x_6	h_6	w_T	q_H	η_{CYCLE}
1	0.9087	2366	1187.9	3359.0	0.3527
0.6	0.9410	2443.5	1228.0	3437.7	0.3563

Notice the very small changes in efficiency.

11.40

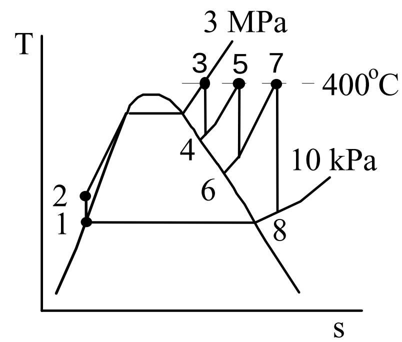
The effect of a number of reheat stages on the ideal steam reheat cycle is to be studied. Repeat Problem 11.37 using two reheat stages, one stage at 1.2 MPa and the second at 0.2 MPa, instead of the single reheat stage at 0.8 MPa.

C.V. Pump reversible, adiabatic and assume incompressible flow, work in

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

$P_4 = P_5 = 1.2$ MPa, $P_6 = P_7 = 0.2$ MPa



3: $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

4: P_4, $s_4 = s_3$ $\Rightarrow$ sup. vap. $h_4 = 2985.3$

5: $h_5 = 3260.7$ kJ/kg, $s_5 = 7.3773$ kJ/kg K

6: P_6, $s_6 = s_5$ $\Rightarrow$ sup. vapor

$h_6 = 2811.2$ kJ/kg

7: $h_7 = 3276.5$ kJ/kg, $s_7 = 8.2217$ kJ/kg K

8: P_8, $s_8 = s_7 \Rightarrow$ sup. vapor $h_8 = 2607.9$ kJ/kg

Total turbine work, same flow rate through all sections

$w_T = (h_3 - h_4) + (h_5 - h_6) + (h_7 - h_8) = 245.5 + 449.5 + 668.6 = 1363.6$ kJ/kg

Total heat transfer in boiler, same flow rate through all sections

$q_H = (h_3 - h_2) + (h_5 - h_4) + (h_7 - h_6) = 3036 + 319.8 + 465.3 = 3821.1$ kJ/kg

Cycle efficiency: $\eta_{TH} = \dfrac{w_T - w_P}{q_H} = \dfrac{1363.6 - 3.02}{3821.1} = \mathbf{0.356}$

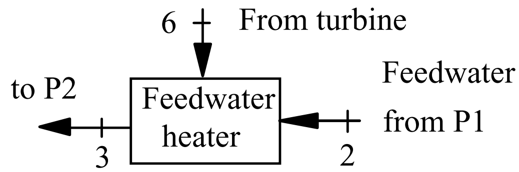
Open Feedwater Heaters

11.41

An open feedwater heater in a regenerative steam power cycle receives 20 kg/s of water at 100°C, 2 MPa. The extraction steam from the turbine enters the heater at 2 MPa, 275°C, and all the feedwater leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The complete diagram is as in Figure 11.8 in main text.



C.V Feedwater heater

Continuity Eq.: $\dot{m}_2 + \dot{m}_6 = \dot{m}_3$

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_3 h_3 = (\dot{m}_2 + \dot{m}_6)\ h_3$

Table B.1.4: $h_2 = 420.45$ kJ/kg, Table B.1.2: $h_3 = 908.77$ kJ/kg

Table B.1.3: $h_6 = 2963$ kJ/kg, this is interpolated

With the values substituted into the energy equation we get

$$\dot{m}_6 = \dot{m}_2 \frac{h_3 - h_2}{h_6 - h_3} = 20 \times \frac{908.77 - 420.45}{2963 - 908.77} = \mathbf{4.754\ kg/s}$$

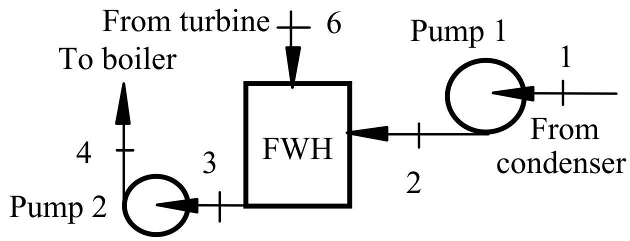
Remark: For lower pressures at state 2 where Table B.1.4 may not have an entry the corresponding saturated liquid at same T from Table B.1.1 is used.

11.42

A power plant with one open feedwater heater has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater is mixed with the feedwater line so the exit is saturated liquid into the second pump. Find the fraction of extraction steam flow and the two specific pump work inputs.

Solution:

The complete diagram is as in Figure 11.8 in the main text.



State out of boiler 5: $h_5 = 4378.82$ kJ/kg, $s_5 = 7.9593$ kJ/kg K

C.V. Turbine reversible, adiabatic: $s_7 = s_6 = s_5$

State 6: P_6 , s_6 => $h_6 = 3640.6$ kJ/kg, $T_6 = 574^{o}C$

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(1000 - 9.6) = \mathbf{1.0\ kJ/kg}$$

$$=> h_2 = h_1 + w_{P1} = 188.42 + 1.0 = 189.42 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x)\, h_2 + x\, h_6 = 1\, h_3$

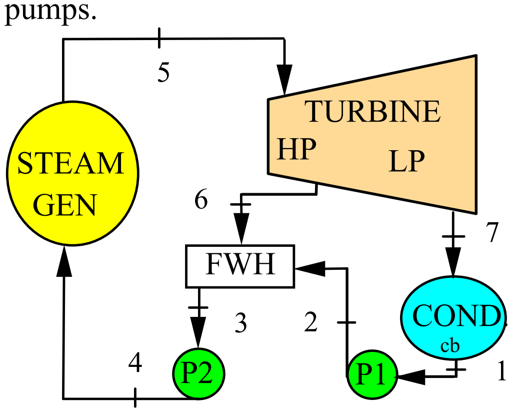
$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = \mathbf{0.1661}$$

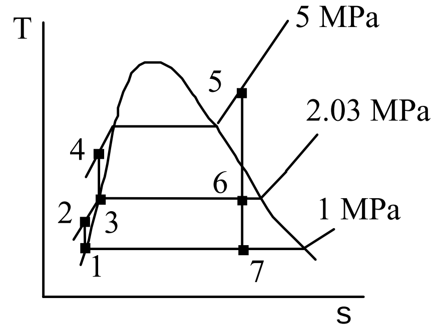
C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001127(5000 - 1000) = \mathbf{4.5\ kJ/kg}$$

11.43

A Rankine cycle operating with ammonia is heated by some low temperature source so the highest T is 120°C at a pressure of 5000 kPa. Its low pressure is 1003 kPa and it operates with one open feedwater heater at 2033 kPa. The total flow rate is 5 kg/s. Find the extraction flow rate to the feedwater heater assuming its outlet state is saturated liquid at 2033 kPa. Find the total power to the two pumps.





State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m^3/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5$ => $x_6 = (s_6 - s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708 \text{ kJ/kg}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.96 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x)\, h_2 + x\, h_6 = 1\, h_3$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = \mathbf{0.1046}$$

$$\dot{m}_{extr} = x\, \dot{m}_{tot} = 0.1046 \times 5 = \mathbf{0.523\ kg/s}$$

$$\dot{m}_1 = (1\text{-}x)\, \dot{m}_{tot} = (1 - 0.1046)\, 5 = 4.477 \text{ kg/s}$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001777(5000 - 2033) = 5.272 \text{ kJ/kg}$$

Total pump work

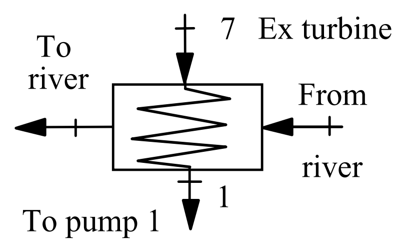
$$\dot{W}_p = \dot{m}_1 w_{P1} + \dot{m}_{tot}\, w_{P2} = 4.477 \times 1.708 + 5 \times 5.272 = \mathbf{34\ kW}$$

11.44

A steam power plant operates with a boiler output of 20 kg/s steam at 2 MPa, 600°C. The condenser operates at 50°C dumping energy to a river that has an average temperature of 20°C. There is one open feedwater heater with extraction from the turbine at 600 kPa and its exit is saturated liquid. Find the mass flow rate of the extraction flow. If the river water should not be heated more than 5°C how much water should be pumped from the river to the heat exchanger (condenser)?

Solution:

The setup is as shown in Fig. 11.10.



1: 50°C sat liq. $v_1 = 0.001012\ m^3/kg$, $h_1 = 209.31$ kJ/kg

2: 600 kPa $s_2 = s_1$

3: 600 kPa, sat liq. $h_3 = h_f = 670.54$ kJ/kg

5: (P, T) $h_5 = 3690.1$ kJ/kg, $s_5 = 7.7023$ kJ/kg K

6: 600 kPa, $s_6 = s_5$ => $h_6 = 3270.0$ kJ/kg

CV P1

$$w_{P1} = v_1(P_2 - P_1) = 0.001012\ (600 - 12.35) = 0.595 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{P1} = 209.9 \text{ kJ/kg}$$

C.V FWH

$$x\ h_6 + (1 - x)\ h_2 = h_3$$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{670.54 - 209.9}{3270.0 - 209.9} = 0.1505$$

$$\dot{m}_6 = x\ \dot{m}_5 = 0.1505 \times 20 = \mathbf{3\ kg/s}$$

CV Turbine: $s_7 = s_6 = s_5$ => $x_7 = 0.9493$, $h_7 = 2471.17$ kJ/kg

CV Condenser

$$q_L = h_7 - h_1 = 2471.17 - 209.31 = 2261.86 \text{ kJ/kg}$$

The heat transfer out of the water from 7 to 1 goes into the river water

$$\dot{Q}_L = (1 - x)\ \dot{m} q_L = 0.85 \times 20 \times 2261.86 = 38\ 429 \text{ kW}$$

$$= \dot{m}_{H2O}\ \Delta h_{H2O} = \dot{m}_{H2O}\ (h_{f25} - h_{f20}) = \dot{m}\ (20.93)$$

$$\dot{m} = 38\ 429 / 20.93 = \mathbf{1836\ kg/s}$$

11.45

Consider an ideal steam regenerative cycle in which steam enters the turbine at 3.0 MPa, 400°C, and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa for an open feedwater heater. The feedwater leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and the feedwater heater. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

This is a standard Rankine cycle with an open FWH as shown in Fig.11.10

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$\Rightarrow \; h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$

C.V. FWH Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

$$(1 - x)\, h_2 + x\, h_6 = 1\, h_3$$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

CV Boiler: $q_H = h_5 - h_4 = 3230.82 - 723.55 = 2507.3$ kJ/kg

CV Turbine

2nd Law $s_7 = s_6 = s_5 = 6.9211$ kJ/kg K

P_6 , s_6 => $h_6 = 2891.6$ kJ/kg (superheated vapor)

$$s_7 = s_6 = s_5 = 6.9211 \quad \Rightarrow \quad x_7 = \frac{6.9211 - 0.6492}{7.501} = 0.83614$$

$$\Rightarrow \quad h_7 = 191.81 + x_7\, 2392.82 = 2192.55 \text{ kJ/kg}$$

Turbine has full flow in HP section and fraction 1-x in LP section

$$\dot{W}_T / \dot{m}_5 = h_5 - h_6 + (1 - x)(h_6 - h_7)$$

$$w_T = 3230.82 - 2891.6 + (1 - 0.1988)(2891.6 - 2192.55) = 899.3$$

P2 has the full flow and P1 has the fraction 1-x of the flow

$$w_{net} = w_T - (1 - x)\, w_{P1} - w_{P2}$$

$$= 899.3 - (1 - 0.1988)0.798 - 2.45 = \mathbf{896.2 \text{ kJ/kg}}$$

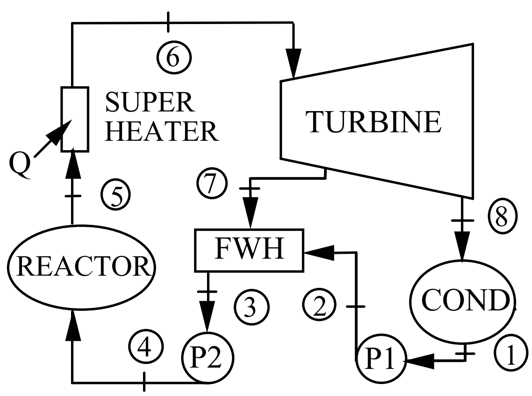
$$\eta_{cycle} = w_{net} / q_H = 896.2 / 2507.3 = \mathbf{0.357}$$

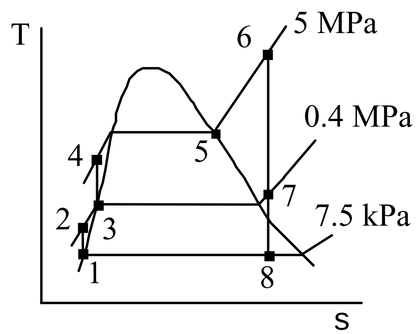
11.46

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this heat exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 0.4 MPa. The isentropic turbine efficiency is 87%, and the condenser pressure is 7.5 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1 MW.

Solution:

The complete cycle diagram is similar to Figure 11.8 except the boiler is sparated into a section heated by the reactor and a super heater section.





CV. Pump P1

$w_{P1} = 0.001008(400 - 7.5) = 0.4$ kJ/kg ;

$h_2 = h_1 + w_{P1} = 168.8 + 0.4 = 169.2$ kJ/kg

CV. Pump P2

$w_{P2} = 0.001084(5000 - 400) = 5.0$ kJ/kg

$h_4 = h_3 + w_{P2} = 604.7 + 5.0 = 609.7$ kJ/kg

C.V. Turbine (to get exit state properties)

$s_7 = s_6 = 7.2589$, $P_7 = 0.4$ MPa => $T_7 = 221.2^{o}C$, $h_7 = 2904.5$ kJ/kg

$s_8 = s_6 = 7.2589 = 0.5764 + x_8 \times 7.6750$ $x_8 = 0.8707$

$h_8 = 168.8 + 0.8707 \times 2406.0 = 2263.7$ kJ/kg

CV: Feedwater heater FWH (to get the extraction fraction x_7)

Divide the equations with the total mass flow rate $\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6$

Continuity: $x_2 + x_7 = x_3 = 1.0$, Energy Eq.: $x_2h_2 + x_7h_7 = h_3$

$x_7 = (604.7-169.2)/(2904.5-169.2) = 0.1592$

CV: Turbine (to get the total specific work)

Full flow from 6 to 7 and the fraction $(1 - x_7)$ from 7 to 8.

$$w_T = (h_6 - h_7) + (1 - x_7)(h_7 - h_8)$$

$$= 3666.5-2904.5 + 0.8408(2904.5-2263.7) = 1300.8 \text{ kJ/kg}$$

CV: Pumps (P1 has $x_1 = 1 - x_7$, P2 has the full flow $x_3 = 1$)

$$w_P = x_1 w_{P1} + x_3 w_{P2} = 0.8408 \times 0.4 + 1 \times 5.0 = 5.3 \text{ kJ/kg}$$

$$w_{NET} = 1300.8 - 5.3 = 1295.5 \quad => \quad \dot{m} = 1000/1295.5 = 0.772 \text{ kg/s}$$

CV: Reactor (this has the full flow)

$$\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 0.772(2794.3 - 609.7) = \mathbf{1686\ kW}$$

CV: Superheater (this has the full flow)

$$\dot{Q}_{SUP} = \dot{m}(h_6 - h_5) = 0.772\,(3666.5 - 2794.3) = \mathbf{673\ kW}$$

11.47

A steam power plant has high and low pressures of 20 MPa and 10 kPa, and one open feedwater heater operating at 1 MPa with the exit as saturated liquid. The maximum temperature is 800°C and the turbine has a total power output of 5 MW. Find the fraction of the flow for extraction to the feedwater and the total condenser heat transfer rate.

The physical components and the T-s diagram is as shown in Fig. 11.10 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5: (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1: (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101$ m^3/kg

State 3: (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127$ m^3/kg

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$$h_2 = h_1 + w_{P1} = 192.81 \text{ kJ/kg}$$

Turbine 5-6: $s_6 = s_5 \;\Rightarrow\; h_6 = 3013.7$ kJ/kg

$$w_{T56} = h_5 - h_6 = 4069.8 – 3013.7 = 1056.1 \text{ kJ/kg}$$

Feedwater Heater ($\dot{m}_{TOT} = \dot{m}_5$): $\quad x\dot{m}_5h_6 + (1 - x)\dot{m}_5h_2 = \dot{m}_5h_3$

$$\Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.8 - 192.81}{3013.7 - 192.81} = \mathbf{0.2021}$$

To get state 7 into condenser consider turbine.

$$s_7 = s_6 = s_5 \;\Rightarrow\; x_7 = (7.0544 - 0.6493)/7.5009 = 0.85391$$

$$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1 \text{ kJ/kg}$$

Find specific turbine work to get total flow rate

$$\dot{W}_T = \dot{m}_{TOT}h_5 - x\dot{m}_{TOT}h_6 - (1 - x)\dot{m}_{TOT}h_7 =$$

$$= \dot{m}_{TOT} \times (h_5 - xh_6 - (1 - x)h_7) = \dot{m}_{TOT} \times 1677.3$$

$$\dot{m}_{TOT} = 5000/1677.3 = 2.98 \text{ kg/s}$$

$$\dot{Q}_L = \dot{m}_{TOT}\,(1-x)\,(h_7-h_1) = 2.98 \times 0.7979(2235.1 - 191.81) = \mathbf{4858\ kW}$$

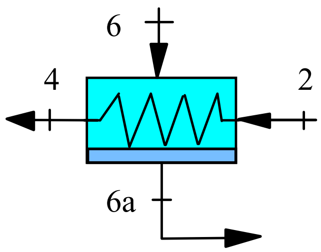
Closed Feedwater Heaters

11.48

A closed feedwater heater in a regenerative steam power cycle heats 20 kg/s of water from 100°C, 20 MPa to 250°C, 20 MPa. The extraction steam from the turbine enters the heater at 4 MPa, 275°C, and leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The schematic is from Figure 11.11 has the feedwater from the pump coming at state 2 being heated by the extraction flow coming from the turbine state 6 so the feedwater leaves as saturated liquid state 4 and the extraction flow leaves as condensate state 6a.



From table B.1	h kJ/kg
B.1.4: 100°C, 20 MPa	$h_2 = 434.06$
B.1.4: 250°C, 20 MPa	$h_4 = 1086.75$
B.1.3: 4 MPa, 275°C	$h_6 = 2886.2$
B.1.2: 4 MPa, sat. liq.	$h_{6a} = 1087.31$

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Since all four state are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6} = \mathbf{7.257\ kg/s}$$

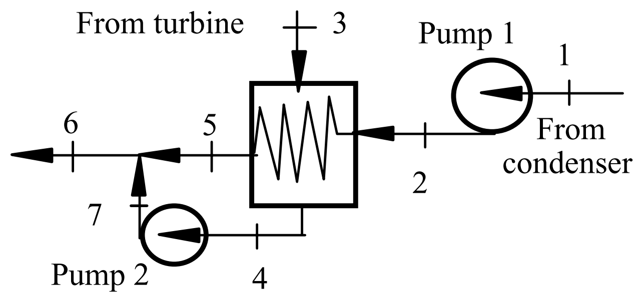
11.49

A power plant with one closed feedwater heater has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater condenses and is pumped up to the 5 MPa feedwater line where all the water goes to the boiler at 200°C. Find the fraction of extraction steam flow and the two specific pump work inputs.

Solution:

s_1 = 0.6387 kJ/kg K,
h_1 = 188.45 kJ/kg
v_1 = 0.00101 m^3/kg,
s_4 = 2.1387 kJ/kg K,
h_4 = 762.81 kJ/kg

T_6 => h_6 = 853.9 kJ/kg



C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$s_3 = s_{IN}$ = 7.9593 kJ/kg K => T_3 = **573.8**, h_3 = 3640.6 kJ/kg

C.V. P1: $w_{P1} = v_1(P_2 - P_1)$ = **5.04 kJ/kg** ⇒ $h_2 = h_1 + w_{P1}$ = 193.49 kJ/kg

C.V. P2: $w_{P2} = v_4(P_7 - P_4)$ = **4.508 kJ/kg** ⇒ $h_7 = h_4 + w_{P2}$ = 767.31 kJ/kg

C.V. Total FWH and pumps:

The extraction fraction is: $x = \dot{m}_3/\dot{m}_6$

Continuity Eq.: $\dot{m}_6 = \dot{m}_1 + \dot{m}_3$, $1 = (1-x) + x$

Energy: $(1 - x)(h_1 + w_{P1}) + x(h_3 + w_{P2}) = h_6$

$$x = \frac{h_6 - h_2}{h_3 + w_{P2} - h_2} = \frac{853.9 - 193.49}{3640.6 + 4.508 - 193.49} = 0.1913$$

$\dot{m}_3/\dot{m}_6$ **= x = 0.1913**

11.50

Repeat Problem 11.45, but assume a closed instead of an open feedwater heater. A single pump is used to pump the water leaving the condenser up to the boiler pressure of 3.0 MPa. Condensate from the feedwater heater is drained through a trap to the condenser.

Solution:

C.V. Turbine, 2nd law:

$s_4 = s_5 = s_6 = 6.9211$ kJ/kg K

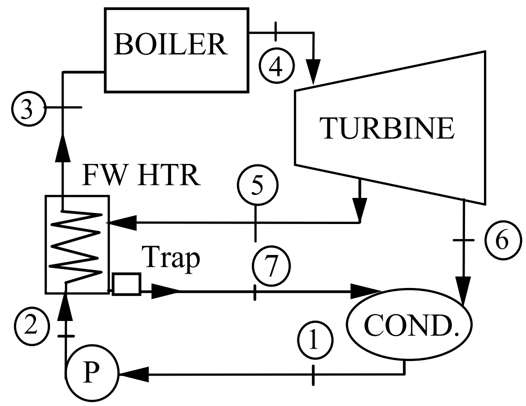
$h_4 = 3230.82$, $h_5 = 2891.6$

=> $x_6 = (6.9211 - 0.6492)/7.501$

$= 0.83614$

$h_6 = 191.81 + x_6\ 2392.82$

$=2192.55$ kJ/kg



Assume feedwater heater exit at the T of the condensing steam

C.V Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

$T_3 = T_{sat}\ (P_5) = 170.43°C$, $h_3 = h_f = h_7 = 721.1$ kJ/kg

C.V FWH

$\dot{m}_5 / \dot{m}_3 = x$, Energy Eq.: $h_2 + x\ h_5 = h_3 + h_7\ x$

$$x = \frac{h_3 - h_2}{h_5 - h_{f\,800}} = \frac{721.1 - 194.83}{2891.6 - 721.1} = 0.2425$$

Turbine work with full flow from 4 to 5 fraction 1-x flows from 5 to 6

$w_T = h_4 - h_5 + (1 - x)(h_5 - h_6)$

$= 3230.82 – 2891.6 + 0.7575\ (2891.6 - 2192.55)$

$= 868.75$ kJ/kg

$w_{net} = w_T - w_P = 868.75 - 3.02 =$ **865.7 kJ/kg**

$q_H = h_4 - h_3 = 3230.82 - 721.1 = 2509.7$ kJ/kg

$\eta_{cycle} = w_{net} / q_H = 865.7 / 2509.7 =$ **0.345**

11.51

Do Problem 11.47 with a closed feedwater heater instead of an open and a drip pump to add the extraction flow to the feed water line at 20 MPa. Assume the temperature is 175°C after the drip pump flow is added to the line. One main pump brings the water to 20 MPa from the condenser.

Solution:

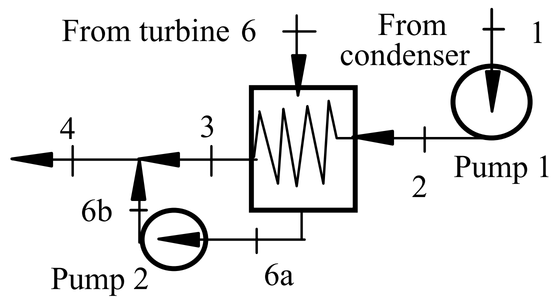
$v_1 = 0.00101\ m^3/kg$,

$h_1 = 191.81$ kJ/kg

$T_4 = 175^oC$; $h_4 = 751.66$ kJ/kg

$h_{6a} = h_{f\ 1MPa} = 762.79$ kJ/kg,

$v_{6a} = 0.001127\ m^3/kg$



Turbine section 1: $s_6 = s_5 = 7.0544$ kJ/kg K

$P_6 = 1$ MPa => $h_6 = 3013.7$ kJ/kg

C.V Pump 1

$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(20\ 000 - 10) = 20.19$ kJ/kg

=> $h_2 = h_1 + w_{P1} = 191.81 + 20.19 = 212.0$ kJ/kg

C.V Pump 2

$w_{P2} = h_{6b} - h_{6a} = v_{6a}(P_{6b} - P_{6a}) = 0.001127(20\ 000 - 1000) = 21.41$ kJ/kg

C.V FWH + P2 select the extraction fraction to be $x = \dot{m}_6 / \dot{m}_4$

$$x\ h_6 + (1 - x)\ h_2 + x\ (w_{P2}) = h_4$$

$$x = \frac{h_4 - h_2}{h_6 - h_2 - w_{P2}} = \frac{751.66 - 212.0}{3013.7 - 212.0 + 21.41} = \mathbf{0.191}$$

Turbine: $s_7 = s_6 = s_5$ & $P_7 = 10$ kPa

$$=> \quad x_7 = \frac{7.0544 - 0.6493}{7.5009} = 0.85391$$

$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

$w_T = [\ h_5 - h_6 + (1 - x)\ (h_6 - h_7)\ ]$

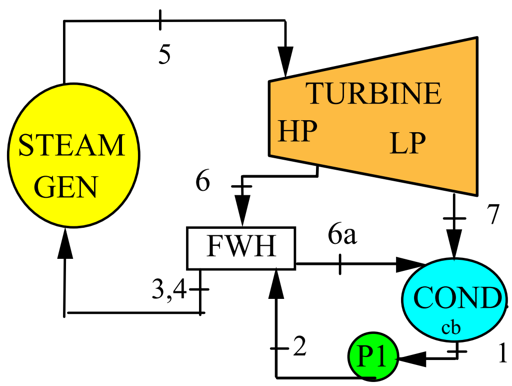
$= [\ 4069.8 – 3013.7 + 0.809\ (3013.7 - 2235.1)] = 1686$ kJ/kg

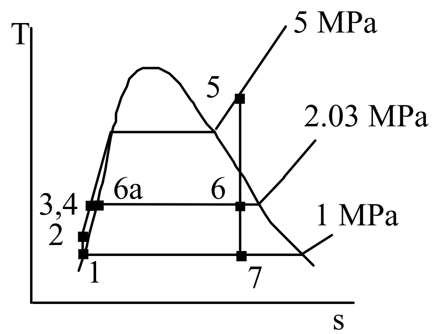
$\dot{W}_T = 5000$ kW $= \dot{m}_5 \times w_T = \dot{m}_5 \times 1686$ kJ/kg => $\dot{m}_5 = 2.966$ kg/s

$\dot{Q}_L = \dot{m}_5(1 - x)\ (h_7 - h_1) = 2.966 \times 0.809\ (2235.1 - 191.81) =$ **4903 kW**

11.52

Assume the powerplant in Problem 11.43 has one closed feedwater heater instead of the open FWH. The extraction flow out of the FWH is saturated liquid at 2033 kPa being dumped into the condenser and the feedwater is heated to 50°C. Find the extraction flow rate and the total turbine power output.





State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658\ m^3/kg$

State 3: $h_3 = h_f + (P_3 – P_{sat})v_f = 421.48 + (5000–2033)0.001777 = 426.75$ kJ/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5 \ \ \Rightarrow \ \ x_6 = (s_6 – s_f)/s_{fg} = 0.99052$, $\ \ h_6 = 1461.53$ kJ/kg

State 6a: $x_{6a} = 0 \ \ \Rightarrow \ \ h_{6a} = 421.48$ kJ/kg

State 7: $s_7 = s_5 \ \ \Rightarrow \ \ x_7 = (s_7 – s_f)/s_{fg} = 0.9236$, $\ \ h_7 = 1374.43$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(5000 - 1003) = 6.627 \text{ kJ/kg}$$

$$\Rightarrow \ h_2 = h_1 + w_{P1} = 298.25 + 6.627 = 304.88 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $h_2 + x\, h_6 = 1\, h_3 + x\, h_{6a}$

$$x = \frac{h_3 - h_2}{h_6 - h_{6a}} = \frac{426.75 - 304.88}{1461.53 - 421.48} = \mathbf{0.1172}$$

$$\dot{m}_{extr} = x\, \dot{m}_{tot} = 0.1172 \times 5 = \mathbf{0.586\ kg/s}$$

Total turbine work

$$\dot{W}_T = \dot{m}_{tot}(h_5 – h_6) + (1 – x)\dot{m}_{tot}\,(h_6 – h)$$

$$= 5(1586.3 – 1461.53) + (5 – 0.586)(1461.53 – 1374.43)$$

$$= \mathbf{1008\ kW}$$

Nonideal Cycles

11.53

Steam enters the turbine of a power plant at 5 MPa and 400°C, and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20 000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant and how does this compare with a Carnot cycle.

Solution: $\dot{W}_T = 20\,000$ kW and $\eta_{Ts} = 85$ %

State 3: $h_3 = 3195.6$ kJ/kg , $s_3 = 6.6458$ kJ/kgK

State 1: $P_1 = P_4 = 10$ kPa , sat liq , $x_1 = 0$

$T_1 = 45.8^oC$, $h_1 = h_f = 191.8$ kJ/kg , $v_1 = v_f = 0.00101$ m^3/kg

C.V Turbine : 1st Law: $q_T + h_3 = h_4 + w_T$; $q_T = 0$

$w_T = h_3 - h_4$, Assume Turbine is isentropic

$s_{4s} = s_3 = 6.6458$ kJ/kgK , $s_{4s} = s_f + x_{4s}\, s_{fg}$, solve for $x_{4s} = 0.7994$

$h_{4s} = h_f + x_{4s}h_{fg} = 1091.0$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 1091$ kJ/kg , $w_T = \eta_{Ts}w_{Ts} = 927.3$ kJ/kg

$\dot{m} = \frac{\dot{W}_T}{w_T} =$ **21.568 kg/s** , $h_4 = h_3 - w_T = 2268.3$ kJ/kg

C.V. Condenser: 1st Law : $h_4 = h_1 + q_c + w_c$; $w_c = 0$

$q_c = h_4 - h_1 = 2076.5$ kJ/kg , $\dot{Q}_c = \dot{m}\, q_c =$ **44 786 kW**

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 5.04$ kJ/kg

1st Law : $h_2 = h_1 + w_p = 196.8$ kJ/kg

C.V Boiler : 1st Law : $q_B + h_2 = h_3 + w_B$; $w_B = 0$

$q_B = h_3 - h_2 = 2998.8$ kJ/kg

$w_{net} = w_T - w_P = 922.3$ kJ/kg

$\eta_{th} = w_{net} / q_B =$ **0.307**

Carnot cycle : $T_H = T_3 = 400^oC$, $T_L = T_1 = 45.8^oC$

$\eta_{th} = \frac{T_H - T_L}{T_H} =$ **0.526**

11.54

A steam power plant has a high pressure of 5 MPa and maintains 50°C in the condenser. The boiler exit temperature is 600°C. All the components are ideal except the turbine which has an actual exit state of saturated vapor at 50°C. Find the cycle efficiency with the actual turbine and the turbine isentropic efficiency.

Solution:

A standard Rankine cycle with an actual non-ideal turbine.

Boiler exit: $h_3 = 3666.5$ kJ/kg, $s_3 = 7.2588$ kJ/kg K

Ideal Turbine: 4s: 50°C, $s = s_3 \Rightarrow x = (7.2588 - 0.7037)/7.3725 = 0.88913$,

$h_{4s} = 209.31 + 0.88913 \times 2382.75 = 2327.88$ kJ/kg

$\Rightarrow w_{Ts} = h_3 - h_{4s} = 1338.62$ kJ/kg

Condenser exit: $h_1 = 209.31$, Actual turbine exit: $h_{4ac} = h_g = 2592.1$

Actual turbine: $w_{Tac} = h_3 - h_{4ac} = 1074.4$ kJ/kg

$\eta_T = w_{Tac} / w_{Ts} =$ **0.803: Isentropic Efficiency**

Pump: $w_P = v_1(P_2 - P_1) = 0.001012(5000-12.35) = 5.05$ kJ/kg

$h_2 = h_1 + w_P = 209.31 + 5.05 = 214.36$ kJ/kg

$q_H = h_3 - h_2 = 3666.5 - 214.36 = 3452.14$ kJ/kg

$\eta_{cycle} = (w_{Tac} - w_P) / q_H =$ **0.31: Cycle Efficiency**

11.55

A steam power cycle has a high pressure of 3.0 MPa and a condenser exit temperature of 45°C. The turbine efficiency is 85%, and other cycle components are ideal. If the boiler superheats to 800°C, find the cycle thermal efficiency.

Solution:

Basic Rankine cycle as shown in Figure 11.3 in the main text.

C.V. Turbine: $w_T = h_3 - h_4$, $s_4 = s_3 + s_{T,GEN}$

Ideal Table B.1.3: $s_4 = s_3 = 7.9862$ kJ/kg K

$\Rightarrow x_{4s} = (7.9862 - 0.6386)/7.5261 = 0.9763$

$h_{4s} = h_f + x\ h_{fg} = 188.42 + 0.9763 \times 2394.77 = 2526.4$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 4146 - 2526.4 = 1619.6$ kJ/kg

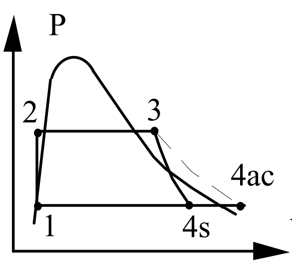
Actual: $w_{T,AC} = \eta \times w_{T,S} = 0.85 \times 1619.6 = 1376.66$ kJ/kg

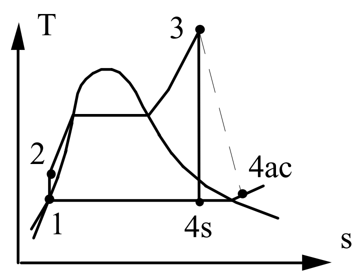
C.V. Pump: $w_P = \int v\ dP \approx v_1(P_2 - P_1) = 0.00101\ (3000 - 9.6) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 4146 - 191.44 = 3954.6$ kJ/kg

$\eta = (w_{T,AC} - w_P)/q_H = (1376.66 - 3.02)/3954.6 =$ **0.347**





11.56

A steam power plant operates with with a high pressure of 5 MPa and has a boiler exit temperature of of 600°C receiving heat from a 700°C source. The ambient at 20°C provides cooling for the condenser so it can maintain 45°C inside. All the components are ideal except for the turbine which has an exit state with a quality of 97%. Find the work and heat transfer in all components per kg water and the turbine isentropic efficiency. Find the rate of entropy generation per kg water in the boiler/heat source setup.

Solution:

Take CV around each component steady state in standard Rankine Cycle.

1: v = 0.00101; h = 188.42, s = 0.6386 (saturated liquid at 45°C).

3: h = 3666.5 kJ/kg, s = 7.2588 kJ/kg K superheated vapor

4_{ac}: h = 188.42 + 0.97 × 2394.8 = 2511.4 kJ/kg

CV Turbine: no heat transfer q = 0

$w_{ac} = h_3 - h_{4ac} = 3666.5 - 2511.4 =$ **1155.1 kJ/kg**

Ideal turbine: $s_4 = s_3 = 7.2588$ => $x_{4s} = 0.88$, $h_{4s} = 2295$ kJ/kg

$w_s = h_3 - h_{4s} = 3666.5 - 2295 = 1371.5$ kJ/kg,

$Eff = w_{ac} / w_s = 1155.1 / 1371.5 =$ **0.842**

CV Condenser: no shaft work w = 0

$q_{out} = h_{4ac} - h_1 = 2511.4 - 188.42 =$ **2323 kJ/kg**

CV Pump: no heat transfer, q = 0 incompressible flow so v = constant

$w = v(P_2 - P_1) = 0.00101(5000-9.59) =$ **5.04 kJ/kg**

CV Boiler: no shaft work, w = 0

$q_H = h_3 - h_2 = h_3 - h_1 - w_P = 3666.5 - 188.42 - 5.04 =$ **3473 kJ/kg**

$s_2 + (q_H / T_H) + s_{Gen} = s_3$ and $s_2 = s_1$ (from pump analysis)

$s_{gen} = 7.2588 - 0.6386 - 3473/(700+273) =$ **3.05 kJ/kg K**

11.57

For the steam power plant described in Problem 11.21, assume the isentropic efficiencies of the turbine and pump are 85% and 80%, respectively. Find the component specific work and heat transfers and the cycle efficiency.

Solution:

This is a standard Rankine cycle with actual non-ideal turbine and pump.

CV Pump, Rev & Adiabatic:

$$w_{Ps} = h_{2s} - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}; \quad s_{2s} = s_1$$

$$w_{Pac} = w_{Ps} / \eta_P = 3.02/0.8 = \mathbf{3.775} \text{ kJ/kg} = h_{2a} - h_1$$

$$h_{2a} = w_{Pac} + h_1 = 3.775 + 191.81 = 195.58 \text{ kJ/kg}$$

CV Boiler: $q_H = h_3 - h_{2a} = 2804.14 – 195.58 = \mathbf{2608.56 \text{ kJ/kg}}$

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \quad => \quad x_4 = 0.7383$$

$$=> \; h_4 = 191.81 + 0.7383 (2392.82) = 1958.34 \text{ kJ/kg}$$

$$w_{Ts} = 2804.14 - 1958.34 = 845.8 \text{ kJ/kg}$$

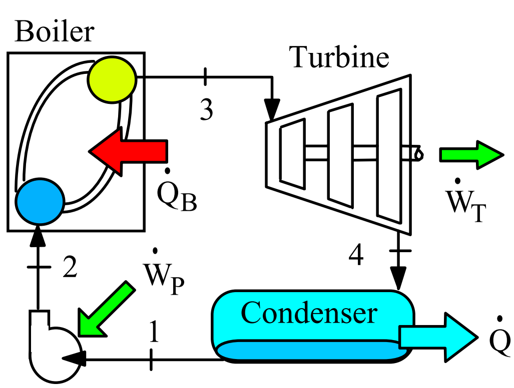
$$w_{Tac} = w_{Ts} \times \eta_T = \mathbf{718.9} = h_3 - h_{4a}$$

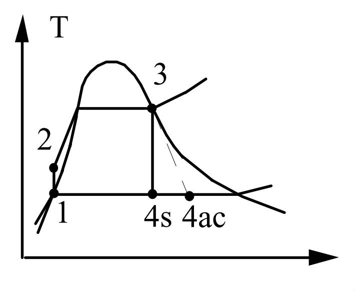
$$h_{4a} = h_3 - w_{Tac} = 2804.14 - 718.9 = 2085.24 \text{ kJ/kg}$$

CV Condenser: $q_L = h_{4a} - h_1 = 2085.24 - 191.81 = \mathbf{1893.4 \text{ kJ/kg}}$

$$\eta_{cycle} = (w_{Tac} - w_{Pac}) / q_H = (718.9 – 3.78) / 2608.56 = \mathbf{0.274}$$

This compares to 0.32 for the ideal case.





state 2s and 2ac nearly the same

11.58

A small steam power plant has a boiler exit of 3 MPa, 400°C while it maintains 50 kPa in the condenser. All the components are ideal except the turbine which has an isentropic efficiency of 80% and it should deliver a shaft power of 9.0 MW to an electric generator. Find the specific turbine work , the needed flow rate of steam and the cycle efficiency.

Solution:

This is a standard Rankine cycle with an actual non-ideal turbine.

CV Turbine (Ideal):

$s_{4s} = s_3 = 6.9211$ kJ/kg K, $x_{4s} = (6.9211 - 1.091)/6.5029 = 0.8965$

$h_{4s} = 2407.35$ kJ/kg, $h_3 = 3230.8$ kJ/kg

=> $w_{Ts} = h_3 - h_{4s} = 823.45$ kJ/kg

CV Turbine (Actual):

$w_{Tac} = \eta_T \times w_{Ts} = \mathbf{658.76} = h_3 - h_{4ac}$, => $h_{4ac} = 2572$ kJ/kg

$\dot{m} = \dot{W} / w_{Tac} = 9000/658.76 =$ **13.66 kg/s**

C.V. Pump:

$w_P = h_2 - h_1 = v_1(P_2 - P_1) = 0.00103\ (3000 - 50) = 3.04$ kJ/kg

=> $h_2 = h_1 + w_P = 340.47 + 3.04 = 343.51$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 3230.8 - 343.51 = 2887.3$ kJ/kg

$\eta_{cycle} = (w_{Tac} - w_P) / q_H = (658.76 - 3.04) / 2887.3 = \mathbf{0.227}$

11.59

Repeat Problem 11.47 assuming the turbine has an isentropic efficiency of 85%.

The physical components and the T-s diagram is as shown in Fig. 11.10 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5: (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1: (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101\ m^3/kg$

State 3: (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127\ m^3/kg$

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$h_2 = h_1 + w_{P1} = 192.81$ kJ/kg

Turbine 5-6: $s_6 = s_5 \Rightarrow h_6 = 3013.7$ kJ/kg

$w_{T56,s} = h_5 - h_6 = 4069.8 – 3013.7 = 1056.1$ kJ/kg

$\Rightarrow w_{T56,AC} = 1056.1 \times 0.85 = 897.69$ kJ/kg

$w_{T56,AC} = h_5 - h_{6AC} \Rightarrow h_{6AC} = h_5 - w_{T56,AC}$

$= 4069.8 - 897.69 = 3172.11$ kJ/kg

Feedwater Heater ($\dot{m}_{TOT} = \dot{m}_5$): $x\dot{m}_5h_{6AC} + (1 - x)\dot{m}_5h_2 = \dot{m}_5h_3$

$$\Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.8 - 192.81}{3172.11 - 192.81} = \mathbf{0.1913}$$

To get the turbine work apply the efficiency to the whole turbine. (i.e. the first section should be slightly different).

$s_{7s} = s_{6s} = s_5 \Rightarrow x_{7s} = (7.0544 – 0.6493)/7.5009 = 0.85391,$

$h_{7s} = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

$w_{T57,s} = h_5 - h_{7s} = 4069.8 - 2235.1 = 1834.7$ kJ/kg

$w_{T57,AC} = w_{T57,s}\eta_T = 1559.5 = h_5 - h_{7AC}$ => $h_{7AC} = 2510.3$ kJ/kg

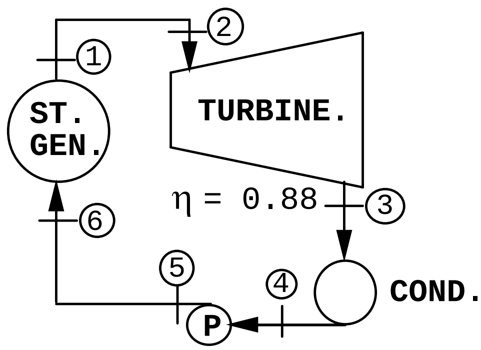
Find specific turbine work to get total flow rate

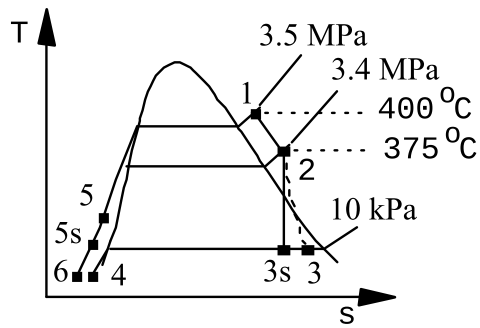
$$\dot{m}_{TOT} = \frac{\dot{W}_T}{xw_{T56} + (1\text{-}x)w_{T57}} = \frac{5000}{0.1913\times897.69 + 0.8087\times1559.5} = \mathbf{3.489\ kg/s}$$

$\dot{Q}_L = \dot{m}_{TOT}(1 - x)(h_7 - h_1) = 3.489 \times 0.8087(2510.3 - 191.81) = \mathbf{6542\ kW}$

11.60

Steam leaves a power plant steam generator at 3.5 MPa, 400°C, and enters the turbine at 3.4 MPa, 375°C. The isentropic turbine efficiency is 88%, and the turbine exhaust pressure is 10 kPa. Condensate leaves the condenser and enters the pump at 35°C, 10 kPa. The isentropic pump efficiency is 80%, and the discharge pressure is 3.7 MPa. The feedwater enters the steam generator at 3.6 MPa, 30°C. Calculate the thermal efficiency of the cycle and the entropy generation for the process in the line between the steam generator exit and the turbine inlet, assuming an ambient temperature of 25°C.





1: $h_1 = 3222.3$ kJ/kg, $s_1 = 6.8405$ kJ/kg K,

2: $h_2 = 3165.7$ kJ/kg, $s_2 = 6.7675$ kJ/kg K

3s: $s_{3S} = s_2 \Rightarrow x_{3S} = 0.8157$, $h_{3S} = 2143.6$ kJ/kg

$w_{T,S} = h_2 - h_{3S} = 3165.7 - 2143.6 = 1022.1$ kJ/kg

$w_{T,AC} = \eta w_{T,S} = 899.4$ kJ/kg, 3ac: $h_3 = h_2 - w_{T,AC} = 2266.3$ kJ/kg

$-w_{P,S} = v_f(P_5 - P_4) = 0.001006(3700 - 10) = 3.7$ kJ/kg

$-w_{P,AC} = -w_{P,S}/\eta_P = 4.6$ kJ/kg

$q_H = h_1 - h_6 = 3222.3 - 129.0 = 3093.3$ kJ/kg

$\eta = w_{NET}/q_H = (899.4 - 4.6)/3093.3 =$ **0.289**

C.V. Line from 1 to 2: $w = \emptyset$,

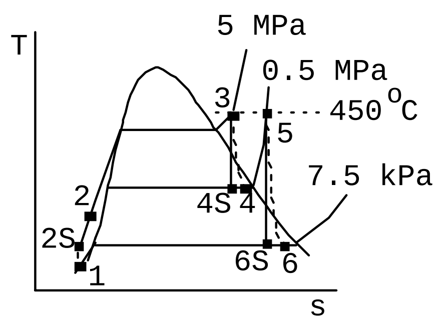
Energy Eq.: $q = h_2 - h_1 = 3165.7 - 3222.3 = -56.6$ kJ/kg

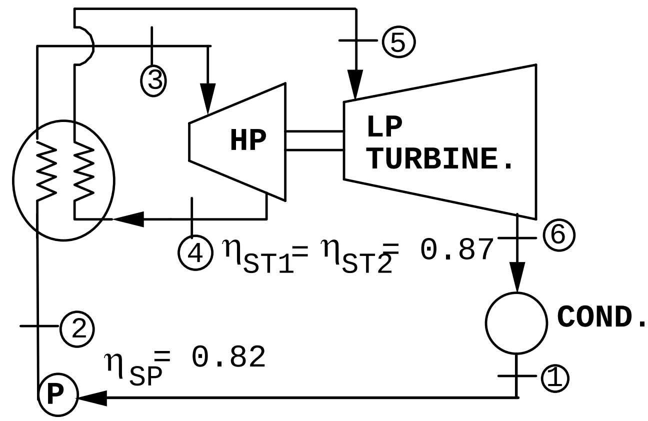
Entropy Eq.: $s_1 + s_{gen} + q/T_0 = s_2$ =>

$s_{gen} = s_2 - s_1 - q/T_0 = 6.7675 - 6.8405 - (-56.6/298.15) =$ **0.117 kJ/kg K**

11.61

In a particular reheat-cycle power plant, steam enters the high-pressure turbine at 5 MPa, 450°C and expands to 0.5 MPa, after which it is reheated to 450°C. The steam is then expanded through the low-pressure turbine to 7.5 kPa. Liquid water leaves the condenser at 30°C, is pumped to 5 MPa, and then returned to the steam generator. Each turbine is adiabatic with an isentropic efficiency of 87% and the pump efficiency is 82%. If the total power output of the turbines is 10 MW, determine the mass flow rate of steam, the pump power input and the thermal efficiency of the power plant.





a) $s_{4S} = s_3 = 6.8185 = 1.8606 + x_{4S} \times 4.9606 \quad => \quad x_{4S} = 0.999$

$h_{4S} = 640.21 + 0.999 \times 2108.5 = 2746.6$ kJ/kg

$w_{T1,S} = h_3 - h_{4S} = 3316.1 - 2746.6 = 569.5$ kJ/kg

$w_{T1} = \eta_{T1,S} \times w_{T1,S} = 0.87 \times 569.5 = 495.5$ kJ/kg

$h_{4ac} = 3316.1 - 495.5 = 2820.6$ kJ/kg

$s_{6S} = s_5 = 7.9406 = 0.5764 + x_{6S} \times 7.675 \quad \Rightarrow \quad x_{6S} = 0.9595$

$h_{6S} = 168.79 + 0.9595 \times 2406 = 2477.3$ kJ/kg

$w_{T2,S} = h_5 - h_{6S} = 3377.9 - 2477.3 = 900.6$ kJ/kg

$w_{T2} = 0.87 \times 900.6 = 783.5$ kJ/kg

$\dot{m} = \dot{W}_T/(w_{T1} + w_{T2}) = 10000/(783.5 + 495.5) =$ **7.82 kg/s**

b) $-w_{P,S} = (0.001004)(5000 - 7.5) = 5.01$ kJ/kg

$-w_P = -w_{SP}/\eta_{SP} = 5.01/0.82 = 6.11$ kJ/kg

$\dot{W}_P = w_P\dot{m} = -7.82 \times 6.11 =$ **-47.8 kW**

c) $q_H = (h_3 - h_2) + (h_5 - h_4) = 3316.1 - 130.2 + 3377.9 - 2820.6 = 3743.2$ kJ/kg

$w_N = 1279.0 - 6.11 = 1272.9$ kJ/kg

$\eta_{TH} = w_N/q_H = 1272.9/3743.2 =$ **0.34**

11.62

A supercritical steam power plant has a high pressure of 30 MPa and an exit condenser temperature of 50°C. The maximum temperature in the boiler is 1000°C and the turbine exhaust is saturated vapor There is one open feedwater heater receiving extraction from the turbine at 1MPa, and its exit is saturated liquid flowing to pump 2. The isentropic efficiency for the first section and the overall turbine are both 88.5%. Find the ratio of the extraction mass flow to total flow into turbine. What is the boiler inlet temperature with and without the feedwater heater?

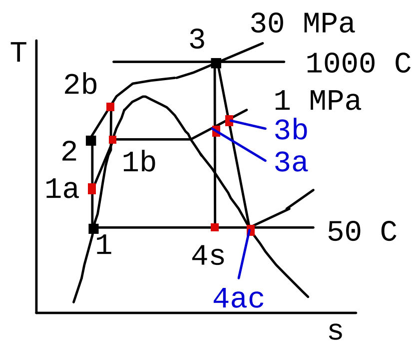
Basically a Rankine Cycle

1: 50°C, 12.35 kPa,
$h = 209.31$ kJ/kg, $s = 0.7037$ kJ/kg K

2: 30 MPa

3: 30 MPa, 1000 °C,
$h = 4554.7$ kJ/kg, $s = 7.2867$ kJ/kg K

4AC: 50°C, $x = 1$, $h = 2592.1$ kJ/kg

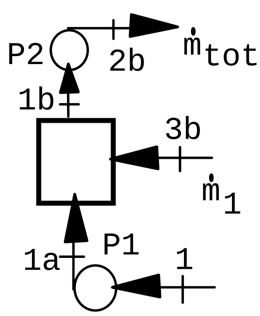


a) C.V. Turbine Ideal: $s_{4S} = s_3 \Rightarrow x_{4S} = 0.8929$,

$h_{4S} = 2336.8$ kJ/kg => $w_{T,S} = h_3 - h_{4S} = 2217.86$ kJ/kg

Actual: $w_{T,AC} = h_3 - h_{4AC} = 1962.6$ kJ/kg, $\eta = w_{T,AC}/w_{T,S} = \mathbf{0.885}$

b)



1b: Sat liq. 179.91°C, $h = 762.81$ kJ/kg

3a: 1 MPa, $s = s_3 \rightarrow h_{3a} = 3149.09$ kJ/kg,
$T_{3a} = 345.96 \rightarrow w_{T1s} = 1405.6$ kJ/kg

3b: 1 MPa, $w_{T1ac} = \eta w_{T1s} = 1243.96$ kJ/kg
$w_{T1ac} = h_3 - h_{3b} \Rightarrow h_{3b} = 3310.74$ kJ/kg

1a: $w_{P1} = v_1(P_{1a} - P_1) \approx 1$ kJ/kg
$h_{1a} = h_1 + w_{P1} = 210.31$ kJ/kg

C.V. Feedwater Heater: $\dot{m}_{TOT}h_{1b} = \dot{m}_1 h_{3b} + (\dot{m}_{TOT} - \dot{m}_1)h_{1a}$

$\Rightarrow \dot{m}_1/\dot{m}_{TOT} = x = (h_{1b} - h_{1a})/(h_{3b} - h_{1a}) = \mathbf{0.178}$

c) C.V. Turbine: $(\dot{m}_{TOT})_3 = (\dot{m}_1)_{3b} + (\dot{m}_{TOT} - \dot{m}_1)_{4AC}$

$\dot{W}_T = \dot{m}_{TOT}h_3 - \dot{m}_1 h_{3b} - (\dot{m}_{TOT} - \dot{m}_1)h_{4AC} = 25 \text{ MW} = \dot{m}_{TOT}w_T$

$w_T = h_3 - xh_{3b} - (1-x)h_{4AC} = 1834.7$ kJ/kg => $\dot{m}_{TOT} = \mathbf{13.63\ kg/s}$

d) C.V. No FWH, Pump Ideal: $w_P = h_{2S} - h_1$, $s_{2S} = s_1$

Steam table $\Rightarrow$ $h_{2S} = 240.1$ kJ/kg, $T_{2S} = \mathbf{51.2°C}$

1 FWH, CV: P2. $s_{2b} = s_{1b} = 2.1386$ kJ/kg K => $T_{2b} = \mathbf{183.9°C}$

Cogeneration

11.63

A cogenerating steam power plant, as in Fig. 11.13, operates with a boiler output of 25 kg/s steam at 7 MPa, 500°C. The condenser operates at 7.5 kPa and the process heat is extracted as 5 kg/s from the turbine at 500 kPa, state 6 and after use is returned as saturated liquid at 100 kPa, state 8. Assume all components are ideal and find the temperature after pump 1, the total turbine output and the total process heat transfer.

Solution:

Pump 1: Inlet state is saturated liquid: $h_1 = 168.79$ kJ/kg, $v_1 = 0.001008$ m^3/kg

$$w_{P1} = \int v\, dP = v_1 (P_2 - P_1) = 0.001008(100 - 7.5) = 0.093 \text{ kJ/kg}$$

$$w_{P1} = h_2 - h_1 \Rightarrow h_2 = h_1 + w_{P1} = 168.88 \text{ kJ/kg}, \quad T_2 = \mathbf{40.3°C}$$

Turbine: $h_5 = 3410.3$ kJ/kg, $s_5 = 6.7974$ kJ/kg K

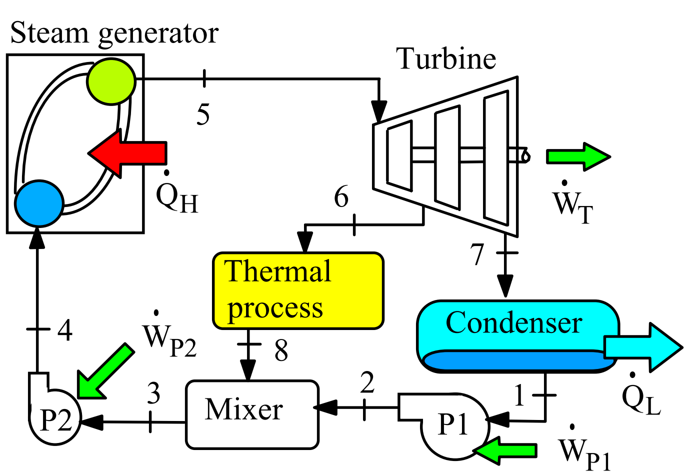
$$P_6,\ s_6 = s_5 \;\Rightarrow\; x_6 = 0.9952, \quad h_6 = 2738.6 \text{ kJ/kg}$$

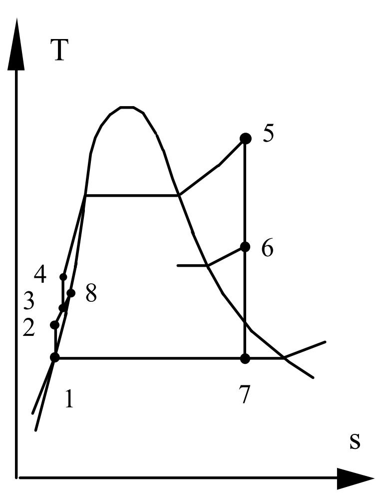
$$P_7,\ s_7 = s_5 \;\Rightarrow\; x_7 = 0.8106, \quad h_7 = 2119.0 \text{ kJ/kg}$$

From the continuity equation we have the full flow from 5 to 6 and the remainder after the extraction flow is taken out flows from 6 to 7.

$$\dot{W}_T = \dot{m}_5 (h_5 - h_6) + 0.80\dot{m}_5 (h_6 - h_7) = 25 (3410.3 - 2738.6)$$
$$+ 20 (2738.6 - 2119) = 16\,792.5 + 12\,392 = \mathbf{29.185\ MW}$$

$$\dot{Q}_{proc} = \dot{m}_6(h_6 - h_8) = 5(2738.6 - 417.46) = \mathbf{11.606\ MW}$$





11.64

A 10 kg/s steady supply of saturated-vapor steam at 500 kPa is required for drying a wood pulp slurry in a paper mill. It is decided to supply this steam by cogeneration, that is, the steam supply will be the exhaust from a steam turbine. Water at 20°C, 100 kPa, is pumped to a pressure of 5 MPa and then fed to a steam generator with an exit at 400°C. What is the additional heat transfer rate to the steam generator beyond what would have been required to produce only the desired steam supply? What is the difference in net power?

Solution:

Desired exit State 4: $P_4 = 500$ kPa, sat. vap. => $x_4 = 1.0$, $T_4 = 151.9°C$

$$h_4 = h_g = 2748.7 \text{ kJ/kg}, \quad s_4 = s_g = 6.8212 \text{ kJ/kg-K}$$

Inlet State: 20°C, 100 kPa $h_1 = h_f = 83.94$ kJ/kg, $v_1 = v_f = 0.001002$ m^3/kg

Without Cogeneration; The water is pumped up to 500 kPa and then heated in the steam generator to the desired exit T.

C.V. Pump: $w_{Pw/o} = v_1(P_4 - P_1) = 0.4$ kJ/kg

$$h_2 = h_1 + w_{Pw/o} = 84.3 \text{ kJ/kg}$$

C.V. Steam Generator: $q_{w/o} = h_4 - h_2 = 2664.4$ kJ/kg

With Cogeneration; The water is pumped to 5 MPa, heated in the steam generator to 400°C and then flows through the turbine with desired exit state.

C.V. Pump: $w_{Pw} = \int vdP = v_1(P_2 - P_1) = 4.91$ kJ/kg

$$h_2 = h_1 + w_{Pw} = 88.85 \text{ kJ/kg}$$

C.V. Steam Generator: Exit 400°C, 5 MPa => $h_3 = 3195.64$ kJ/kg

$$q_w = h_3 - h_2 = 3195.64 - 88.85 = 3106.8 \text{ kJ/kg}$$

C.V.: Turbine, Inlet and exit states given

$$w_t = h_3 - h_4 = 3195.64 - 2748.7 = 446.94 \text{ kJ/kg}$$

Comparison

Additional Heat Transfer: $q_w - q_{w/o} = 3106.8 - 2664.4 = 442.4$ kJ/kg

$$\dot{Q}_{extra} = \dot{m}(q_w - q_{w/o}) = \mathbf{4424\ kW}$$

Difference in Net Power: $w_{diff} = (w_t - w_{Pw}) + w_{Pw/o}$,

$$w_{diff} = 446.94 - 4.91 + 0.4 = 442.4 \text{ kJ/kg}$$

$$\dot{W}_{diff} = \dot{m}w_{diff} = \mathbf{4424\ kW}$$

By adding the extra heat transfer at the higher pressure and a turbine all the extra heat transfer can come out as work (it appears as a 100% efficiency)

11.65

In a cogenerating steam power plant the turbine receives steam from a high-pressure steam drum and a low-pressure steam drum as shown in Fig. P11.65. The condenser is made as two closed heat exchangers used to heat water running in a separate loop for district heating. The high-temperature heater adds 30 MW and the low-temperature heaters adds 31 MW to the district heating water flow. Find the power cogenerated by the turbine and the temperature in the return line to the deaerator.

Solution:

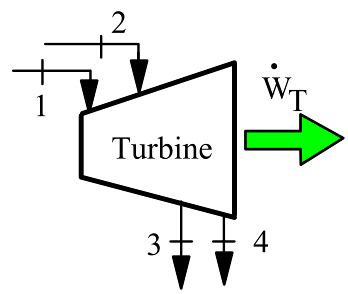
Inlet states from Table B.1.3

$h_1 = 3445.9$ kJ/kg, $s_1 = 6.9108$ kJ/kg K

$h_2 = 2855.4$ kJ/kg, $s_2 = 7.0592$ kJ/kg K

$\dot{m}_{TOT} = \dot{m}_1 + \dot{m}_2 = 27$ kg/s

Assume a reversible turbine and the two flows can mix without s generation.



Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_4 h_4 + \dot{W}_T$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 = \dot{m}_{TOT} s_{mix} \;\Rightarrow\; s_{MIX} = 6.9383$ kJ/kg K

State 3: $s_3 = s_{MIX} \;\Rightarrow\; h_3 = 2632.4$ kJ/kg, $x_3 = 0.966$

State 4: $s_4 = s_{MIX} \;\Rightarrow\; h_4 = 2413.5$ kJ/kg, $x_4 = 0.899$

$$\dot{W}_T = 22 \times 3445.9 + 5 \times 2855.4 - 13 \times 2632.4 - 14 \times 2413.5$$
$$= 22\ 077 \text{ kW} = \mathbf{22\ MW}$$

District heating line $\dot{Q}_{TOT} = \dot{m}(h_{95} - h_{60}) = 60\ 935$ kW

OK, this matches close enough

C.V. Both heaters: $\dot{m}_3 h_3 + \dot{m}_4 h_4 - \dot{Q}_{TOT} = \dot{m}_{TOT} h_{EX}$

$13 \times 2632.4 - 14 \times 2413.5 – 60\ 935 = 7075.2 = 27 \times h_{EX}$

$h_{EX} = 262 \approx h_f \;\Rightarrow\; T_{EX} = \mathbf{62.5°C}$

11.66

A boiler delivers steam at 10 MPa, 550°C to a two-stage turbine as shown in Fig. 11.17. After the first stage, 25% of the steam is extracted at 1.4 MPa for a process application and returned at 1 MPa, 90°C to the feedwater line. The remainder of the steam continues through the low-pressure turbine stage, which exhausts to the condenser at 10 kPa. One pump brings the feedwater to 1 MPa and a second pump brings it to 10 MPa. Assume the first and second stages in the steam turbine have isentropic efficiencies of 85% and 80% and that both pumps are ideal. If the process application requires 5 MW of power, how much power can then be cogenerated by the turbine?

Solution:

5: $h_5 = 3500.9$, $s_5 = 6.7567$ kJ/kg K

First ideal turbine T1

6s: $s_{6S} = s_5 \Rightarrow h_{6S} = 2932.1$ kJ/kg

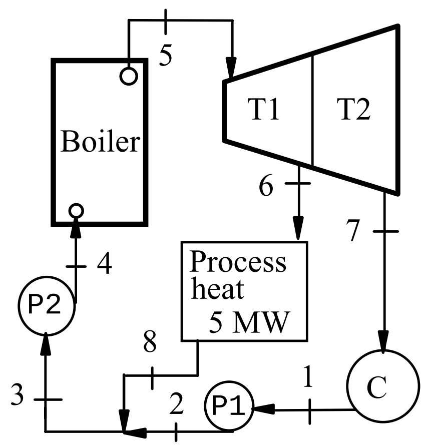
$w_{T1,S} = h_5 - h_{6S} = 568.8$ kJ/kg

Now the actual turbine T1

$\Rightarrow w_{T1,AC} = 483.5$ kJ/kg

$h_{6AC} = h_5 - w_{T1,AC} = 3017.4$

6ac: $P_6, h_{6AC} \Rightarrow s_{6AC} = 6.9129$ kJ/kg K



First ideal turbine T2 (it follows the actual T1)

State 7s: $s_{7S} = s_{6AC} \quad \Rightarrow \quad h_{7S} = 2189.9$ kJ/kg

$w_{T2,S} = h_{6AC} - h_{7S} = 827.5$ kJ/kg

$w_{T2,AC} = \eta w_{T2,S} = 622 = h_{6AC} - h_{7AC} \quad \Rightarrow \quad h_{7AC} = 2355.4$ kJ/kg

Now do the process heat requirement

8: $h_8 = 377.6$ kJ/kg, $\quad q_{PROC} = h_{6AC} - h_8 = 2639.8$ kJ/kg

$$\dot{m}_6 = \dot{Q}/q_{PROC} = 5000/2639.8 = 1.894 \text{ kg/s} = 0.25\, \dot{m}_{TOT}$$

$$\Rightarrow \dot{m}_{TOT} = \dot{m}_5 = 7.576 \text{ kg/s}, \ \dot{m}_7 = \dot{m}_5 - \dot{m}_6 = 5.682 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_{6AC} - \dot{m}_7 h_{7AC} = \mathbf{7424\ kW}$$

11.67

A smaller power plant produces 25 kg/s steam at 3 MPa, 600 C, in the boiler. It cools the condenser to an exit of 45C and the cycle is shown in Fig. P11.67. There is an extraction done at 500 kPa to an open feedwater heater, and in addition a steam supply of 5 kg/s is taken out and not returned. The missing 5 kg/s water is added to the feedwater heater from a 20C, 500 kPa source. Find the needed extraction flow rate to cover both the feedwater heater and the steam supply. Find the total turbine power output.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45^oC, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

5: 3.0 MPa, 600^oC: $h_5 = 3682.34$ kJ/kg, $s_5 = 7.5084$ kJ/kg K

3: 500 kPa, $x = 0$: $h_3 = 640.21$ kJ/kg 8: $h_8 = 84.41$ kJ/kg

6: 500 kPa, $s_6 = s_5$ from HP turbine, $h_6 = 3093.26$ kJ/kg

C.V. Pump 1. Reversible and adiabatic. Incompressible so v = constant

Energy: $w_{p1} = h_2 - h_1 = \int v\, dP = v_1(P_2 - P_1)$

$= 0.00101\ (500 - 9.6) = 0.495$ kJ/kg

$h_2 = h_1 + w_{p1} = 188.42 + 0.495 = 188.915$ kJ/kg

C.V. Turbine sections

Entropy Eq.: $s_7 = s_5 = 7.5084$ kJ/kg K => two-phase state

$s_7 = 7.5084 = 0.6386 + x_7 \times 7.5261 \;\Rightarrow\; x_7 = 0.9128$

$h_7 = 188.42 + 0.9128 \times 2394.77 = 2374.4$ kJ/kg

C.V. Feedwater heater, including the make-up water flow, $x = \dot{m}_6/\dot{m}_5$.

Energy eq.: $\dot{m}_8h_8 + (\dot{m}_5 - \dot{m}_6)h_2 + (\dot{m}_6 - \dot{m}_8)h_6 = \dot{m}_5h_3$

Divide by $\dot{m}_5$ and solve for x

$$x = \frac{h_3 - h_2 + (h_6 - h_8)\ \dot{m}_8/\ \dot{m}_5}{h_6 - h_2} = \frac{640.21 - 188.915 + (3093.26 - 84.41)5/25}{3093.26 - 188.915}$$

$= 0.3626$

$\dot{m}_6 = x\ \dot{m}_5 = 0.3626 \times 25 =$ **9.065 kg/s**

C.V. Turbine energy equation

$\dot{W}_T = \dot{m}_5h_5 - \dot{m}_6h_6 - \dot{m}_7h_7$

$= 25 \times 3682.34 - 9.065 \times 3093.26 - 16.935 \times 2374.4$

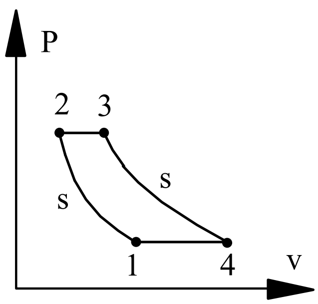
= 26 182 kW

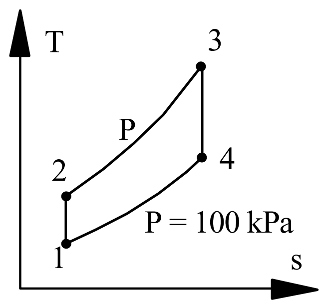
Brayton Cycles, Gas Turbines

11.68

Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:





Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$$T_3 = 1100^oC$$

$$\dot{m} = 10 \text{ kg/s}$$

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\dot{W}_C = \dot{m}w_C = \mathbf{3048\ kW}, \qquad \dot{W}_T = \dot{m}w_T = \mathbf{7013\ kW}$$

Energy added by the combustion process

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1373.2 - 596.8) = 779.5 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/779.5 = \mathbf{0.509}$$

11.69

Repeat Problem 11.68, but assume variable specific heat for the air, table A.7. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100^{o}C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:

From A.7: $h_1 = 293.6$ kJ/kg, $s^o_{T1} = 6.84597$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.84597 + 0.287\ln 12 = 7.55914$$

$$\Rightarrow\; T_2 = 590 \text{ K}, \quad h_2 = 597.2 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = h_2 - h_1 = 597.2 - 293.6 = 303.6 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.28

From A.7: $h_3 = 1483.1$, $s^o_{T3} = 8.50554$

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.50554 + 0.287\ln(1/12) = 7.79237$$

$$\Rightarrow\; T_4 = 734.8 \text{ K}, \quad h_4 = 751.1 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1483.1 - 751.1 = 732 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\Rightarrow\; \dot{W}_C = \dot{m}w_C = \mathbf{3036\ kW}, \quad \dot{W}_T = \dot{m}w_T = \mathbf{7320\ kW}$$

Energy added by the combustion process

$$q_H = h_3 - h_2 = 1483.1 - 597.2 = 885.9 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_C = 732 - 303.6 = 428.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 428.4/885.9 = \mathbf{0.484}$$

11.70

A Brayton cycle inlet is at 300 K, 100 kPa and the combustion adds 670 kJ/kg. The maximum temperature is 1200 K due to material considerations. What is the maximum allowed compression ratio? For this calculate the net work and cycle efficiency assuming variable specific heat for the air, table A.7.

Solution:

Combustion: $h_3 = h_2 + q_H$; ${}_2w_3 = 0$ and $T_{max} = T_3 = 1200$ K

$$h_2 = h_3 - q_H = 1277.8 - 670 = 607.8 \text{ kJ/kg}$$

From Table A.7.1

$$T_2 \approx 600 \text{ K};\quad s^o_{T2} = 7.57638 \text{ ; } T_1 = 300 \text{ K};\quad s^o_{T1} = 6.86926 \text{ kJ/kg K}$$

Reversible adiabatic compression leads to constant s, from Eq.8.28:

$$P_2 / P_1 = \exp[\ (s^o_{T2} - s^o_{T1})/R\ ] = \exp(2.4638) = \mathbf{11.75}$$

Reversible adiabatic expansion leads to constant s, from Eq.8.28

$$s^o_{T4} = s^o_{T3} + R \ln(P_4 / P_3) = 8.34596 + 0.287 \ln(1 / 11.75) = 7.6388 \text{ kJ/kgK}$$

From Table A.7.1 by linear interpolation $T_4 \approx 636.6$ K, $h_4 = 645.97$ kJ/kg

$$w_T = h_3 - h_4 = 1277.8 - 645.97 = 631.8 \text{ kJ/kg}$$

$$w_C = h_2 - h_1 = 607.8 - 300.47 = 307.3 \text{ kJ/kg}$$

$$w_{net} = w_T - w_C = 631.8 - 307.3 = \mathbf{324.5 \text{ kJ/kg}}$$

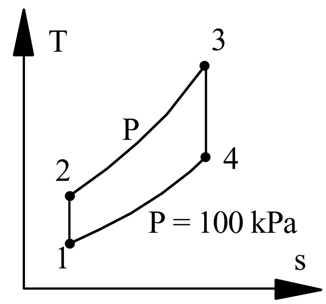
$$\eta = w_{net} / q_H = 324.5 / 670 = \mathbf{0.484}$$

11.71

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Brayton cycle so this means:
Minimum T: $T_1 = 300$ K
Maximum T: $T_3 = 1600$ K
Pressure ratio: $P_2/P_1 = 14$



Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Expansion in turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

$$w_{NET} = 851.2 - 339.5 = 511.7 \text{ kJ/kg}$$

Do the overall net and cycle efficiency

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/511.7 = 195.4 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 = \mathbf{166.32\ MW}$$

$$w_C/w_T = 339.5/851.2 = \mathbf{0.399}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 638.1) = 965.7 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 511.7/965.7 = \mathbf{0.530}$$

11.72

A Brayton cycle produces 14 MW with an inlet state of $17^{o}C$, 100 kPa, and a compression ratio of 16:1. The heat added in the combustion is 960 kJ/kg. What are the highest temperature and the mass flow rate of air, assuming cold air properties?

Solution:

Efficiency is from Eq.11.8

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = \mathbf{0.547}$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\ 000}{0.547} = 25\ 594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\ 594 \text{ kW}/\ 960 \text{ kJ/kg} = \mathbf{26.66\ kg/s}$$

Temperature after compression is

$$T_2 = T_1\ r_p^{(k-1)/k} = 290 \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 + \frac{960}{1.004} = \mathbf{1596.5\ K}$$

11.73

Do the previous problem with properties from table A.7.1 instead of cold air properties.

Solution:

With the variable specific heat we must go through the processes one by one to get net work and the highest temperature T_3.

From A.7.1: $h_1 = 290.43$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.83521 + 0.287 \ln 16 = 7.63094$$

$$\Rightarrow\; T_2 = 631.9 \text{ K}, \quad h_2 = 641 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = h_2 - h_1 = 641 - 290.43 = 350.57 \text{ kJ/kg}$$

Energy Eq. combustor: $h_3 = h_2 + q_H = 641 + 960 = 1601$ kJ/kg

State 3: (P, h): $\mathbf{T_3 = 1471\ K}$, $s^o_{T3} = 8.58811$ kJ/kg K

The expansion is reversible and adiabatic so constant s. From Eq.8.28

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.58811 + 0.287\ln(1/16) = 7.79238$$

$$\Rightarrow\; T_4 = 734.8 \text{ K}, \quad h_4 = 751.11 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1601 - 751.11 = 849.89 \text{ kJ/kg}$$

Now the net work is

$$w_{net} = w_T - w_C = 849.89 - 350.57 = 499.32 \text{ kJ/kg}$$

The total required power requires a mass flow rate as

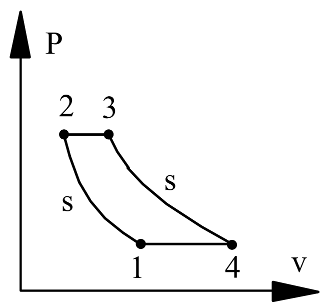
$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{14\,000}{499.32} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{28.04\ kg/s}$$

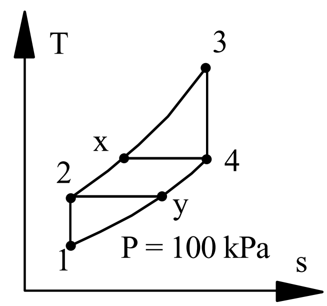
Regenerators, Intercoolers, and Nonideal Cycles

11.74

An ideal regenerator is incorporated into the ideal air-standard Brayton cycle of Problem 11.68. Find the thermal efficiency of the cycle with this modification. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:





Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$T_3 = 1100^oC$

$\dot{m} = 10$ kg/s

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Ideal regenerator: $T_X = T_4 = 674.7$ K

$$q_H = h_3 - h_X = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/701.3 = \mathbf{0.565}$$

11.75

The gas-turbine cycle shown in Fig. P11.75 is used as an automotive engine. In the first turbine, the gas expands to pressure P_5, just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown in the figure and assume that all processes are ideal. Determine the intermediate pressure P_5, the net specific work output of the engine, and the mass flow rate through the engine. Find also the air temperature entering the burner T_3, and the thermal efficiency of the engine.

a) Consider the compressor

$$s_2 = s_1 \;\Rightarrow\; T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8 \text{ K}$$

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$$

Consider then the first turbine work

$$w_{T1} = -w_C = 201.6 = C_{P0}(T_4 - T_5) = 1.004(1600 - T_5)$$

$$\Rightarrow\; T_5 = 1399.2 \text{ K}$$

$$s_5 = s_4 \;\Rightarrow\; P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k-1}{k}} = 600\left(\frac{1399.2}{1600}\right)^{3.5} = \mathbf{375\ kPa}$$

b) $$s_6 = s_5 \;\Rightarrow\; T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\frac{100}{375}\right)^{0.286} = 958.8 \text{ K}$$

The second turbine gives the net work out

$$w_{T2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 958.8) = 442.2 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{T2} = 150/442.2 = \mathbf{0.339\ kg/s}$$

c) Ideal regenerator $\Rightarrow$ $T_3 = T_6 = \mathbf{958.8\ K}$

$$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 958.8) = 643.8 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 442.2/643.8 = \mathbf{0.687}$$

11.76

Repeat Problem 11.71, but include a regenerator with 75% efficiency in the cycle. A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Both compressor and turbine are reversible and adiabatic so constant s, Eq.8.32 relates then T to P assuming constant heat capacity.

Compressor: $\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1$ K

$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5$ kJ/kg

Turbine $s_4 = s_3 \Rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2$ K

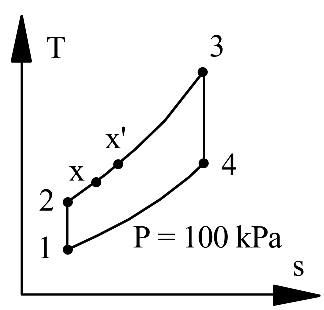
$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2$ kJ/kg

$w_{NET} = 851.2 - 339.5 = 511.7$ kJ/kg

$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\ 000/511.7 = 195.4$ kg/s

$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 =$ **166.32 MW**

$w_C/w_T = 339.5/851.2 =$ **0.399**



For the regenerator

$$\eta_{REG} = 0.75 = \frac{h_X - h_2}{h_{X'} - h_2} = \frac{T_X - T_2}{T_4 - T_2} = \frac{T_X - 638.1}{752.2 - 638.1}$$

$\Rightarrow T_X = 723.7$ K

Turbine and compressor work not affected by regenerator.

Combustor needs to add less energy with the regenerator as

$q_H = C_{P0}(T_3 - T_X) = 1.004(1600 - 723.7) = 879.8$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 511.7/879.8 =$ **0.582**

11.77

A two-stage air compressor has an intercooler between the two stages as shown in Fig. P11.77. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant pressure intercooler cools the air to the inlet temperature, $T_3 = T_1$. It can be shown that the optimal pressure, $P_2 = (P_1P_4)^{1/2}$, for minimum total compressor work. Find the specific compressor works and the intercooler heat transfer for the optimal P_2.

Solution:

Optimal intercooler pressure $P_2 = \sqrt{100 \times 1600} = 400$ kPa

1: $h_1 = 290.43, \quad s^o_{T1} = 6.83521$

C.V. C1: $w_{C1} = h_2 - h_1, \quad s_2 = s_1$ leading to Eq.8.28

$\Rightarrow \quad s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 4 = 7.2331$

$\Rightarrow T_2 = 430.3$ K, $\quad h_2 = 432.05$ kJ/kg

$w_{C1} = 432.05 - 290.43 =$ **141.6 kJ/kg**

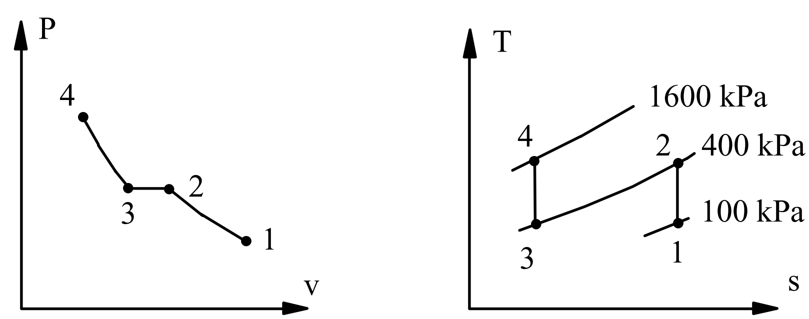
C.V. Cooler: $T_3 = T_1 \quad \Rightarrow \quad h_3 = h_1$

$q_{OUT} = h_2 - h_3 = h_2 - h_1 = w_{C1} =$ **141.6 kJ/kg**

C.V. C2: $T_3 = T_1, \quad s_4 = s_3$ and since $s^o_{T3} = s^o_{T1}$, $P_4/P_3 = P_2/P_1$

$\Rightarrow \quad s^o_{T4} = s^o_{T3} + R \ln(P_4/P_3) = s^o_{T2}$, so we have $T_4 = T_2$

Thus we get $w_{C2} = w_{C1} =$ **141.6 kJ/kg**



11.78

A two-stage compressor in a gas turbine brings atmospheric air at 100 kPa, 17°C to 500 kPa, then cools it in an intercooler to 27°C at constant P. The second stage brings the air to 1000 kPa. Assume both stages are adiabatic and reversible. Find the combined specific work to the compressor stages. Compare that to the specific work for the case of no intercooler (i.e. one compressor from 100 to 1000 kPa).

Solution:

C.V. Stage 1: 1 => 2

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 290 (500/100)^{0.2857} = 459.3 \text{ K}$$

$$w_{c1in} = C_P(T_2 - T_1) = 1.004(459.3 – 290) = 187.0 \text{ kJ/kg}$$

C.V. Stage 2: 3 => 4

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 300 (1000/500)^{0.2857} = 365.7 \text{ K}$$

$$w_{c2in} = C_P(T_4 - T_3) = 1.004(365.7 – 300) = 65.96 \text{ kJ/kg}$$

$$w_{tot} = w_{c1} + w_{c2} = 187 + 65.96 = \mathbf{253\ kJ/kg}$$

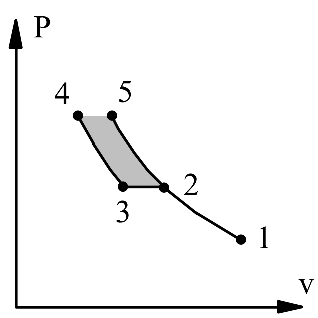
The intercooler reduces the work for stage 2 as T is lower and so is specific volume.

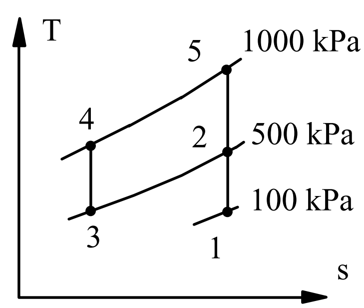
C.V. One compressor 1 => 5

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_5 = T_1 (P_5/P_1)^{(k-1)/k} = 290 (1000/100)^{0.2857} = 559.88 \text{ K}$$

$$w_{in} = C_P(T_5 - T_1) = 1.004(559.88 – 290) = \mathbf{271\ kJ/kg}$$





The reduction in work due to the intercooler is shaded in the P-v diagram.

11.79

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P11.79, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, *x*, bypasses the burner and the rest $(1 - x)$ goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction *x*. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1;\ \ s_2 = s_1$

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_2 = T_1\,(P_2/P_1)^{(k-1)/k} = 290\,(450/100)^{\,0.2857} = 445.7\text{ K}$$

$$h_2 = 447.75\text{ kJ/kg}, \quad -w_C = 447.75 - 290.43 = 157.3\text{ kJ/kg}$$

C.V.Burner: $h_3 = h_2 + q_H = 447.75 + 1200 = 1647.75$ kJ/kg

$$\Rightarrow\ T_3 = 1510\text{ K}$$

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22$ kJ/kg

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1647.75 - 1046.22}{1647.75 - 447.75} = \mathbf{0.5013}$$

$$\dot{W}_{T1} = \dot{W}_{C,in} \ \Rightarrow\ \dot{w}_{T1} = -w_C = 157.3 = h_3 - h_4$$

$$h_4 = 1046.22 - 157.3 = 888.9\text{ kJ/kg} \ \Rightarrow\ T_4 = \mathbf{860\ K}$$

$$P_4 = P_{MIX}(T_4/T_{MIX})^{k/(k-1)} = 450 \times (860/1000)^{3.5} = \mathbf{265\ kPa}$$

$$s_4 = s_5 \ \Rightarrow\ T_5 = T_4\,(P_5/P_4)^{(k-1)/k} = 860\,(100/265)^{0.2857} = 651\text{ K}$$

$$h_5 = 661.2\text{ kJ/kg}$$

$$w_{T2} = h_4 - h_5 = 888.9 - 661.2 = \mathbf{227.7\ kJ/kg}$$

11.80

Repeat Problem 11.71, but assume that the compressor has an isentropic efficiency of 85% and the turbine an isentropic efficiency of 88%.

Solution:

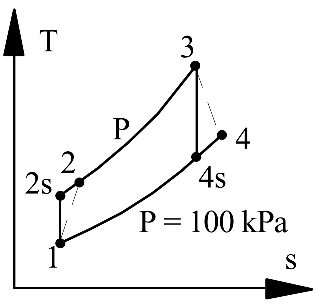
Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}



Ideal compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Actual compressor

$$\Rightarrow w_C = w_{SC}/\eta_{SC} = 339.5/0.85 = 399.4 \text{ kJ/kg} = C_{P0}(T_2\text{-}T_1)$$

$$\Rightarrow T_2 = T_1 + w_c/C_{P0} = 300 + 399.4/1.004 = 697.8 \text{ K}$$

Ideal turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

Actual turbine

$$\Rightarrow w_T = \eta_{ST}\ w_{ST} = 0.88 \times 851.2 = 749.1 \text{ kJ/kg} = C_{P0}(T_3\text{-}T_4)$$

$$\Rightarrow T_4 = T_3 - w_T/C_{P0} = 1600 - 749.1/1.004 = 853.9 \text{ K}$$

Do the overall net and cycle efficiency

$$w_{NET} = 749.1 - 399.4 = 349.7 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/349.7 = 286.0 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 286.0 \times 749.1 = \mathbf{214.2\ MW}$$

$$w_C/w_T = 399.4/749.1 = \mathbf{0.533}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1600 - 697.8) = 905.8 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 349.7/905.8 = \mathbf{0.386}$$

11.81

Repeat Problem 11.77 when the intercooler brings the air to T_3 = 320 K. The corrected formula for the optimal pressure is $P_2 = [\ P_1P_4\ (T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 9.184, where *n* is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so n/(n - 1) = 1.4/0.4 = 3.5

$$P_2 = 400 \sqrt{(320/290)^{3.5}} = 475.2 \text{ kPa}$$

C.V. C1: $s_2 = s_1 \Rightarrow T_2 = T_1\ (P_2/P_1)^{\frac{k-1}{k}} = 290\ (475.2/100)^{0.2857} = 452.67$ K

$-w_{C1} = h_2 - h_1 = C_p(T_2 - T_1) = 1.004(452.67 - 290) =$ **163.3 kJ/kg**

C.V. Cooler: $q_{OUT} = h_2 - h_3 = 1.004(452.67 - 320) =$ **133.2 kJ/kg**

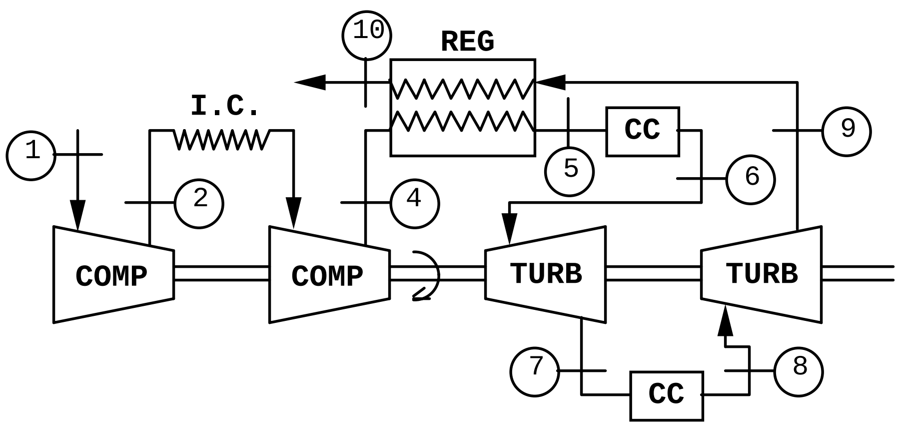
C.V. C2: $s_4 = s_3 \Rightarrow T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 320\ (1600/475.2)^{0.2857} = 452.67$ K

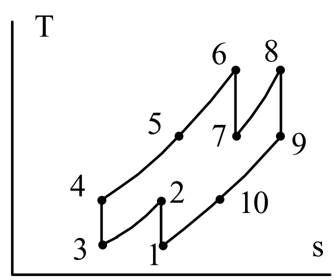
$-w_{C2} = h_4 - h_3 = C_p(T_2 - T_1) = 1.004(452.67 - 320) =$ **133.2 kJ/kg**

11.82

Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance to the first compressor is 100 kPa, the temperature entering each compressor is 20°C, and the temperature entering each turbine is 1100°C. An ideal regenerator is also incorporated into the cycle. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:





$P_2/P_1 = P_4/P_3 = P_6/P_7 = P_8/P_9 = 8.0$

$P_1 = 100$ kPa

$T_1 = T_3 = 20^oC, \quad T_6 = T_8 = 1100^oC$

Assume constant specific heat

$s_2 = s_1$ and $s_4 = s_3 \quad \Rightarrow$

$$T_4 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(8)^{0.286} = 531.4 \text{ K}$$

Total $w_C = 2 \times w_{12} = 2C_{P0}(T_2 - T_1) = 2 \times 1.004(531.4 - 293.2) =$ **478.1 kJ/kg**

Also $s_6 = s_7$ and $s_8 = s_9$: $\Rightarrow T_7 = T_9 = T_6\left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{8}\right)^{0.286} = 757.6$ K

Total $w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 1.004(1373.2 - 756.7) =$ **1235.5 kJ/kg**

$w_{NET} = 1235.5 - 478.1 = 757.4$ kJ/kg

Ideal regenerator: $T_5 = T_9$, $T_{10} = T_4$

$\Rightarrow q_H = (h_6 - h_5) + (h_8 - h_7) = 2C_{P0}(T_6 - T_5)$

$= 2 \times 1.004(1373.2 - 757.6) = 1235.5$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 757.4/1235.5 =$ **0.613**

11.83

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 5 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. Calculate the temperature at the exit of each compressor stage and the total specific work required.

Solution:

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K

State 3: $T_3 = 330$ K

$P_2 = 5\ P_1 = 500$ kPa; $\quad P_4 = 5\ P_3 = 2500$ kPa

Energy Eq.: $w_{c1} + h_1 = h_2 \;\; => \;\; w_{c1} = h_2 - h_1 = C_P(T_2 - T_1)$

Ideal C1 constant s, Eq.8.32: $\quad T_{2s} = T_1\ (P_2/P_1)^{(k-1)/k} = 475.4$ K

$w_{c1\ s} = C_P(T_{2s} - T_1) = 176.0$ kJ/kg,

Actual Eq.9.28: $\quad w_{c1} = w_{c1\ s}/\eta = 176/0.82 = 214.6$ kJ/kg

$T_2 = T_1 + w_{c1}/C_P =$ **513.7 K**

Ideal C2 constant s, Eq.8.32: $\quad T_{4s} = T_3\ (P_4/P_3)^{(k-1)/k} = 552.6$ K

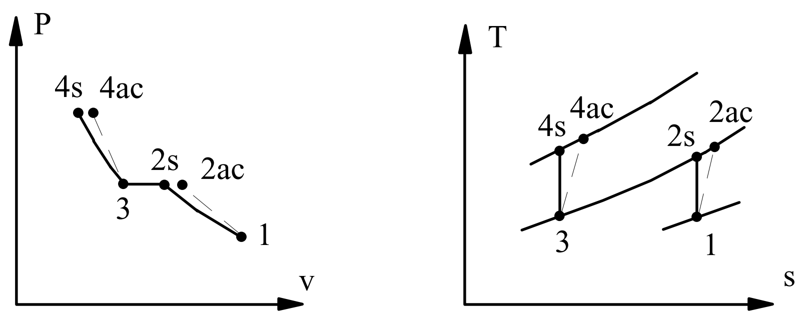
$w_{c2\ s} = C_P(T_{4s} - T_3\ ) = 193.4$ kJ/kg;

Actual Eq.9.28: $\quad w_{c2} = w_{c2\ s}/\eta = 235.9$ kJ/kg

$T_4 = T_3 + w_{c2}\ /\ C_P =$ **565 K**

Total work in:

$w = w_{c1} + w_{c2} = 214.6 + 235.9 =$ **450.5 kJ/kg**



11.84

Repeat the questions in Problem 11.75 when we assume that friction causes pressure drops in the burner and on both sides of the regenerator. In each case, the pressure drop is estimated to be 2% of the inlet pressure to that component of the system, so P_3 = 588 kPa, P_4 = 0.98 P_3 and P_6 = 102 kPa.

Solution:

a) From solution 11.75: $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8 \text{ K}$

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$$

$$P_3 = 0.98 \times 600 = 588 \text{ kPa}, \quad P_4 = 0.98 \times 588 = 576.2 \text{ kPa}$$

$$s_5 = s_4 \Rightarrow P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 576.2\left(\frac{1399.2}{1600}\right)^{3.5} = 360.4 \text{ kPa}$$

b) $P_6 = 100/0.98 = 102 \text{ kPa}, \quad s_{6S} = s_5$

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\frac{102}{292.8}\right)^{0.286} = 975.2 \text{ K}$$

$$w_{ST2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 975.2) = 425.7 \text{ kJ/kg}$$

$\dot{m} = \dot{W}_{NET}/w_{NET} = 150/425.7 =$ **0.352 kg/s**

c) $T_3 = T_6 = 975.2 \text{ K}$

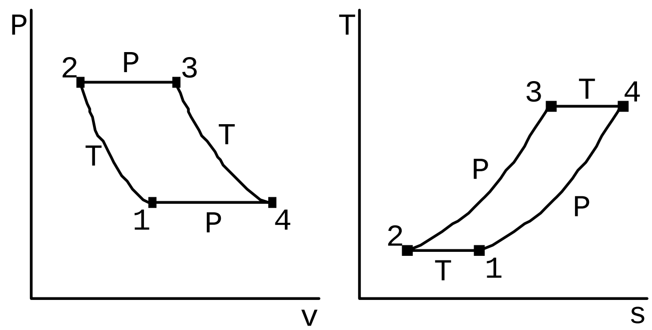
$$q_H = C_{P0}(T_4 - T_3) = 1.004\,(1600 - 975.2) = 627.3 \text{ kJ/kg}$$

$\eta_{TH} = w_{NET}/q_H = 425.7/627.3 =$ **0.678**

Ericsson Cycles

11.85

Consider an ideal air-standard Ericsson cycle that has an ideal regenerator as shown in Fig. P11.85. The high pressure is 1 MPa and the cycle efficiency is 70%. Heat is rejected in the cycle at a temperature of 300 K, and the cycle pressure at the beginning of the isothermal compression process is 100 kPa. Determine the high temperature, the compressor work, and the turbine work per kilogram of air.



$P_2 = P_3 = 1$ MPa

$T_1 = T_2 = 300$ K

$P_1 = 100$ kPa

${}_2q_3 = -{}_4q_1$ (ideal reg.)

$\Rightarrow q_H = {}_3q_4$ & $w_T = q_H$

$r_p = P_2/P_1 = 10$

$$\eta_{TH} = \eta_{CARNOT\ TH.} = 1 - T_L/T_H = 0.7 \;\Rightarrow\; T_3 = T_4 = T_H = \mathbf{1000\ K}$$

$$q_L = -w_C = \int v\, dP = RT_1 \ln\left(\frac{P_2}{P_1}\right) = 0.287 \times 300 \times \ln\left(\frac{1000}{100}\right) = \mathbf{198.25}$$

$$w_T = q_H = -\int v\, dP = -RT_3 \ln(P_4/P_3) = \mathbf{660.8\ kJ/kg}$$

11.86

An air-standard Ericsson cycle has an ideal regenerator. Heat is supplied at 1000°C and heat is rejected at 20°C. Pressure at the beginning of the isothermal compression process is 70 kPa. The heat added is 600 kJ/kg. Find the compressor work, the turbine work, and the cycle efficiency.

Solution:

Identify the states

Heat supplied at high temperature $T_3 = T_4 = 1000°C = 1273.15$ K

Heat rejected at low temperature $T_1 = T_2 = 20°C = 293.15$ K

Beginning of the compression: $P_1 = 70$ kPa

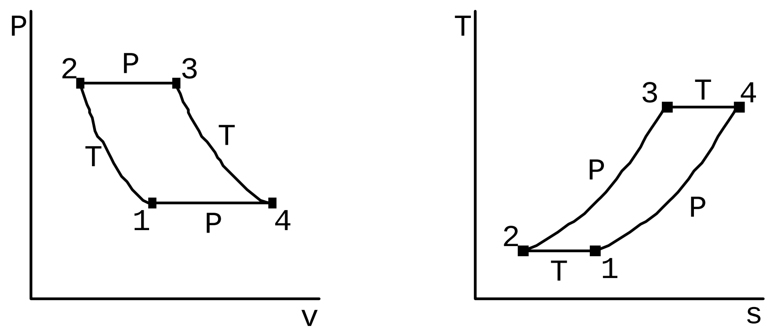
Ideal regenerator: ${}_2q_3 = -{}_4q_1 \quad \Rightarrow \quad q_H = {}_3q_4 = 600$ kJ/kg

$\Rightarrow \quad w_T = q_H =$ **600 kJ/kg**

$$\eta_{TH} = \eta_{CARNOT} = 1 - \frac{293.15}{1273.15} = \mathbf{0.7697}$$

$$w_{NET} = \eta_{TH} q_H = 0.7697 \times 600 = 461.82 \text{ kJ/kg}$$

$$q_L = -w_C = 600 - 461.82 = \mathbf{138.2\ kJ/kg}$$

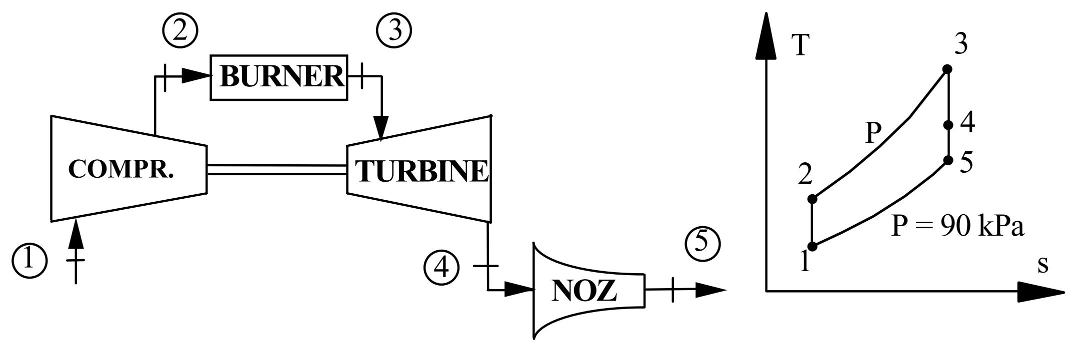


Jet Engine Cycles

11.87

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 11.27. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.

Solution:



C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.28

$\Rightarrow \quad s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 14 = 7.59262$ kJ/kg K

From A.7 $h_2 = 617.2$ kJ/kg, $T_2 = 609.4$ K

$w_C = h_2 - h_1 = 617.2 - 290.43 = 326.8$ kJ/kg

C.V. Turbine: $w_T = h_3 - h_4 = w_C$ and $s_4 = s_3 \Rightarrow$

$h_4 = h_3 - w_C = 1635.8 - 326.8 = 1309$

$\Rightarrow \quad s^o_{T4} = 8.37142$ kJ/kg K, $T_4 = 1227$ K

$P_4 = P_3 \exp[(s^o_{T4} - s^o_{T3})/R] = 1260 \exp[\,(8.37142 - 8.61208)/0.287\,]$

$= 1260 \exp(-0.83854) =$ **544.8 kPa**

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.28

$\Rightarrow s^o_{T5} = s^o_{T3} + R \ln(P_5/P_3) = 8.61208 + 0.287 \ln (1/14) = 7.85467$ kJ/kgK

=> From A.7 $T_5 = 778$ K, $h_5 = 798.2$ kJ/kg

Now the energy equation

$(1/2)\mathbf{V}_5^2 = h_4 - h_5 = 510.8 \quad \Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 510.8}$ = **1011 m/s**

11.88

The turbine section in a jet engine receives gas (assume air) at 1200 K, 800 kPa with an ambient atmosphere at 80 kPa. The turbine is followed by a nozzle open to the atmosphere and all the turbine work drives a compressor receiving air at 85 kPa, 270 K with the same flow rate. Find the turbine exit pressure so the nozzle has an exit velocity of 800 m/s. To what pressure can the compressor bring the incomming air?

Solution:

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s, from Eq.8.32 we can relate the T's and P's

State 1: 1200 K, 800 kPa State 3: 80 kPa; $s_3 = s_1$

Eq.8.32: $T_3 = T_1 (P_3/P_1)^{(k-1)/k} = 1200(80/800)^{0.2857} = 621.56$ K

Energy: $h_1 + 0 = h_3 + (1/2)\mathbf{V}_3^2 + w_T = h_2 + w_T$

$w_T = h_1 - h_3 - (1/2)\mathbf{V}_3^2 \cong C_P(T_1 - T_3) - (1/2)\mathbf{V}_3^2$

$= 1.004(1200 - 621.56) - (1/2) \times 800^2/1000$

$= 580.75 - 320 = 260.75$ kJ/kg

C.V. Nozzle alone to establish state 2.

$h_2 = h_3 + (1/2)\mathbf{V}_3^2$

$T_2 = T_3 + (1/2)\mathbf{V}_3^2/C_P = 621.56 + 320/1.004 = 940.29$ K

$P_2 = P_1 + (T_2/T_1)^{k/(k-1)} = 800 \times (940.29/1200)^{3.5} =$ **340.7 kPa**

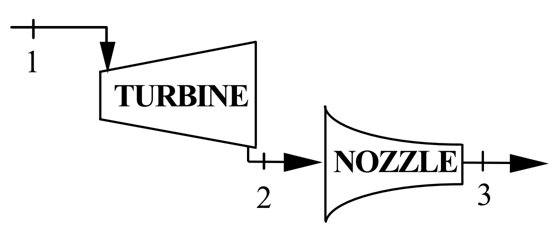
C.V. Compressor

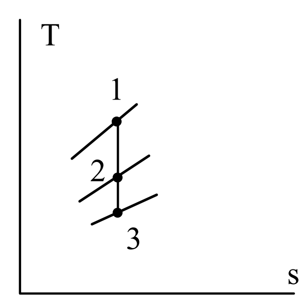
$w_c = h_e - h_i = w_T = 260.75$ kJ/kg

$T_e = T_i + w_c/\ C_P = 270 + 260.75/1.004 = 529.71$ K

Reversible adiabatic compressor, constant s gives relation in Eq.8.32

$P_e = P_i \times (T_e/T_i)^{k/(k-1)} = 85 \times (529.71/270)^{3.5} =$ **899 kPa**





11.89

The turbine in a jet engine receives air at 1250 K, 1.5 MPa. It exhausts to a nozzle at 250 kPa, which in turn exhausts to the atmosphere at 100 kPa. The isentropic efficiency of the turbine is 85% and the nozzle efficiency is 95%. Find the nozzle inlet temperature and the nozzle exit velocity. Assume negligible kinetic energy out of the turbine.

Solution:

C.V. Turbine: $h_i = 1336.7$, $s^o_{Ti} = 8.3940$, $s_{es} = s_i$ then from Eq.8.28

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 8.3940 + 0.287 \ln (250/1500) = 7.8798$ kJ/kg K

Table A.7.1 $T_{es} = 796$ K, $h_{es} = 817.9$ kJ/kg,

Energy Eq.: $w_{T,s} = h_i - h_{es} = 1336.7 - 817.9 = 518.8$ kJ/kg

Eq.9.27: $w_{T,AC} = w_{T,s} \times \eta_T = 441$ kJ/kg $= h_{e,AC} - h_i$

$\Rightarrow h_{e,AC} = 895.7 \Rightarrow$ $\mathbf{T_{e,AC} = 866\ K}$, $s^o_{Te} = 7.9730$ kJ/kg K

C.V. Nozzle: $h_i = 895.7$ kJ/kg, $s^o_{Ti} = 7.9730$ KJ/kgK, $s_{es} = s_i$

then from Eq.8.28

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R \ln(P_e/P_i) = 7.9730 + 0.287 \ln (100/250) = 7.7100$ kJ/kgK

Table A.7.1 $\Rightarrow T_{e,s} = 681$ K, $h_{e,s} = 693.1$ kJ/kg

Energy Eq.: $(1/2)\mathbf{V}^2_{e,s} = h_i - h_{e,s} = 895.7 - 693.1 = 202.6$ kJ/kg

Eq.9.30: $(1/2)\mathbf{V}^2_{e,AC} = (1/2)\mathbf{V}^2_{e,s} \times \eta_{NOZ} = 192.47$ kJ/kg

$\mathbf{V}_{e,AC} = \sqrt{2 \times 1000 \times 192.47} = \mathbf{620\ m/s}$

11.90

Consider an air standard jet engine cycle operating in a 280K, 100 kPa environment. The compressor requires a shaft power input of 4000 kW. Air enters the turbine state 3 at 1600 K, 2 MPa, at the rate of 9 kg/s, and the isentropic efficiency of the turbine is 85%. Determine the pressure and temperature entering the nozzle at state 4. If the nozzle efficiency is 95%, determine the temperature and velocity exiting the nozzle at state 5.

Solution:

C.V. Shaft: $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{W}_C$

CV Turbine: $h_3 - h_4 = \dot{W}_C / \dot{m} = 4000/9 = 444.4$ kJ/kg

$h_4 = 1757.3 - 444.4 = 1312.9$ kJ/kg

Work back to the ideal turbine conditions

Eq.9.27: $w_{Ta} = w_C = 444.4 \quad \Rightarrow \quad w_{Ts} = w_{Ta} / \eta = 522.82 = h_3 - h_{4s}$

$h_{4s} = 1234.5 \quad \Rightarrow \quad T_{4s} \approx 1163$ K, $s^o_{T4s} = 8.3091$ kJ/kg K

$s_{4s} - s_3 = 0 = s^o_{T4s} - s^o_{T3} - R \ln(P_4/P_3)$

$0 = 8.3091 - 8.6905 - 0.287 \ln(P_4/2000) \Rightarrow$ $\mathbf{P_4 = 530\ kPa}$

State 4 from A.7.1: $h_4 = 1312.9$, $T_4 = 1229.8$ K, $s^o_{T4} = 8.3746$ kJ/kg K

First consider the reversible adiabatic (isentropic) nozzle so from Eq.8.28

$s_{5s} - s_4 = 0 = s^o_{T5s} - s^o_{T4} - R \ln(P_5/P_4)$

$s^o_{T5s} = 8.3746 + 0.287 \ln(100/530) = 7.8960$ kJ/kg K

Table A.7.1: $T_{5s} = 808.1$ K, $h_{5s} = 831.0$ kJ/kg

$\Rightarrow \quad 0.5\mathbf{V}^2_{5s} = h_4 - h_{5s} = 1312.9 - 831.0 = 481.9$ kJ/kg

Now consider the actual nozzle

Eq.9.30: $0.5\mathbf{V}^2_{5a} = \eta(0.5\mathbf{V}^2_{5s}) = 457.81 \quad \Rightarrow \quad$ $\mathbf{V}_{5a}$= **957 m/s**

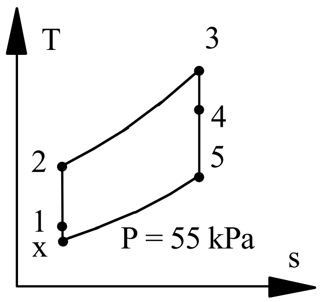
$h_{5a} = h_4 - 0.5\mathbf{V}^2_{5a} = 1312.9 - 457.81 = 855.1$ kJ/kg

$\Rightarrow \quad T_{5a} \approx$ **830 K**

11.91

A jet aircraft is flying at an altitude of 4900 m, where the ambient pressure is approximately 55 kPa and the ambient temperature is −18°C. The velocity of the aircraft is 280 m/s, the pressure ratio across the compressor is 14:1 and the cycle maximum temperature is 1450 K. Assume the inlet flow goes through a diffuser to zero relative velocity at state 1. Find the temperature and pressure at state 1 and the velocity (relative to the aircraft) of the air leaving the engine at 55 kPa.

Solution:



Ambient

T_X = -18°C = 255.2 K, P_X = 55 kPa = P_5

also $\mathbf{V}_X$ = 280 m/s

Assume that the air at this state is reversibly decelerated to zero velocity and then enters the compressor at 1.

$P_2/P_1 = 14$ & $T_3 = 1450$ K

C.V. Diffuser section.

EnergyEq.: $T_1 = T_X + \frac{V_X^2}{2 \times 1000} = 255.2 + \frac{(280)^2}{2 \times 1000 \times 1.0035} = \mathbf{294.3\ K}$

Eq.8.32: $P_1 = P_X\left(\frac{T_1}{T_X}\right)^{\frac{k}{k-1}} = 55\left(\frac{294.3}{255.2}\right)^{3.5} = \mathbf{90.5\ kPa}$

C.V. Compressor, isentropic so use Eq.8.32 and then energy equation

$$T_2 = T_1\ (P_2/P_1)^{\frac{k-1}{k}} = 294.3(14)^{0.286} = 626.0\ \text{K}$$

$$w_C = -{}_1w_2 = C_{P0}(T_2 - T_1) = 1.004(1450 - T_4) \quad \Rightarrow \quad T_4 = 1118.3\ \text{K}$$

Pressure ratio: $P_3 = P_2 = 14 \times 90.5 = 1267$ kPa

C.V. Turbine, isentropic so use Eq.8.32

$$P_4 = P_3\ (T_4/T_3)^{\frac{k}{k-1}} = 1267(1118.3/1450)^{3.5} = 510\ \text{kPa}$$

C.V. Nozzle, isentropic so use Eq.8.32 and energy equation

$$T_5 = T_4\ (P_5/P_4)^{\frac{k-1}{k}} = 1118.3(55/510)^{0.286} = 591.5\ \text{K}$$

$$\frac{V_5^2}{2 \times 1000} = C_{P0}(T_4 - T_5) = 1.004(1118.3 - 591.5) = 528.7\ \text{kJ/kg}$$

$\Rightarrow$ $\mathbf{V_5 = 1028\ m/s}$

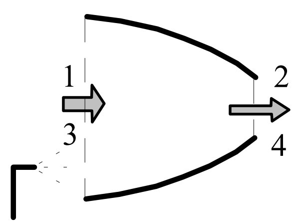
11.92

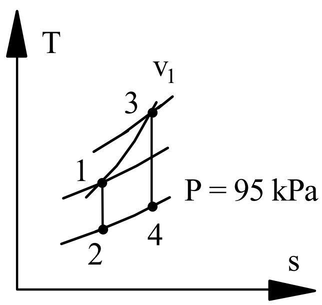
An afterburner in a jet engine adds fuel after the turbine thus raising the pressure and temperature due to the energy of combustion. Assume a standard condition of 800 K, 250 kPa after the turbine into the nozzle that exhausts at 95 kPa. Assume the afterburner adds 450 kJ/kg to that state with a rise in pressure for same specific volume, and neglect any upstream effects on the turbine. Find the nozzle exit velocity before and after the afterburner is turned on.

Solution:

Before afterburner is on: 1: 800 K; 250 kPa and 2: 95 kPa

After afterburner is on: 3: $v = v_1$ and 4: 95 kPa





Assume reversible adiabatic nozzle flow, then constant s from Eq.8.32

$$T_2 = T_1\ (P_2/P_1)^{(k-1)/k} = 800 \times (95/250)^{\ 0.2857} = 606.8\ \text{K}$$

Energy Eq.: $(1/2)\mathbf{V}_2^2 = C_P(T_1 - T_2)$

$$\mathbf{V}_2 = \sqrt{2\ C_P(T_1 - T_2)} = \sqrt{2 \times 1004(800 - 606.8)} = \mathbf{622.8\ m/s}$$

Add the q_{AB} at assumed constant volume then energy equation gives

$$T_3 = T_1 + q_{AB}/C_v = 800 + 450/0.717 = 1427.6\ \text{K}$$

$$v_3 = v_1 \ \Rightarrow\ P_3 = P_1(\ T_3/T_1) = 250 \times 1427.6/800 = 446.1\ \text{kPa}$$

Reversible adiabatic expansion, again from Eq.8.32

$$T_4 = T_3\ (P_4/P_3)^{(k-1)/k} = 1427.6 \times (95/446.1)^{\ 0.2857} = 917.7\ \text{K}$$

$$\mathbf{V}_2 = \sqrt{2\ C_P(T_3 - T_4)} = \sqrt{2 \times 1004(1427.6 - 917.7)} = \mathbf{1012\ m/s}$$

Otto Cycles

11.93

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

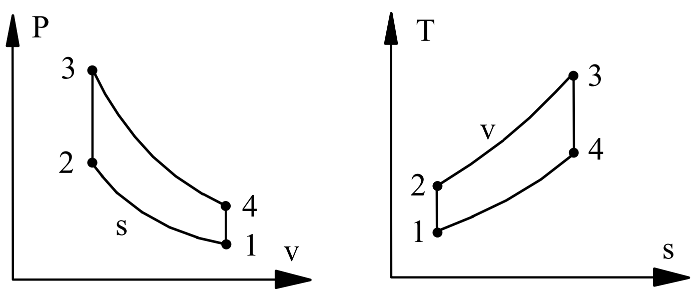
$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 8^{0.4} = 689.2 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 95 \times 8^{1.4} = 1746 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 689.2 + 1300/0.717 = \mathbf{2502\ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1746\ (2502 / 689.2) = \mathbf{6338\ kPa}$$



11.94

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.5.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 290 \times 9^{0.4} = 698.4 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 90 \times 9^{1.4} = 1950.7 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.717 (1800 - 698.4) = 789.85 \text{ kJ/kg}$$

$$P_3 = P_2 \times (T_3/T_2) = 1950.7 (1800 / 698.4) = 5027.6 \text{ kPa}$$

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_4 = T_3 (v_3/v_4)^{k-1} = 1800 \times (1/9)^{0.4} = 747.4 \text{ K}$$

$$P_4 = P_3(T_4/T_3)(v_3/v_4) = 5027.6 (747.4/1800) (1/9) = \mathbf{232 \text{ kPa}}$$

Find now the net work

$$_1w_2 = u_1 - u_2 = C_v(T_1 - T_2) = 0.717(290 - 698.4) = -292.8 \text{ kJ/kg}$$

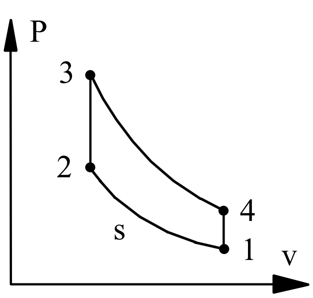
$$_3w_4 = u_3 - u_4 = C_v(T_3 - T_4) = 0.717(1800 - 747.4) = 754.7 \text{ kJ/kg}$$

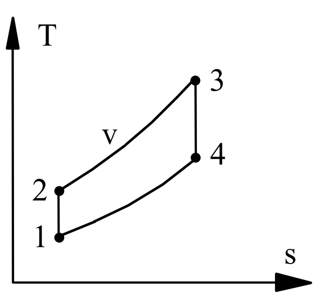
Net work and overall efficiency

$$w_{NET} = {}_3w_4 + {}_1w_2 = 754.7 - 292.8 = \mathbf{461.9 \text{ kJ/kg}}$$

$$\eta = w_{NET}/q_H = 461.9/789.85 = \mathbf{0.585}$$

Comment: We could have found η from Eq.11.18 and then $w_{NET} = \eta q_H$.

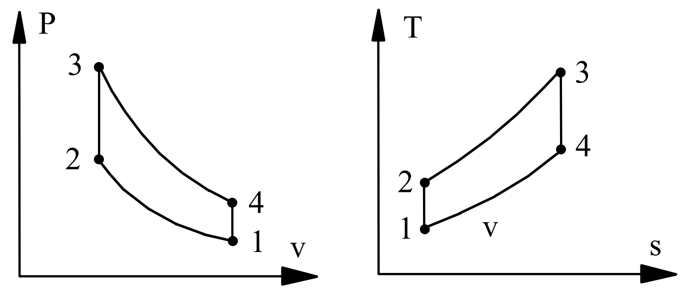




11.95

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 1800 kJ/kg of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 90 kPa, 10°C. Assuming constant specific heat, with the value from Table A.5, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 90(7)^{1.4} = 1372 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \times (7)^{0.4} = 616.6 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_{V0} = 616.6 + 1800/0.717 = \mathbf{3127\ K}$$

$$P_3 = P_2T_3/T_2 = 1372 \times 3127 / 616.6 = \mathbf{6958\ kPa}$$

Efficiency and net work

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.2/616.5 = \mathbf{0.541}$$

$$w_{net} = \eta_{TH} \times q_H = 0.541 \times 1800 = 973.8 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029 \text{ m}^3\text{/kg}$$

$$v_2 = (1/7)\, v_1 = 0.1290 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{973.8}{0.9029 - 0.129} = \mathbf{1258\ kPa}$$

11.96

A gasoline engine has a volumetric compression ratio of 8 and before compression has air at 280 K, 85 kPa. The combustion generates a peak pressure of 6500 kPa. Find the peak temperature, the energy added by the combustion process and the exhaust temperature.

Solution:

Solve the problem with cold air properties.

Compression. Isentropic so we use Eqs.8.33-8.34

$$P_2 = P_1(v_1/v_2)^k = 85(8)^{1.4} = 1562 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 280(8)^{0.4} = 643.3 \text{ K}$$

Combustion. Constant volume

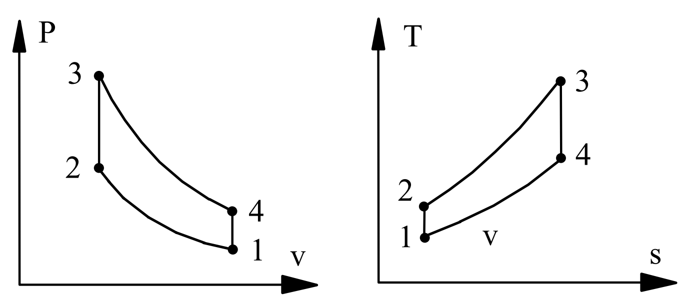
$$T_3 = T_2\,(P_3/P_2) = 643.3 \times 6500/1562 = \mathbf{2677\ K}$$

$$q_H = u_3 - u_2 \approx C_v(T_3 - T_2)$$

$$= 0.717\,(2677 - 643.3) = \mathbf{1458\ kJ/kg}$$

Exhaust. Isentropic expansion so from Eq.8.33

$$T_4 = T_3/8^{0.4} = 2677/2.2974 = \mathbf{1165\ K}$$



11.97

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 290 K, 85 kPa in the cylinder. The combustion peak pressure is 6000 kPa. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:

Compression. Isentropic so we use Eqs.8.33-8.34

$$P_2 = P_1(v_1/v_2)^k = 85\ (10)^{1.4} = 2135.1\ \text{kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 290\ (10)^{0.4} = 728.45\ \text{K}$$

Combustion. Constant volume

$$T_3 = T_2\ (P_3/P_2) = 728.45 \times 6000/2135.1 = \mathbf{2047\ K}$$

Exhaust. Isentropic expansion so from Eq.8.33

$$T_4 = T_3\ /\ (v_1/v_2)^{k-1} = T_3\ /\ 10^{0.4} = 2047\ /\ 2.5119 = \mathbf{814.9\ K}$$

Overall cycle efficiency is from Eq.11.18, $r_v = v_1/v_2$

$$\eta = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \mathbf{0.602}$$

Comment: No actual gasoline engine has an efficiency that high, maybe 35%.

11.98

A for stroke gasoline engine has a compression ratio of 10:1 with 4 cylinders of total displacement 2.3 L. the inlet state is 280 K, 70 kPa and the engine is running at 2100 RPM with the fuel adding 1800 kJ/kg in the combustion process. What is the net work in the cycle and how much power is produced?

solution:

Overall cycle efficiency is from Eq.11.18, $r_v = v_1/v_2$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \mathbf{0.602}$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 1800 = 1083.6 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.11.15 to 11.17

$$v_1 = RT_1 / P_1 = 0.287 \times 280 / 70 = 1.148 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1 \left(1 - \frac{1}{r_v}\right)} = \frac{1083.6}{1.148 \times 0.9} = 1048.8 \text{ kPa}$$

Now we can find the power from Eq.11.17

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 1048.8 \times 0.0023 \times \frac{2100}{60} \times \frac{1}{2} = \mathbf{42.2\ kW}$$

11.99

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto Cycle

Combustion process: $T_3 = 2050$ K; $u_2 = u_3 - q_H$

$$T_2 = T_3 - q_H / C_{vo} = 2050 - 1000 / 0.717 = 655.3 \text{ K}$$

Compression process

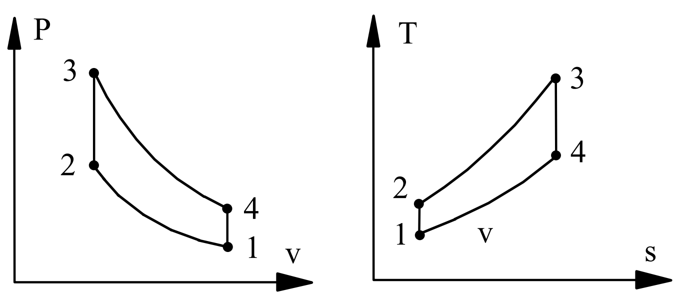
$$P_2 = P_1(T_2 / T_1)^{k/(k-1)} = 90(655.3/290)^{3.5} = 1561 \text{ kPa}$$

$$CR = v_1 / v_2 = (T_2 / T_1)^{1/(k-1)} = (655.3 / 290)^{2.5} = \mathbf{7.67}$$

$$-{}_1w_2 = u_2 - u_1 = C_{vo}(T_2 - T_1) = 0.717(655.3 - 290) = \mathbf{262\ kJ / kg}$$

Highest pressure is after the combustion

$$P_3 = P_2T_3 / T_2 = 1561 \times 2050 / 655.3 = \mathbf{4883\ kPa}$$



11.100

Answer the same three questions for the previous problem, but use variable heat capacities (use table A.7).

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \qquad s^o_{T2} = 8.0889 \text{ kJ/kg K}$$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.28

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2 v_1/T_1 v_2)$$

$$= 8.0889 - 6.8352 - 0.287 \ln(960.5/290) - 0.287 \ln(v_1/v_2)$$

Solving => $v_1 / v_2 =$ **23.78**

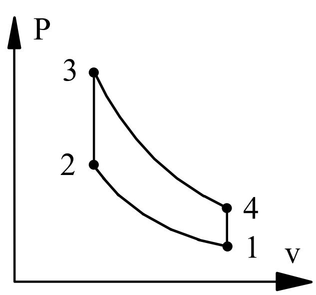
Comment: This is much too high for an actual Otto cycle.

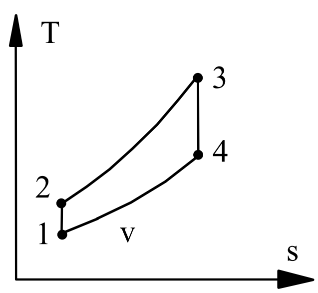
$$-{}_1w_2 = u_2 - u_1 = 725.7 - 207.2 = \mathbf{518.5\ kJ/kg}$$

Highest pressure is after combustion

$$P_3 = P_2 T_3 / T_2 = P_1 (T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 23.78 = \mathbf{15\ 129\ kPa}$$

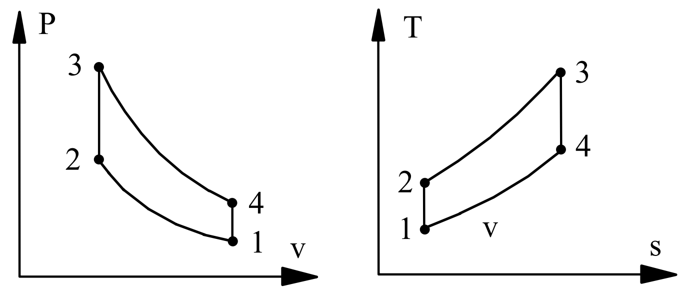




11.101

When methanol produced from coal is considered as an alternative fuel to gasoline for automotive engines, it is recognized that the engine can be designed with a higher compression ratio, say 10 instead of 7, but that the energy release with combustion for a stoichiometric mixture with air is slightly smaller, about 1700 kJ/kg. Repeat Problem 11.95 using these values.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 90(10)^{1.4} = 2260.7 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.15(10)^{0.4} = 711.2 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H / C_{vo} = 711.2 + 1700 / 0.717 = \mathbf{3082\ K}$$

$$P_3 = P_2(T_3 / T_2) = 2260.7 \times 3082 / 711.2 = \mathbf{9797\ kPa}$$

Efficiency, net work, displacement and P_{meff}

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.15/711.2 = \mathbf{0.602}$$

$$w_{net} = \eta_{TH} \times q_H = 0.6 \times 1700 = 1023.4 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \times 283.15/90 = 0.9029 \text{ m}^3\text{/kg},$$

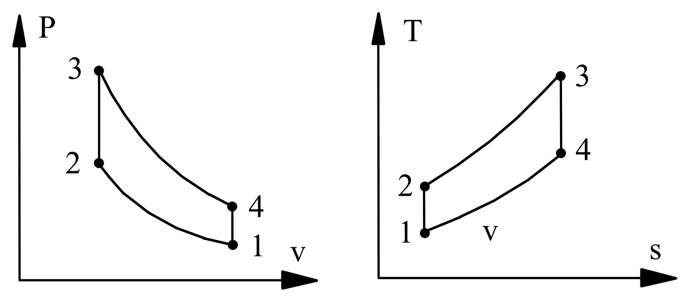
$$v_2 = v_1/10 = 0.0903 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 1023.4 / (0.9029 - 0.0903) = \mathbf{1255\ kPa}$$

11.102

A gasoline engine receives air at 10 C, 100 kPa, having a compression ratio of 9:1 by volume. The heat addition by combustion gives the highest temperature as 2500 K. use cold air properties to find the highest cycle pressure, the specific energy added by combustion, and the mean effective pressure.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$P_2 = P_1(v_1/v_2)^k = 100\ (9)^{1.4} = 2167.4$ kPa

$T_2 = T_1(v_1/v_2)^{k-1} = 283.15\ (9)^{0.4} = 681.89$ K

Combustion: constant volume

$P_3 = P_2(T_3 / T_2) = 2167.4 \times 2500 / 681.89 =$ **7946.3 kPa**

$q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 0.717\ (2500 - 681.89) =$ **1303.6 kJ/kg**

Efficiency, net work, displacement and P_{meff}

$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.15/681.89 =$ **0.5847**

$w_{net} = \eta_{TH} \times q_H = 0.5847 \times 1303.6 = 762.29$ kJ/kg

$v_1 = RT_1/P_1 = 0.287 \times 283.15 / 100 = 0.81264$ m^3/kg,

$v_2 = v_1/10 = 0.081264$ m^3/kg

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{762.29}{0.81264 - 0.081264} = \mathbf{1055\ kPa}$$

11.103

Repeat Problem 11.95, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (and the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $s^o_{T1} = 6.8113$ kJ/kg K

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.28

$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$

$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.8113 + 0.287 \ln 7 = 7.3698$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$LHS_{600} = 7.5764 - 0.287 \ln(600/283.2) = 7.3609$ (too low)

$LHS_{620} = 7.6109 - 0.287 \ln(620/283.2) = 7.3860$ (too high)

Interpolate to get: $T_2 = 607.1$ K, $u_2 = 440.5$ kJ/kg

=> $-{}_1w_2 = u_2 - u_1 = 238.2$ kJ/kg,

$u_3 = 440.5 + 1800 = 2240.5$ => $T_3 =$ **2575.8 K** , $s^o_{T3} = 9.2859$ kJ/kgK

$P_3 = 90 \times 7 \times 2575.8 / 283.2 =$ **5730 kPa**

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.28 as before

$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.2859 + 0.287 \ln(1/7) = 8.7274$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$LHS_{1400} = 8.5289 - 0.287 \ln(1400/2575.8) = 8.7039$ (too low)

$LHS_{1450} = 8.5711 - 0.287 \ln(1450/2575.8) = 8.7360$ (too high)

Interpolation $\Rightarrow$ $T_4 = 1436.6$ K, $u_4 = 1146.9$ kJ/kg

${}_3w_4 = u_3 - u_4 = 2240.5 - 1146.9 = 1093.6$ kJ/kg

Net work, efficiency and mep

➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1093.6 - 238.2 = 855.4$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 855.4 / 1800 =$ **0.475**

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029$ m^3/kg

$v_2 = (1/7)\, v_1 = 0.1290$ m^3/kg

$P_{meff} = \dfrac{w_{net}}{v_1 - v_2} = 855.4 / (0.9029 - 0.129) =$ **1105 kPa**

11.104

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent *n* somewhat larger than the specific heat ratio *k*. Repeat Problem 11.95 but assume that the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with *n* equal to 1.50.

See solution to 11.95 except for process 3 to 4.

T_3 = **3127 K**, P_3 = **6.958 MPa**

$v_3 = RT_3/P_3 = v_2 = 0.129\ m^3/kg, \quad v_4 = v_1 = 0.9029\ m^3/kg$

Process: $Pv^{1.5}$ = constant.

$P_4 = P_3(v_3/v_4)^{1.5} = 6958\ (1/7)^{1.5} = 375.7$ kPa

$T_4 = T_3(v_3/v_4)^{0.5} = 3127(1/7)^{0.5} = 1181.9$ K

$_1w_2 = \int Pdv = \frac{R}{1\text{-}1.4}(T_2 - T_1) = \frac{0.287}{-0.4}(606.6 - 283.15) = -239.3$ kJ/kg

$_3w_4 = \int Pdv = R(T_4 - T_3)/(1 - 1.5)$

$= -0.287(1181.9-3127)/0.5 = 1116.5$ kJ/kg

$w_{NET} = 1116.5 - 239.3 = 877.2$ kJ/kg

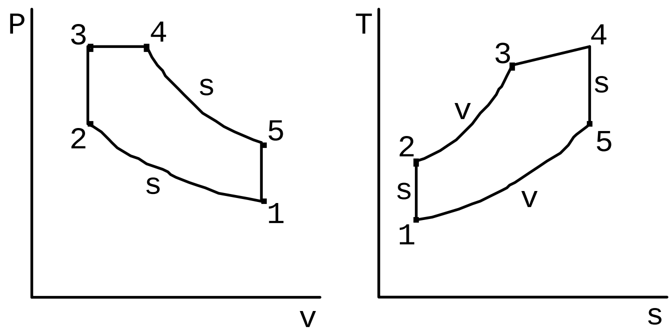
$\eta_{CYCLE} = w_{NET}/q_H = 877.2/1800 =$ **0.487**

$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 877.2/(0.9029 - 0.129) =$ **1133 kPa**

Note a smaller w_{NET}, η_{CYCLE}, P_{meff} compared to an ideal cycle.

11.105

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2100 kJ/kg, that the state at the beginning of the compression process is 90 kPa, 20°C, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.



$P_1 = 90$ kPa, $T_1 = 20^oC$

$r_V = v_1/v_2 = 7$

a) $q_{23} = (2/3) \times 2100 = 1400$ kJ/kg;

$q_{34} = 2100/3 = 700$ kJ/kg

b) $P_2 = P_1(v_1/v_2)^k = 90(9)^{1.4} = 1951$ kPa

$T_2 = T_1(v_1/v_2)^{k-1} = 293.15(9)^{0.4} = 706$ K

$T_3 = T_2 + q_{23}/C_{V0} = 706 + 1400/0.717 = \mathbf{2660\ K}$

$P_3 = P_2T_3/T_2 = 1951(2660/706) = \mathbf{7350.8\ kPa} = P_4$

$T_4 = T_3 + q_{34}/C_{P0} = 2660 + 700/1.004 = 3357$ K

$$\frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{P_4}{P_1} \times \frac{T_1}{T_4} = \frac{7350.8}{90} \times \frac{293.15}{3357} = 7.131$$

$T_5 = T_4(v_4/v_5)^{k-1} = 3357(1/7.131)^{0.4} = 1530$ K

$q_L = C_{V0}(T_5\text{-}T_1) = 0.717(1530 - 293.15) = 886.2$ kJ/kg

$\eta_{TH} = 1 - q_L/q_H = 1 - 886.2/2100 = \mathbf{0.578}$

Std. Otto Cycle: $\eta_{TH} = 1 - (9)^{-0.4} = \mathbf{0.585}$, small difference

Diesel Cycles

11.106

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Standard Diesel cycle and we will use cold air properties.

Compression process (isentropic) from Eqs.8.32-8.34:

$$(P_2/P_1) = (v_1/v_2)^k = CR^{1.4}$$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/95)^{1/1.4} = \mathbf{19.32}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 290 \times (6000/95)^{0.2857} = 947.9 \text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\ T_3/(T_2 \times CR)\ ; \quad v_4 = v_1$$

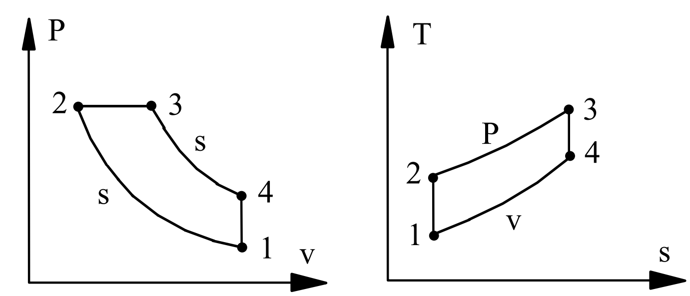
Expansion process, isentropic from Eq.8.32

$$T_4 = T_3\ (v_3/v_4)^{k-1} = T_3\ [T_3/\ (CR \times T_2)]^{k-1}$$

$$= 2400 \times [\ 2400/(19.32 \times 947.9)\ ]^{0.4} = 1064.6 \text{ K}$$

Efficiency from Eq.11.7

$$\eta = 1 - \frac{1}{k}\frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{1.4}\frac{1064.6 - 290}{2400 - 947.9} = \mathbf{0.619}$$



11.107

A diesel engine has a bore of 0.1 m, a stroke of 0.11 m and a compression ratio of 19:1 running at 2000 RPM (revolutions per minute). Each cycle takes two revolutions and has a mean effective pressure of 1400 kPa. With a total of 6 cylinders find the engine power in kW and horsepower, hp.

Solution:

Work from mean effective pressure, Eq.11.15.

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} \quad => \quad w_{net} = P_{meff}\,(v_{max} - v_{min})$$

The displacement is

$$\Delta V = \pi \text{Bore}^2 \times 0.25 \times S = \pi \times 0.1^2 \times 0.25 \times 0.11 = 0.000864 \text{ m}^3$$

Work per cylinder per power stroke, Eq.11.16

$$W = P_{meff}(V_{max} - V_{min}) = 1400 \times 0.000864 \text{ kPa m}^3 = 1.2096 \text{ kJ/cycle}$$

Only every second revolution has a power stroke so we can find the power, see also Eq.11.17

$$\dot{W} = W \times N_{cyl} \times \text{RPM} \times 0.5 \text{ (cycles / min)}\times\text{(min / 60 s)}\times\text{(kJ / cycle)}$$

$$= 1.2096 \times 6 \times 2000 \times 0.5 \times (1/60) = \mathbf{121 \text{ kW} = 162 \text{ hp}}$$

The conversion factor from kW to hp is from Table A.1 under power.

11.108

A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa, 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work and the thermal efficiency.

Solution:

Compression process (isentropic) from Eqs.8.33-34

$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961$ K

$P_2 = 95 \times (20)^{1.4} = 6297.5$ kPa ; $v_2 = v_1/20 = RT_1/(20\ P_1) = 0.043805$

$-_1w_2 = u_2 - u_1 \approx C_{vo}(T_2 - T_1) = 0.717\ (961 - 290) = 481.1$ kJ/kg

Combustion at constant P which is the maximum presssure

$P_3 = P_2 =$ **6298 kPa** ; $v_3 = v_2\ T_3 / T_2 = 0.043805 \times 1800/961 = 0.08205$

${}_2w_3 = P\ (v_3 - v_2) = 6298 \times (0.08215 - 0.043805) = 241.5$ kJ/kg

${}_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2) = 1.004(1800 - 961) = 842.4$

Expansion process (isentropic) from Eq.8.33

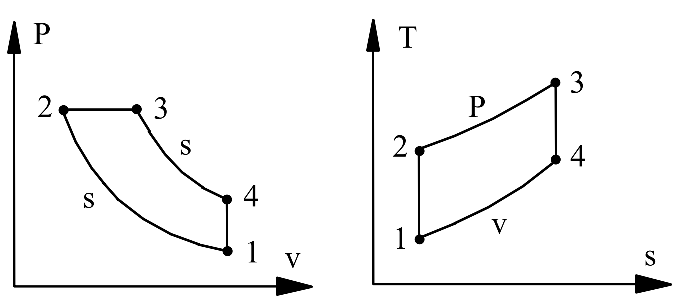
$T_4 = T_3(v_3 / v_4)^{0.4} = 1800\ (0.08205 / 0.8761)^{0.4} = 698$ K

${}_3w_4 = u_3 - u_4 \approx C_{vo}(T_3 - T_4) = 0.717\ (1800 - 698) = 790.1$ kJ/kg

Cycle net work and efficiency

$w_{net} = {}_2w_3 + {}_3w_4 + {}_1w_2 = 241.5 + 790.1 - 481.1 =$ **550.5 kJ/kg**

$\eta = w_{net} / q_H = 550.5/ 842.4 =$ **0.653**



11.109

At the beginning of compression in a diesel cycle T = 300 K, P = 200 kPa and after combustion (heat addition) is complete T = 1500 K and P = 7.0 MPa. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Standard Diesel cycle. See P-v and T-s diagrams for state numbers.

Compression process (isentropic) from Eqs.8.33-8.34

$P_2 = P_3 = 7000$ kPa => $v_1 / v_2 = (P_2/P_1)^{1/k} = (7000 / 200)^{0.7143} = \mathbf{12.67}$

$T_2 = T_1(P_2 / P_1)^{(k-1)/k} = 300(7000 / 200)^{0.2857} = 828.4$ K

Expansion process (isentropic) first get the volume ratios

$v_3 / v_2 = T_3 / T_2 = 1500 / 828.4 = 1.81$

$v_4 / v_3 = v_1 / v_3 = (v_1 / v_2)(v_2 / v_3) = 12.67 / 1.81 = 7$

The exhaust temperature follows from Eq.8.33

$T_4 = T_3(v_3 / v_4)^{k-1} = (1500 / 7)^{0.4} = 688.7$ K

$q_L = C_{vo}(T_4 - T_1) = 0.717(688.7 - 300) = 278.5$ kJ/kg

$q_H = h_3 - h_2 \approx C_{po}(T_3 - T_2) = 1.004(1500 - 828.4) = 674$ kJ/kg

Overall performance

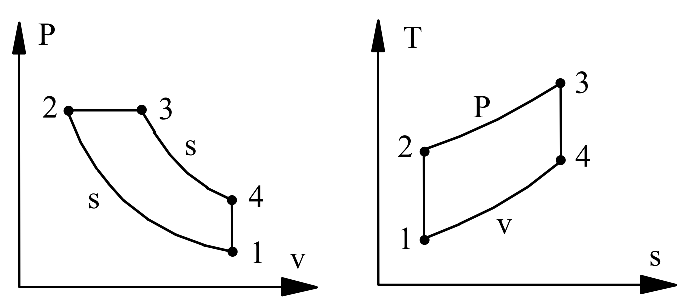
$\eta = 1 - q_L / q_H = 1 - 278.5 / 674 = \mathbf{0.587}$

$w_{net} = q_{net} = q_H - q_L = 674 - 278.5 = 395.5$ kJ/kg

$v_{max} = v_1 = R\,T_1 / P_1 = 0.287 \times 300 / 200 = 0.4305$ m^3/kg

$v_{min} = v_{max} / (v_1 / v_2) = 0.4305 / 12.67 = 0.034$ m^3/kg

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = 395.5 / (0.4305 - 0.034) = \mathbf{997\ kPa}$$



Remark: This is a too low compression ratio for a practical diesel cycle.

11.110

Do problem 11.106, but use the properties from A.7 and not the cold air properties.

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1$ => from Eq.8.28

$$s^o_{T2} = s^o_{T1} + R \ln(P_2 / P_1) = 6.8352 + 0.287 \ln(6000/95) = 8.025 \text{ kJ/kg K}$$

A.7.1 => $T_2 = 907.6$ K; $h_2 = 941.16$;

$h_3 = 2755.8$ kJ/kg; $s^o_{T3} = 9.19586$ kJ/kg K

$q_H = h_3 - h_2 = 2755.8 – 941.16 = 1814.2$ kJ/kg

$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907.6) \times (6000/ 95) = 20.18$

Expansion process

$$s^o_{T4} = s^o_{T3} + R \ln(P_4 / P_3) = s^o_{T3} + R \ln(T_4 / T_3) + R \ln(v_3/v_4)$$

$$v_3/v_4 = v_3/v_1= (v_2/v_1) \times (T_3/T_2) = (T_3/T_2)\ (1/CR)$$
$$= (2400/907.6)\ (1/20.18) = 0.13104$$

$$s^o_{T4} - R \ln(T_4 / T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.1958 + 0.287 \ln 0.13104 = 8.61254$$

Trial and error on T_4 since it appears both in s^o_{T4} and the ln function

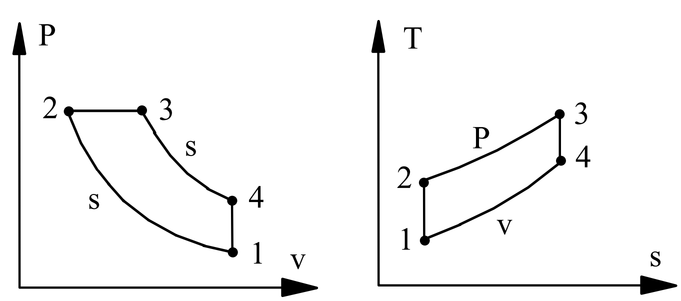
$T_4 =1300$ LHS = 8.4405 – 0.287 ln (1300/2400) = 8.616

$T_4 = 1250$ LHS = 8.3940 – 0.287 ln (1250/2400) = 8.5812

Now Linear interpolation $T_4 = 1295$ K, $u_4 = 1018.26$ kJ/kg

$q_L = u_4 - u_1 = 1018.26 – 207.19 = 811.08$ kJ/kg

$\eta = 1 – (q_L/ q_H) = 1 – (811.08/1814.2) =$ **0.553**



11.111

A diesel engine has air before compression at 280 K, 85 kPa. The highest temperature is 2200 K and the highest pressure is 6 MPa. Find the volumetric compression ratio and the mean effective pressure using cold air properties at 300 K.

Solution:

Compression $(P_2/P_1) = (v_1/v_2)^k = CR^k$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/85)^{1/1.4} = \mathbf{20.92}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 280 \times (6000/85)^{0.2857} = 944.8 \text{ K}$$

Combustion. Highest temperature is after combustion.

$$q_H = h_3 - h_2 = C_P(T_3 - T_2) = 1.004(2200 - 944.8) = 1260.2 \text{ kJ/kg}$$

Expansion

$$T_4 = T_3 (v_3/v_4)^{k-1} = T_3 [\, T_3/ (CR \times T_2) \,]^{k-1}$$

$$= 2200 \times (2200/20.92 \times 944.8)^{0.4} = 914.2 \text{ K}$$

$$q_L = u_4 - u_1 = C_V(T_4 - T_1) = 0.717(914.2 - 280) = 454.7 \text{ kJ/kg}$$

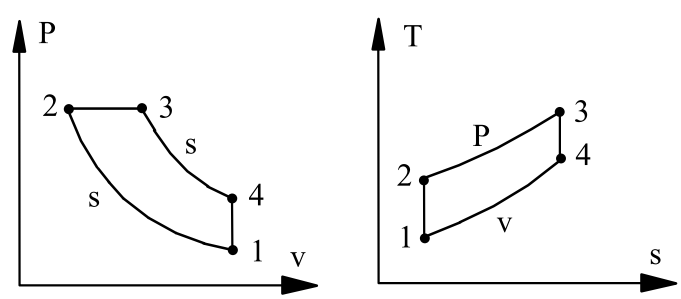
$$v_1 = RT_1/P_1 = 0.287 \times 280/85 = 0.9454 \text{ m}^3\text{/kg}$$

Displacement and mep from net work

$$v_1 - v_2 = v_1 - v_1/CR = v_1[1 - (1/CR)] = 0.9002 \text{ m}^3\text{/kg}$$

$$P_{meff} = w_{net}/(v_1 - v_2) = (q_H - q_L)/(v_1 - v_2)$$

$$= (1260.2 - 454.7)/0.9002 = \mathbf{894.8 \text{ kPa}}$$



11.112

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the maximum temperature (by iteration) in the cycle to have a thermal efficiency of 60%?

Solution:

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $v_1/v_2 = 20$, $\eta_{TH} = 0.6$

Since the efficiency depends on T_3 and T_4, which are connected through the expansion process in a nonlinear manner we have an iterative problem.

$$T_2 = T_1(v_1/v_2)^{k-1} = 290(20)^{0.4} = 961.2 \text{ K}$$

$$v_1 = 0.287 \times 290/95 = 0.876 \text{ m}^3/\text{kg} = v_4,$$

$$v_2 = v_1/CR = 0.876 / 20 = 0.0438 \text{ m}^3/\text{kg}$$

$$v_3 = v_2(T_3/T_2) = 0.0438\ (T_3/961.2) = 0.0000456\ T_3$$

$$T_3 = T_4\ (v_4/v_3)^{k-1} = \left(\frac{0.876}{0.0000456\ T_3}\right)^{0.4} \quad \Rightarrow \quad T_4 = 0.019345\ T_3^{1.4}$$

Now substitute this into the formula for the efficiency

$$\eta_{TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.019345 \times T_3^{1.4} - 290}{1.4(T_3 - 961.2)}$$

$$\Rightarrow\ 0.019345 \times T_3^{1.4} - 0.56 \times T_3 + 248.272 = 0$$

Trial and error on this non-linear equation in T_3

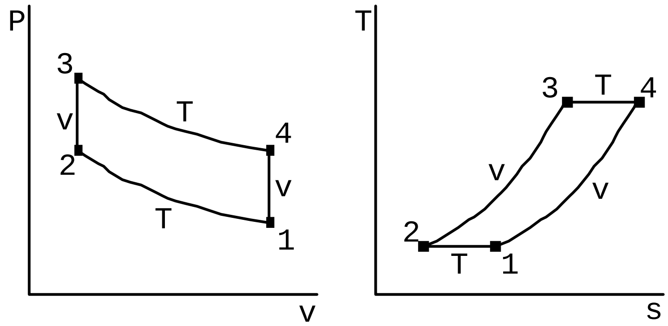
3050 K: LHS = +1.06   3040 K: LHS = -0.036,

Linear interpolation $T_3 =$ **3040 K**

Stirling-cycle engine

11.113

Consider an ideal Stirling-cycle engine in which the state at the beginning of the isothermal compression process is 100 kPa, 25°C, the compression ratio is 6, and the maximum temperature in the cycle is 1100°C. Calculate the maximum cycle pressure and the thermal efficiency of the cycle with and without regenerators.



Ideal Stirling cycle

$T_1 = T_2 = 25\ ^oC$

$P_1 = 100$ kPa

$CR = v_1/v_2 = 6$

$T_3 = T_4 = 1100\ ^oC$

Isothermal compression (heat goes out)

$$T_1 = T_2 \;\Rightarrow\; P_2 = P_1(v_1/v_2) = 100 \times 6 = 600 \text{ kPa}$$

$$_1w_2 = {}_1q_2 = -RT_1 \ln(v_1/v_2) = -0.287\times 298.2 \ln(6) = -153.3 \text{ kJ/kg}$$

Constant volume heat addition

$$V_2 = V_3 \;\Rightarrow\; P_3 = P_2T_3/T_2 = 600\times1373.2/298.2 = \mathbf{2763\ kPa}$$

$$q_{23} = u_3 - u_2 = C_{v\,o}(T_3 - T_2) = 0.717\ (1100 - 25) = 770.8 \text{ kJ/kg}$$

Isothermal expansion (heat comes in)

$$w_{34} = q_{34} = RT_3 \ln(v_4/v_3) = 0.287 \times 1373.2 \times \ln 6 = 706.1 \text{ kJ/kg}$$

$$w_{net} = 706.1 - 153.3 = 552.8 \text{ kJ/kg}$$

Efficiency without regenerator, (q_{23} and q_{34} are coming in from source)

$$\eta_{\text{NO REGEN}} = \frac{w_{net}}{q_{23} + q_{34}} = \frac{552.8}{770.8 + 706.1} = \mathbf{0.374,}$$

Efficiency with regenerator, (Now only q_{34} is coming in from source)

$$\eta_{\text{WITH REGEN}} = \frac{w_{net}}{q_{34}} = \frac{552.8}{706.1} = \mathbf{0.783}$$

11.114

An air-standard Stirling cycle uses helium as the working fluid. The isothermal compression brings helium from 100 kPa, 37°C to 600 kPa. The expansion takes place at 1200 K and there is no regenerator. Find the work and heat transfer in all of the 4 processes per kg helium and the thermal cycle efficiency.

Helium table A.5: $R = 2.077$ kJ/kg K, $C_{vo} = 3.1156$ kJ/kg K

Compression/expansion: $v_4 / v_3 = v_1 / v_2 = P_2 / P_1 = 600 / 100 = 6$

1 -> 2 $-_1w_2 = -q_{12} = \int P\,dv = R\,T_1 \ln(v_1 / v_2) = RT_1 \ln(P_2 / P_1)$

$= 2.077 \times 310 \times \ln 6 = \mathbf{1153.7\ kJ/kg}$

2 -> 3 : $_2w_3 = \mathbf{0}$; $q_{23} = C_{vo}(T_3 - T_2) = 3.1156(1200 - 310) = \mathbf{2773\ kJ/kg}$

3 -> 4: $_3w_4 = q_{34} = R\,T_3 \ln\frac{v_4}{v_3} = 2.077 \times 1200 \ln 6 = \mathbf{4465.8\ kJ/kg}$

4 -> 1 $_4w_1 = \mathbf{0}$; $q_{41} = C_{vo}(T_4 - T_1) = \mathbf{-2773\ kJ/kg}$

$$\eta_{cycle} = \frac{_1w_2 + {_3w_4}}{q_{23} + q_{34}} = \frac{-1153.7 + 4465.8}{2773 + 4465.8} = \mathbf{0.458}$$

11.115

Consider an ideal air-standard Stirling cycle with an ideal regenerator. The minimum pressure and temperature in the cycle are 100 kPa, 25°C, the compression ratio is 10, and the maximum temperature in the cycle is 1000°C. Analyze each of the four processes in this cycle for work and heat transfer, and determine the overall performance of the engine.

Ideal Stirling cycle diagram as in Fig. 11.31, with

$P_1 = 100$ kPa, $T_1 = T_2 = 25^oC$, $v_1/v_2 = 10$, $T_3 = T_4 = 1000^oC$

From 1-2 at const T: ${}_1w_2 = {}_1q_2 = T_1(s_2 - s_1)$

$$= -RT_1 \ln(v_1/v_2) = -0.287 \times 298.2 \times \ln(10) = -197.1 \text{ kJ/kg}$$

From 2-3 at const V: ${}_2w_3 = \mathbf{0}$

$q_{23} = C_{V0}(T_3 - T_2) = 0.717\ (1000 - 25) = \mathbf{699\ kJ/kg}$

From 3-4 at const T; ${}_3w_4 = {}_3q_4 = T_3(s_4 - s_3)$

$$= +RT_3 \times \ln\frac{v_4}{v_3} = 0.287 \times 1237.2 \times \ln(10) = \mathbf{841.4\ kJ/kg}$$

From 4-1 at const V; ${}_4w_1 = \mathbf{0}$

$q_{41} = C_{V0}(T_1 - T_4) = 0.717\ (25 - 1000) = \mathbf{-699\ kJ/kg}$

$w_{NET} = -197.1 + 0 + 841.4 + 0 = 644.3$ kJ/kg

Since q_{23} is supplied by $-q_{41}$ (regenerator)

$$q_H = q_{34} = 841.4 \text{ kJ/kg}, \quad \eta_{TH} = \frac{w_{NET}}{q_H} = \frac{644.3}{841.4} = \mathbf{0.766}$$

NOTE: $q_H = q_{34} = RT_3 \times \ln(10)$, $q_L = -{}_1q_2 = RT_1 \times \ln(10)$

$$\eta_{TH} = \frac{q_H - q_L}{q_H} = \frac{T_3 - T_1}{T_3} = \frac{975}{1273.2} = 0.766 = \text{Carnot efficiency}$$

11.116

The air-standard Carnot cycle was not shown in the text; show the *T–s* diagram for this cycle. In an air-standard Carnot cycle the low temperature is 280 K and the efficiency is 60%. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Solution:

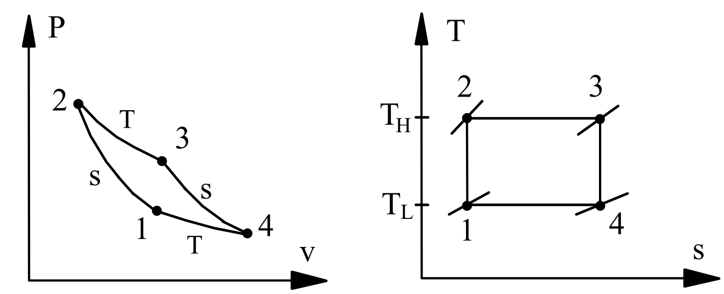
Carnot cycle efficiency from Eq.7.5

$$\eta = 0.6 = 1 - T_H/T_L$$

$$\Rightarrow \; T_H = T_L/0.4 = \mathbf{700\ K}$$

Just before heat addition is state 2 and after heat rejection is state 1 so P_1 = 100 kPa and the isentropic compression is from Eq.8.32

$$P_2 = P_1(T_H/T_L)^{\frac{1}{k-1}} = \mathbf{2.47\ MPa}$$



11.117

Air in a piston/cylinder goes through a Carnot cycle in which T_L = 26.8°C and the total cycle efficiency is η = 2/3. Find T_H, the specific work and volume ratio in the adiabatic expansion for constant C_P, C_v.

Solution:

Carnot cycle efficiency Eq.7.5:

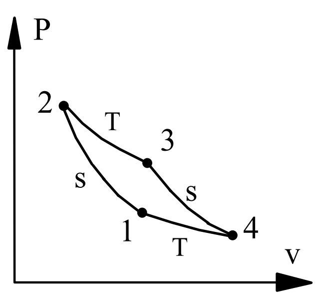
$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

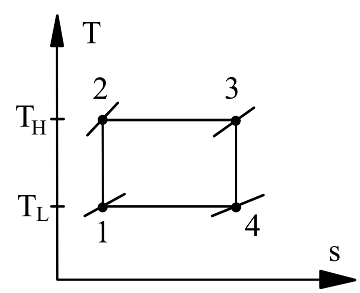
Adiabatic expansion 3 to 4: Pv^k = constant, work from Eq.8.38 (n = k)

$${}_3w_4 = (P_4v_4 - P_3v_3)/(1 - k) = \frac{R}{1 - k}(T_4 - T_3) = u_3 - u_4$$

$$= C_v(T_3 - T_4) = 0.717(900 - 300) = \mathbf{429.9\ kJ/kg}$$

$$v_4/v_3 = (T_3/T_4)^{1/(k - 1)} = 3^{2.5} = \mathbf{15.6}$$





11.118

Do the previous problem 11.117 using values from Table A.7.1.

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 26.8°C$ and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

From A.7.1: $u_3 = 674.82$ kJ/kg, $s^{\circ}_{T3} = 8.0158$ kJ/kg K

$u_4 = 214.36$ kJ/kg, $s^{\circ}_{T4} = 6.8693$ kJ/kg K

Energy equation with q = 0

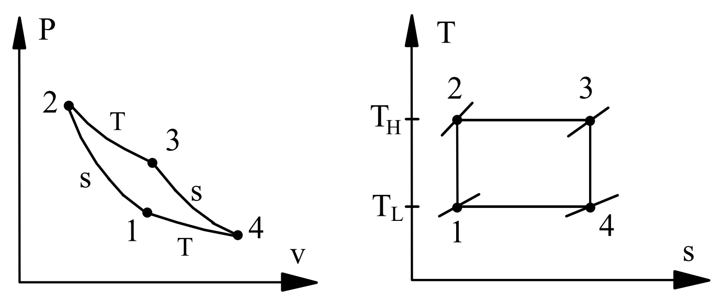
$$_3w_4 = u_3 - u_4 = 674.82 - 214.36 = \mathbf{460.5\ kJ/kg}$$

Entropy equation, constant s

$$s^{\circ}_{T4} = s^{\circ}_{T3} + R \ln(P_4 / P_3) = s^{\circ}_{T3} + R \ln(T_4 / T_3) + R \ln(v_3/v_4)$$

$$\Rightarrow \quad 6.8693 = 8.0158 + 0.287 \ln(300/900) + 0.287 \ln(v_3/v_4)$$

$$\Rightarrow \quad v_4/v_3 = \mathbf{18.1}$$



Refrigeration cycles

11.119

A refrigerator with R-12 as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. Assume an ideal refrigeration cycle as in Fig. 11.24. Find the specific heat transfer from the cold space and that to the hot space, and the coefficient of performance.

Solution:

Exit evaporator sat. vapor −10°C from B.3.1: $h_1 = 183.19$, $s_1 = 0.7019$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.3.1: $h_3 = 76.22$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.3.2 $\Rightarrow$ $h_2 \approx 210.1$ kJ/kg

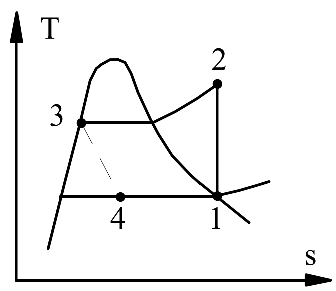
Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 183.19 - 76.22 =$ **107 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 210.1 - 76.22 =$ **133.9 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **3.98**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.3



11.120

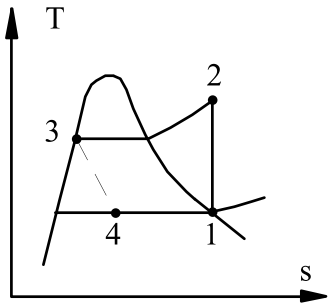
Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluids R-12 and R-22.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



	Property for:	R-12, B.3	R-22, B.4
Compressor	h_1, kJ/kg	180.97	244.13
	$s_2 = s_1$, kJ/kg K	0.7051	0.9505
	P_2, MPa	1.0843	1.729
	T_2, ^oC	54.7	74.4
	h_2, kJ/kg	212.63	289.26
	$w_C = h_2 - h_1$	31.66	45.13
Exp. valve	$h_3 = h_4$, kJ/kg	79.71	100.98
Evaporator	$q_L = h_1 - h_4$	101.26	143.15
	$\beta = q_L/w_C$	**3.198**	**3.172**

The value of h_2 is taken from the computer program as it otherwise will be a double interpolation due to the value of P_2.

11.121

The environmentally safe refrigerant R-134a is one of the replacements for R-12 in refrigeration systems. Repeat Problem 11.120 using R-134a and compare the result with that for R-12.

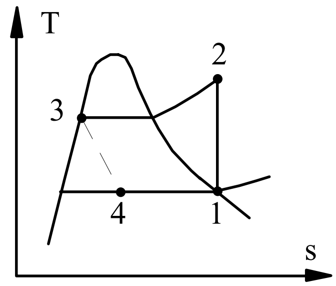
Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluids R-12 and R-22.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



	Property for:	R-12, B.3	R-134a, B.5
Compressor	h_1, kJ/kg	180.97	389.2
	$s_2 = s_1$, kJ/kg K	0.7051	1.7354
	P_2, MPa	1.0843	1.16
	T_2, ^oC	54.7	51.8*
	h_2, kJ/kg	212.63	429.9*
	$w_C = h_2 - h_1$	31.66	40.7
Exp. valve	$h_3 = h_4$, kJ/kg	79.71	264.11
Evaporator	$q_L = h_1 - h_4$	101.26	125.1
	$\beta = q_L/w_C$	**3.198**	**3.07**

* To get state 2 an interpolation is needed:

At 1 MPa, s = 1.7354 : T = 45.9 and h = 426.8 kJ/kg

At 1.2 MPa, s = 1.7354 : T = 53.3 and h = 430.7 kJ/kg

make a linear interpolation to get properties at 1.16 MPa

11.122

A refrigerator using R-22 is powered by a small natural gas fired heat engine with a thermal efficiency of 25%, as shown in Fig.P11.122. The R-22 condenses at 40°C and it evaporates at –20°C and the cycle is standard. Find the two specific heat transfers in the refrigeration cycle. What is the overall coefficient of performance as Q_L/Q_1?

Solution:

Evaporator: Inlet State is saturated liq-vap with $h_4 = h_3 = 94.27$ kJ/kg

The exit state is saturated vapor with $h_1 = 242.06$ kJ/kg

$q_L = h_1 - h_4 = h_1 - h_3 =$ **147.79 kJ/kg**

Compressor: Inlet State 1 and Exit State 2 about 1.6 MPa

$w_C = h_2 - h_1$; $s_2 = s_1 = 0.9593$ kJ/kgK

2: $T_2 \approx 70°C$ $h_2 = 287.2$ kJ/kg

$w_C = h_2 - h_1 = 45.14$ kJ/kg

Condenser: Brings it to saturated liquid at state 3

$q_H = h_2 - h_3 = 287.2 - 94.27 =$ **192.9 kJ/kg**

Overall Refrigerator:

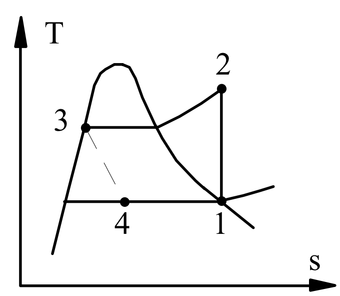
$$\beta = q_L / w_C = 147.79 / 45.14 = 3.274$$

Heat Engine:

$$\dot{W}_{HE} = \eta_{HE}\dot{Q}_1 = \dot{W}_C = \dot{Q}_L / \beta$$

$$\dot{Q}_L / \dot{Q}_1 = \eta\beta = 0.25 \times 3.274 = \mathbf{0.819}$$

Ideal refrigeration cycle
$T_{cond} = 40^oC = T_3$
$T_{evap} = -20^oC = T_1$
Properties from Table B.4



11.123

A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses R-12 as the refrigerant which must remove 5 kW from the cold space. Find the flow rate of the R-12 needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:

Basic refrigeration cycle: $T_1 = T_4 = -15°C$, $T_3 = 20°C$

Table B.3: $h_4 = h_3 = 54.87$ kJ/kg; $h_1 = h_g = 180.97$ kJ/kg

$$\dot{Q}_L = \dot{m}_{R-12} \times q_L = \dot{m}_{R-12}(h_1 - h_4)$$

$$q_L = 180.97 - 54.87 = 126.1 \text{ kJ/kg}$$

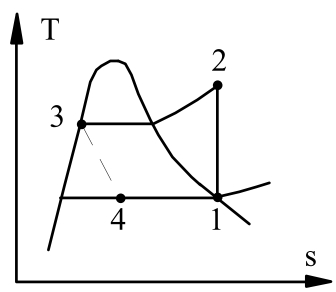
$$\dot{m}_{R-12} = 5.0 / 126.1 = \mathbf{0.03965\ kg/s}$$

Ideal refrigeration cycle

$T_{cond} = 20^oC$

$T_{evap} = -15^oC = T_1$

Properties from Table B.3



11.124

A refrigerator with R-12 as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. The actual adiabatic compressor exit temperature is 60°C. Assume no pressure loss in the heat exchangers. Find the specific heat transfer from the cold space and that to the hot space, the coefficient of performance and the isentropic efficiency of the compressor.

Solution:

State 1: Inlet to compressor, sat. vapor -10°C,

$h_1 = 183.19$ kJ/kg, $s_1 = 0.7019$ kJ/kg K

State 2: Actual compressor exit, $h_{2AC} = 217.97$ kJ/kg

State 3: Exit condenser, sat. liquid 1MPa, $h_3 = 76.22$ kJ/kg

State 4: Exit valve, $h_4 = h_3$

C.V. Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 =$ **107 kJ/kg**

C.V. Ideal Compressor: $w_{C,S} = h_{2,S} - h_1$, $s_{2,S} = s_1$

State 2s: 1 MPa, s = 0.7019 kJ/kg K; $T_{2,S} = 49.66°C$, $h_{2,S} = 210.1$ kJ/kg

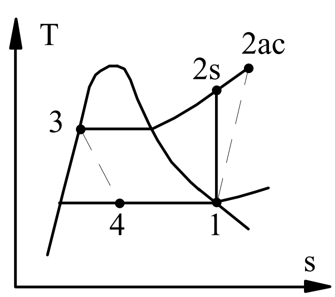
$w_{C,S} = h_{2,S} - h_1 = 26.91$ kJ/kg

C.V. Actual Compressor: $w_C = h_{2,AC} - h_1 = 34.78$ kJ/kg

$$\beta = \frac{q_L}{w_C} = \mathbf{3.076}, \quad \eta_C = w_{C,S}/w_C = \mathbf{0.774}$$

C.V. Condenser: $q_H = h_{2,AC} - h_3 =$ **141.75 kJ/kg**

Ideal refrigeration cycle with actual compressor
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_2 = 60^oC$
$T_{evap} = -10^oC = T_1$
Properties from Table B.3



11.125

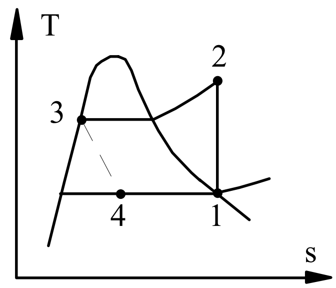
Consider an ideal heat pump that has a condenser temperature of 50°C and an evaporator temperature of 0°C. Determine the coefficient of performance of this heat pump for the working fluids R-12, R-22, and ammonia.

Solution:

Ideal heat pump

$T_{cond} = 50^oC = T_3$

$T_{evap} = 0^oC = T_1$



C.V.	Property for: From Table:	R-12 B.3	R-22 B.4	NH_3 B.2
	h_1, kJ/kg	187.53	249.95	1442.32
Compressor	$s_2 = s_1$, kJ/kgK	0.6965	0.9269	5.3313
	P_2, MPa	1.2193	1.9423	2.0333
	T_2, ^oC	56.7	72.2	115.6
	h_2, kJ/kg	211.95	284.25	1672.84
	$w_C = h_2 - h_1$	24.42	34.3	230.52
Exp. valve	$h_3 = h_4$, kJ/kg	84.94	107.85	421.58
Condenser	$q_H = h_2 - h_3$	127.01	176.4	1251.26
	$\beta' = q_H/w_C$	**5.201**	**5.143**	**5.428**

11.126

The air conditioner in a car uses R-134a and the compressor power input is 1.5 kW bringing the R-134a from 201.7 kPa to 1200 kPa by compression. The cold space is a heat exchanger that cools atmospheric air from the outside 30°C down to 10°C and blows it into the car. What is the mass flow rate of the R-134a and what is the low temperature heat transfer rate. How much is the mass flow rate of air at 10°C?

Standard Refrigeration Cycle

Table B.5: h_1 = 392.28 kJ/kg; s_1 = 1.7319 kJ/kg K; $h_4 = h_3$ = 266

C.V. Compressor (assume ideal)

$$\dot{m}_1 = \dot{m}_2 \qquad w_C = h_2 - h_1; \quad s_2 = s_1 + s_{gen}$$

$$P_2,\ s = s_1 \;\Rightarrow\; h_2 = 429.5 \text{ kJ/kg} \;\Rightarrow\; w_C = 37.2 \text{ kJ/kg}$$

$$\dot{m}\, w_C = \dot{W}_C \;\Rightarrow\; \dot{m} = 1.5 / 37.2 = \mathbf{0.0403\ kg/s}$$

C.V. Evaporator

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.0405(392.28 - 266) = \mathbf{5.21\ kW}$$

C.V. Air Cooler

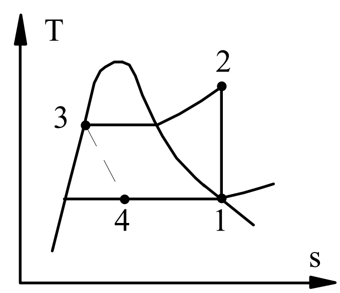
$$\dot{m}_{air}\Delta h_{air} = \dot{Q}_L \approx \dot{m}_{air} C_p \Delta T$$

$$\dot{m}_{air} = \dot{Q}_L / (C_p \Delta T) = 5.21 / (1.004 \times 20) = \mathbf{0.26\ kg / s}$$

Ideal refrigeration cycle

P_{cond} = 1200 kPa = P_3

P_{evap} = 201.7 kPa = P_1



11.127

A refrigerator using R-134a is located in a 20°C room. Consider the cycle to be ideal, except that the compressor is neither adiabatic nor reversible. Saturated vapor at -20°C enters the compressor, and the R-134a exits the compressor at 50°C. The condenser temperature is 40°C. The mass flow rate of refrigerant around the cycle is 0.2 kg/s, and the coefficient of performance is measured and found to be 2.3. Find the power input to the compressor and the rate of entropy generation in the compressor process.

Solution:

Table B.5: $P_2 = P_3 = P_{sat\ 40C} = 1017$ kPa, $h_4 = h_3 = 256.54$ kJ/kg

$s_2 \approx 1.7472$ kJ/kg K, $h_2 \approx 430.87$ kJ/kg;

$s_1 = 1.7395$ kJ/kg K, $h_1 = 386.08$ kJ/kg

$\beta = q_L / w_C \;\; \text{->} \;\; w_C = q_L / \beta = (h_1 - h_4) / \beta = (386.08 - 256.54) / 2.3 = 56.32$

$\dot{W}_C = \dot{m}\, w_C =$ **11.26 kW**

C.V. Compressor $h_1 + w_C + q = h_2$ ->

$q_{in} = h_2 - h_1 - w_C = 430.87 - 386.08 - 56.32 = -11.53$ kJ/kg i.e. a heat loss

$s_1 + \int dQ/T + s_{gen} = s_2$

$s_{gen} = s_2 - s_1 - q / T_o = 1.7472 - 1.7395 + (11.53 / 293.15) = 0.047$ kJ/kg K

$\dot{S}_{gen} = \dot{m}\, s_{gen} = 0.2 \times 0.047 =$ **0.0094 kW / K**

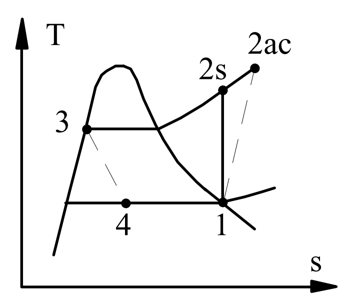
Ideal refrigeration cycle with actual compressor

$T_{cond} = 40^oC$

$T_2 = 50^oC$

$T_{evap} = -20^oC = T_1$

Properties from Table B.5



11.128

A refrigerator has a steady flow of R-22 as saturated vapor at –20°C into the adiabatic compressor that brings it to 1000 kPa. After the compressor, the temperature is measured to be 60°C. Find the actual compressor work and the actual cycle coefficient of performance.

Solution:

Table B.4.1: $h_1 = 242.06$ kJ/kg, $s_1 = 0.9593$ kJ/kg K

$P_2 = P_3 = 1000$ kPa, $h_4 = h_3 = h_f = 72.86$ kJ/kg

$h_{2\ ac} = 286.97$ kJ/kg

C.V. Compressor (actual)

Energy Eq.: $w_{C\ ac} = h_{2\ ac} - h_1 = 286.97 - 242.06 = \mathbf{44.91\ kJ/kg}$

C.V. Evaporator

Energy Eq.: $q_L = h_1 - h_4 = h_1 - h_3 = 242.06 - 72.86 = 169.2$ kJ/kg

$$\beta = \frac{q_L}{w_{C\ ac}} = \frac{169.2}{44.91} = \mathbf{3.77}$$

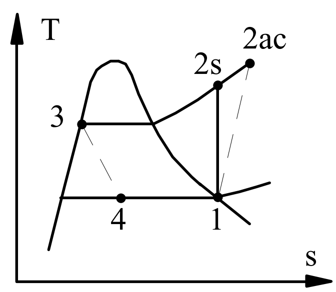
Ideal refrigeration cycle with actual compressor

$T_{cond} = 23.4^oC = T_{sat\ 1000\ kPa}$

$T_2 = 60^oC$

$T_{evap} = -20^oC = T_1$

Properties from Table B.4



11.129

A small heat pump unit is used to heat water for a hot-water supply. Assume that the unit uses R-22 and operates on the ideal refrigeration cycle. The evaporator temperature is 15°C and the condenser temperature is 60°C. If the amount of hot water needed is 0.1 kg/s, determine the amount of energy saved by using the heat pump instead of directly heating the water from 15 to 60°C.

Solution:

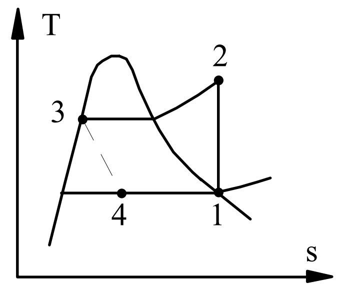
Ideal R-22 heat pump

$T_1 = 15^oC$, $T_3 = 60^oC$

From Table B.4.1

$h_1 = 255.02$ kJ/kg, $s_2 = s_1 = 0.9062$ kJ/kg K

$P_2 = P_3 = 2.427$ MPa, $h_3 = 122.18$ kJ/kg



Entropy compressor: $s_2 = s_1$ => $T_2 = 78.4^oC$, $h_2 = 282.86$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 27.84$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 160.68$ kJ/kg

To heat 0.1 kg/s of water from 15°C to 60°C,

$$\dot{Q}_{H2O} = \dot{m}(\Delta h) = 0.1(251.11 - 62.98) = 18.81 \text{ kW}$$

Using the heat pump

$$\dot{W}_{IN} = \dot{Q}_{H2O}(w_C/q_H) = 18.81(27.84/160.68) = 3.26 \text{ kW}$$

a saving of **15.55 kW**

11.130

The refrigerant R-22 is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 10°C; its exit temperature from the compressor is measured and found to be 85°C. If the compressor exit is at 2 MPa what is the compressor isentropic efficiency and the cycle COP?

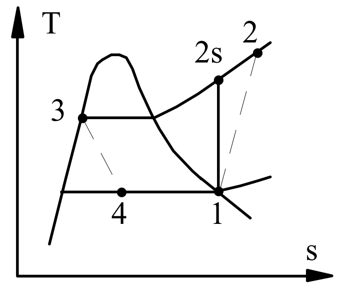
Solution:

R-22 heat pump:
Table B.4

State 1: $T_{EVAP} = 10^oC$, $x = 1$

$h_1 = 253.42$ kJ/kg, $s_1 = 0.9129$ kJ/kg K

State 2: T_2, P_2: $h_2 = 295.17$ kJ/kg



C.V. Compressor

Energy Eq.: $w_{C\,ac} = h_2 - h_1 = 295.17 - 253.42 = \mathbf{41.75\ kJ/kg}$

State 2s: 2 MPa , $s_{2S} = s_1 = 0.9129$ kJ/kg $T_{2S} = 69^oC$, $h_{2S} = 280.2$ kJ/kg

Efficiency: $$\eta = \frac{w_{C\,s}}{w_{C\,ac}} = \frac{h_{2S} - h_1}{h_2 - h_1} = \frac{280.2 - 253.42}{295.17 - 253.42} = \mathbf{0.6414}$$

C.V. Condenser

Energy Eq.: $q_H = h_2 - h_3 = 295.17 - 109.6 = 185.57$ kJ/kg

COP Heat pump: $$\beta = \frac{q_H}{w_{C\,ac}} = \frac{185.57}{41.75} = \mathbf{4.44}$$

11.131

A refrigerator in a laboratory uses R-22 as the working substance. The high pressure is 1200 kPa, the low pressure is 201 kPa, and the compressor is reversible. It should remove 500 W from a specimen currently at –20°C (not equal to T in the cycle) that is inside the refrigerated space. Find the cycle COP and the electrical power required.

Solution:

State 1: 201 kPa, x = 1, Table B.4.1: $h_1 = 239.92$ kJ/kg, $s_1 = 0.9685$ kJ/kg K

State 3: 1200 kPa, x = 0, Table B.4.1: $h_3 = 81.57$ kJ/kg

C.V. Compressor

Energy Eq.: $w_C = h_2 - h_1$

Entropy Eq.: $s_2 = s_1 + s_{gen} = s_1$

State 2: 1.2 MPa, $s_2 = s_1 = 0.9685$ kJ/kg, $T_2 \approx 60^{o}C$, $h_2 = 285.21$ kJ/kg

$$w_C = h_2 - h_1 = 285.21 - 239.92 = 45.29 \text{ kJ/kg}$$

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 239.92 - 81.57 = 158.35$ kJ/kg

COP Refrigerator: $\beta = \dfrac{q_L}{w_C} = \dfrac{158.35}{45.29} = \mathbf{3.5}$

Power: $\dot{W}_{IN} = \dot{Q}_L / \beta = 500 \text{ W}/ 3.5 = \mathbf{142.9\ W}$

11.132

Consider the previous problem and find the two rates of entropy generation in the process and where they occur.

Solution:

From the basic cycle we know that entropy is generated in the valve as the throttle process is irreversible.

State 1: 201 kPa, x = 1, Table B.4.1: $h_1 = 239.92$ kJ/kg, $s_1 = 0.9685$ kJ/kg K

State 3: 1200 kPa, x = 0, Table B.4.1: $h_3 = 81.57$ kJ/kg, $s_3 = 0.30142$ kJ/kg K

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 239.92 - 81.57 = 158.35$ kJ/kg

Mass flow rate: $\dot{m} = \dot{Q}_L / q_L = 0.5 / 158.35 = 0.00316$ kg/s

C.V. Valve

Energy Eq.: $h_4 = h_3 = 81.57$ kJ/kg $\Rightarrow$ $x_4 = (h_4 - h_f)/h_{fg}$

$$x_4 = \frac{81.57 - 16.19}{223.73} = 0.29223$$

$$s_4 = s_f + x_4 s_{fg} = 0.067 + x_4 \times 0.9015 = 0.33045 \text{ kJ/kg K}$$

Entropy Eq.: $s_{gen} = s_4 - s_3 = 0.33045 - 0.30142 = 0.02903$ kJ/kg K

$$\dot{S}_{gen\ valve} = \dot{m}s_{gen} = 0.00316 \times 0.02903 = \mathbf{0.0917\ W/K}$$

There is also entropy generation in the heat transfer process from the specimen at –20°C to the refrigerant T = -25°C = T_{sat} (201 kPa).

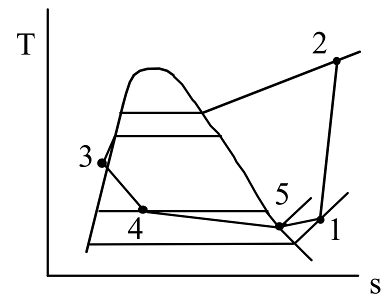
$$\dot{S}_{gen\ inside} = \dot{Q}_L \left[\frac{1}{T_{specimen}} - \frac{1}{T_L} \right] = 500 \left(\frac{1}{248} - \frac{1}{253}\right) = \mathbf{0.04\ W/K}$$

11.133

In an actual refrigeration cycle using R-12 as the working fluid, the refrigerant flow rate is 0.05 kg/s. Vapor enters the compressor at 150 kPa, −10°C, and leaves at 1.2 MPa, 75°C. The power input to the compressor is measured and found be 2.4 kW. The refrigerant enters the expansion valve at 1.15 MPa, 40°C, and leaves the evaporator at 175 kPa, −15°C. Determine the entropy generation in the compression process, the refrigeration capacity and the coefficient of performance for this cycle.

Solution:

Actual refrigeration cycle



1: compressor inlet $T_1 = -10^oC$, $P_1 = 150$ kPa

2: compressor exit $T_2 = 75^oC$, $P_2 = 1.2$ MPa

3: Expansion valve inlet $T_3 = 40^oC$
$P_3 = 1.15$ MPa

5: evaporator exit $T_5 = -15^oC$, $P_5 = 175$ kPa

Table B.3 $h_1 = 184.619$, $s_1 = 0.7318$, $h_2 = 226.543$, $s_2 = 0.7404$

CV Compressor: $h_1 + q_{COMP} + w_{COMP} = h_2$; $s_1 + \int dq/T + s_{gen} = s_2$

$$w_{COMP} = \dot{W}_{COMP}/\dot{m} = 2.4/0.05 = 48.0 \text{ kJ/kg}$$

$$q_{COMP} = h_2 - w_{COMP} - h_1 = 226.5 - 48.0 - 184.6 = -6.1 \text{ kJ/kg}$$

$$s_{gen} = s_2 - s_1 - q / T_o = 0.7404 - 0.7318 + 6.1/298.15 = \mathbf{0.029\ kJ / kg\ K}$$

C.V. Evaporator

$$q_L = h_5 - h_4 = 181.024 - 74.527 = 106.5 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_L = \dot{m}q_L = 0.05 \times 106.5 = \mathbf{5.325\ kW}$$

COP: $\beta = q_L/w_{COMP} = 106.5/48.0 = \mathbf{2.219}$

Ammonia absorption cycles

11.134

Consider a small ammonia absorption refrigeration cycle that is powered by solar energy and is to be used as an air conditioner. Saturated vapor ammonia leaves the generator at 50°C, and saturated vapor leaves the evaporator at 10°C. If 7000 kJ of heat is required in the generator (solar collector) per kilogram of ammonia vapor generated, determine the overall performance of this system.

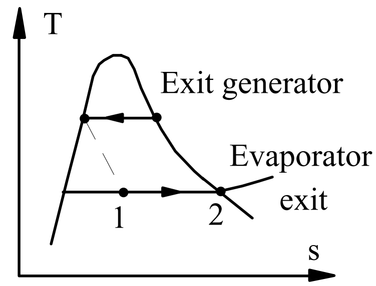
Solution;

NH_3 absorption cycle:

sat. vapor at 50^oC exits the generator

sat. vapor at 10^oC exits the evaporator

$q_H = q_{GEN} = 7000$ kJ/kg NH_3 out of gen.



C.V. Evaporator

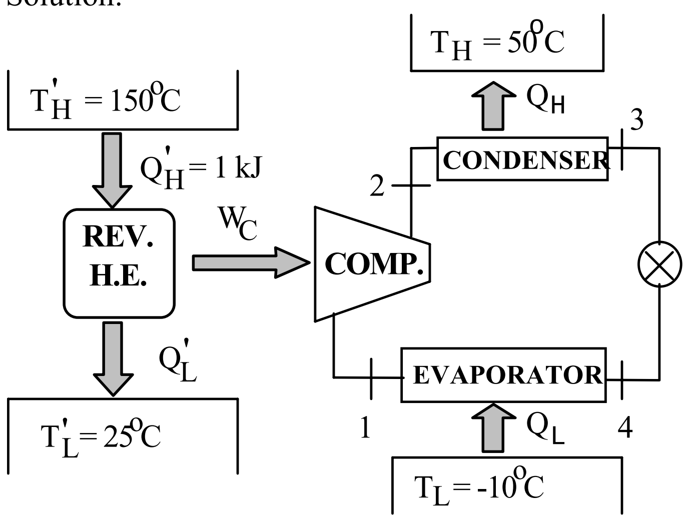
$$q_L = h_2 - h_1 = h_{g\ 10^oC} - h_{f\ 50^oC} = 1452.2 - 421.6 = 1030.6 \text{ kJ/kg}$$

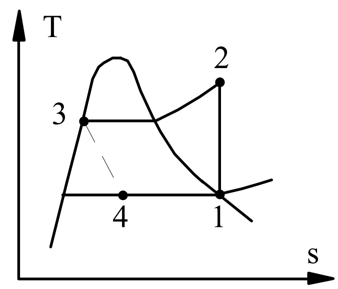
COP $\Rightarrow$ $q_L/q_H = 1030.6/7000 =$ **0.147**

11.135

The performance of an ammonia absorption cycle refrigerator is to be compared with that of a similar vapor-compression system. Consider an absorption system having an evaporator temperature of −10°C and a condenser temperature of 50°C. The generator temperature in this system is 150°C. In this cycle 0.42 kJ is transferred to the ammonia in the evaporator for each kilojoule transferred from the high-temperature source to the ammonia solution in the generator. To make the comparison, assume that a reservoir is available at 150°C, and that heat is transferred from this reservoir to a reversible engine that rejects heat to the surroundings at 25°C. This work is then used to drive an ideal vapor-compression system with ammonia as the refrigerant. Compare the amount of refrigeration that can be achieved per kilojoule from the high-temperature source with the 0.42 kJ that can be achieved in the absorption system.

Solution:





$T_1 = -10\ ^oC$

$h_1 = 1430.8\ ,\ \ s_1 = 5.4673$

$h_4 = h_3 = 421.48$

For the rev. heat engine: $\quad \eta_{TH} = 1 - T_L'/T_H' = 1 - \dfrac{298.2}{423.2} = 0.295$

$$\Rightarrow\ W_C = \eta_{TH}\, Q_H' = 0.295 \text{ kJ}$$

For the NH_3 refrig. cycle: $\quad P_2 = P_3 = 2033$ kPa , $\quad$ Use 2000 kPa Table

$$s_2 = s_1 = 5.4673 \quad => \quad T_2 \approx 135°C \qquad h_2 \approx 1724$$

$$w_C = h_2 - h_1 = 1724 - 1430.8 = 293.2 \text{ kJ/kg}$$

$$q_L = h_1 - h_4 = 1430.8 - 421.48 = 1009.3 \text{ kJ/kg}$$

$$\beta = q_L/w_C = 1009.3 / 293.2 = 3.44$$

$$\Rightarrow\ Q_L = \beta w_C = 3.44 \times 0.295 = \mathbf{1.015\ kJ}$$

based on assumption of ideal heat engine & refrigeration cycle.

Air standard refrigeration cycles

11.136

The formula for the coefficient of performance when we use cold air properties is not given in the text. Derive the expression for COP as function of the compression ratio similar to how the Brayton cycle efficiency was found.

Definition of COP: $$\beta = \frac{q_L}{w_{net}} = \frac{q_L}{q_H - q_L} = \frac{1}{\frac{q_H}{q_L} - 1}$$

From the refrigeration cycle we get the ratio of the heat transfers as

$$\frac{q_H}{q_L} = \frac{C_p(T_2 - T_3)}{C_p(T_1 - T_4)} = \frac{T_2(1 - T_3/T_2)}{T_1(1 - T_4/T_1)}$$

The pressure ratios are the same and we have isentropic compression/expansion

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_3}{T_4}\right)^{k/(k-1)}$$

so now we get

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \qquad \text{or} \qquad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The heat transfer ratio simplifies to

$$\frac{q_H}{q_L} = \frac{T_2}{T_1}$$

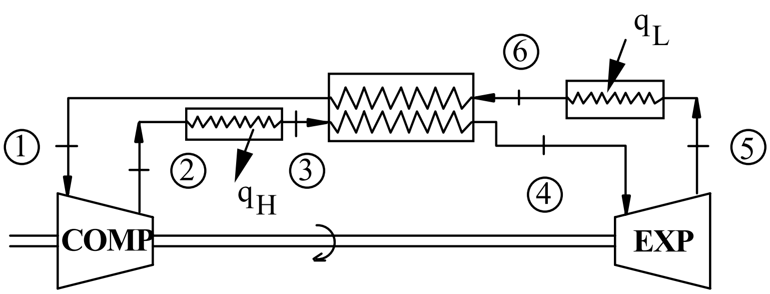
and so the COP reduces to

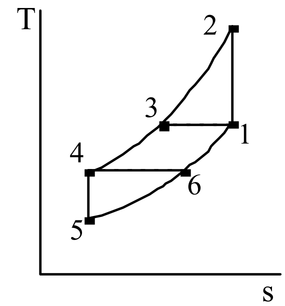
$$\beta = \frac{1}{\frac{T_2}{T_1} - 1} = \frac{1}{\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1}$$

11.137

A heat exchanger is incorporated into an ideal air-standard refrigeration cycle, as shown in Fig. P11.137. It may be assumed that both the compression and the expansion are reversible adiabatic processes in this ideal case. Determine the coefficient of performance for the cycle.

Solution:





Standard air refrigeration cycle with

$T_1 = T_3 = 15\ ^oC = 288.2\ K$, $P_1 = 100\ kPa$, $P_2 = 1.4\ MPa$

$T_4 = T_6 = -50\ ^oC = 223.2\ K$

We will solve the problem with cold air properties.

Compressor, isentropic $s_2 = s_1$ so from Eq.8.32

$$\Rightarrow\ T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2(1400/100)^{0.286} = 613\ K$$

$$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(613 - 288.2) = 326\ kJ/kg$$

Expansion in expander (turbine)

$$s_5 = s_4 \Rightarrow\ T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2(100/1400)^{0.286} = 104.9\ K$$

$$w_E = C_{P0}(T_4 - T_5) = 1.004(223.2 - 104.9) = 118.7\ kJ/kg$$

Net cycle work

$$w_{NET} = w_E - w_C = 118.7 - 326.0 = -207.3\ kJ/kg$$

$$q_L = C_{P0}(T_6 - T_5) = w_E = 118.7\ kJ/kg$$

Overall cycle performance, COP

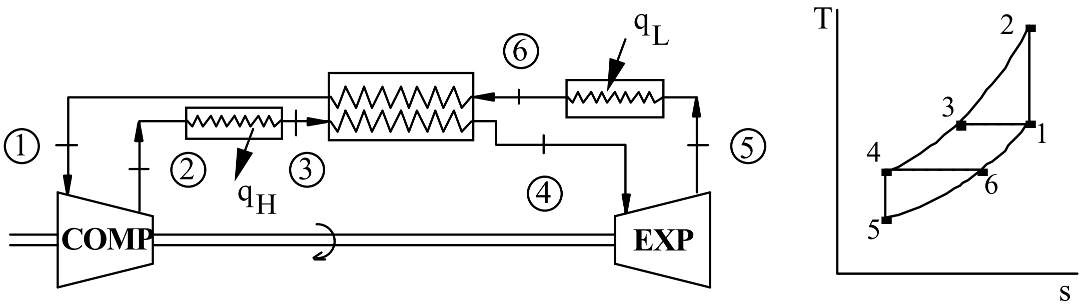
$$\beta = q_L/w_{NET} = 118.7 / 207.3 = \mathbf{0.573}$$

11.138

Repeat Problems 11.137, but assume that helium is the cycle working fluid instead of air. Discuss the significance of the results.

A heat exchanger is incorporated into an ideal air-standard refrigeration cycle, as shown in Fig. P11.137. It may be assumed that both the compression and the expansion are reversible adiabatic processes in this ideal case. Determine the coefficient of performance for the cycle.

Solution:



Standard air refrigeration cycle with helium and states as

$T_1 = T_3 = 15\ ^oC = 288.2\ K,\quad P_1 = 100\ kPa,\quad P_2 = 1.4\ MPa$

$T_4 = T_6 = -50\ ^oC = 223.2\ K$

Compressor, isentropic $s_2 = s_1$ so from Eq.8.32

$$\Rightarrow\ T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2\left(\frac{1400}{100}\right)^{0.40} = 828.2\ K$$

$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 5.193(828.2 - 288.2) = 2804.1\ kJ/kg$

Expansion in expander (turbine)

$$s_5 = s_4 \quad\Rightarrow\quad T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2\left(\frac{100}{1400}\right)^{0.40} = 77.7\ K$$

$w_E = C_{P0}(T_4 - T_5) = 15.193(223.2 - 77.7) = 755.5\ kJ/kg$

Net cycle work

$w_{NET} = 755.5 - 2804.1 = -2048.6\ kJ/kg$

$q_L = C_{P0}(T_6 - T_5) = 5.193(223.2 - 77.7) = 755.5\ kJ/kg$

Overall cycle performance, COP

$\beta = q_L/w_{NET} = 755.5/2048.6 =$ **0.369**

Notice that the low temperature is lower and work terms higher than with air. It is due to the higher heat capacity C_{P0} and ratio of specific heats (k = 1 2/3). The expense is a lower COP requiring more work input per kJ cooling.

11.139

Repeat Problem 11.137, but assume an isentropic efficiency of 75% for both the compressor and the expander.

Standard air refrigeration cycle with

$T_1 = T_3 = 15\ ^oC = 288.2$ K, $P_1 = 100$ kPa, $P_2 = 1.4$ MPa

$T_4 = T_6 = -50\ ^oC = 223.2$ K

We will solve the problem with cold air properties.

Ideal compressor, isentropic $s_{2S} = s_1$ so from Eq.8.32

$$\Rightarrow\ T_{2S} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2(1400/100)^{0.286} = 613 \text{ K}$$

$$w_{SC} = -w_{12} = C_{P0}(T_{2S} - T_1) = 1.004(613 - 288.2) = 326 \text{ kJ/kg}$$

The actual compressor

$$w_C = w_{SC} / \eta_{SC} = 326/0.75 = 434.6 \text{ kJ/kg}$$

Expansion in ideal expander (turbine)

$$s_5 = s_4 \Rightarrow\ T_{5S} = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2(100/1400)^{0.286} = 104.9 \text{ K}$$

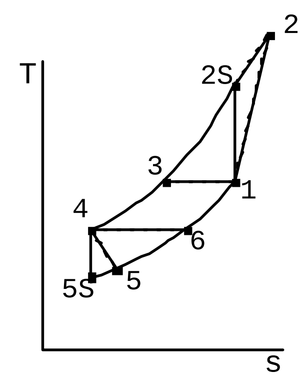
$$w_E = C_{P0}(T_4 - T_5) = 1.004(223.2 - 104.9) = 118.7 \text{ kJ/kg}$$

The actual expander (turbine)

$$w_E = \eta_{SE} \times w_{SE} = 0.75 \times 118.7 = 89.0 \text{ kJ/kg}$$
$$= C_{P0}(T_4 - T_5) = 1.004(223.2 - T_5)$$

$$\Rightarrow\ T_5 = 134.5 \text{ K}$$

$$w_{NET} = 89.0 - 434.6 = -345.6 \text{ kJ/kg}$$



$$q_L = C_{P0}(T_6 - T_5) = 1.004(223.2 - 134.5) = 89.0 \text{ kJ/kg}$$

$$\beta = q_L/(-w_{NET}) = 89.0/345.6 = \mathbf{0.258}$$

Combined Cycles

11.140

A binary system power plant uses mercury for the high-temperature cycle and water for the low-temperature cycle, as shown in Fig. 11.39. The temperatures and pressures are shown in the corresponding *T–s* diagram. The maximum temperature in the steam cycle is where the steam leaves the superheater at point 4 where it is 500°C. Determine the ratio of the mass flow rate of mercury to the mass flow rate of water in the heat exchanger that condenses mercury and boils the water and the thermal efficiency of this ideal cycle.

The following saturation properties for mercury are known

P, MPa	T_g, °C	h_f, kJ/kg	h_g, kJ/kg	s_f kJ/kgK	s_g, kJ/kgK
0.04	309	42.21	335.64	0.1034	0.6073
1.60	562	75.37	364.04	0.1498	0.4954

Solution:

For the mercury cycle:

$$s_d = s_c = 0.4954 = 0.1034 + x_d \times 0.5039, \quad x_d = 0.7779$$

$$h_b = h_a - w_{P\ HG} \approx h_a \text{ (since } v_F \text{ is very small)}$$

$$q_H = h_c - h_a = 364.04 - 42.21 = 321.83 \text{ kJ/kg}$$

$$q_L = h_d - h_a = 270.48 - 42.21 = 228.27 \text{ kJ/kg}$$

For the steam cycle:

$$s_5 = s_4 = 7.0097 = 0.6493 + x_5 \times 7.5009, \quad x_5 = 0.8480$$

$$h_5 = 191.83 + 0.848 \times 2392.8 = 2220.8$$

$$w_P \approx v_1(P_2 - P_1) = 0.00101(4688 - 10) = 4.7 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.8 + 4.7 = 196.5$$

$$q_H \text{ (from Hg)} = h_3 - h_2 = 2769.9 - 196.5 = 2600.4$$

$$q_H \text{ (ext. source)} = h_4 - h_3 = 3437.4 - 2796.9 = 640.5$$

CV: Hg condenser - H_2O boiler: 1st law: $m_{Hg}(h_d - h_a) = m_{H2O}(h_3 - h_2)$

$$m_{Hg}/m_{H2O} = \frac{2796.9 - 196.5}{270.48 - 42.21} = \mathbf{11.392}$$

$$q_{H\ TOTAL} = (m_{Hg}/m_{H2O})(h_c - h_b) + (h_4 - h_3) \text{ (for 1 kg } H_2O)$$

$$= 11.392 \times 321.83 + 640.5 = 4306.8 \text{ kJ}$$

All q_L is from the H_2O condenser:

$$q_L = h_5 - h_1 = 2220.8 - 191.8 = 2029.0 \text{ kJ}$$

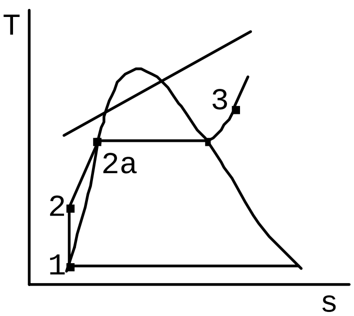
$$w_{NET} = q_H - q_L = 4306.8 - 2029.0 = 2277.8 \text{ kJ}$$

$$\eta_{TH} = w_{NET}/q_H = 2277.8/4306.8 = \mathbf{0.529}$$

11.141

A Rankine steam power plant should operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and the boiler exit temperature should be 500°C. The available high-temperature source is the exhaust of 175 kg/s air at 600°C from a gas turbine. If the boiler operates as a counterflowing heat exchanger where the temperature difference at the pinch point is 20°C, find the maximum water mass flow rate possible and the air exit temperature.

Solution:



C.V. Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1)$

$= 0.00101(3000 - 10) = 3.02$ kJ/kg

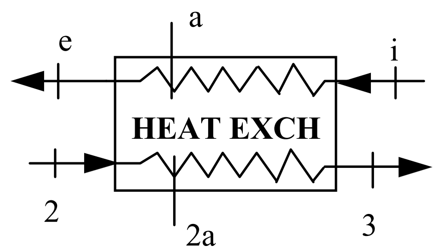
$h_2 = h_1 + w_P = 191.83 + 3.02 = 194.85$ kJ/kg

Heat exchanger water states

State 2a: $T_{2a} = T_{SAT} = 233.9$ °C

$h_{2a} = 1008.42$ kJ/kg

State 3: $h_3 = 3456.5$ kJ/kg



Heat exchanger air states

inlet: $h_{air,in} = 903.16$ kJ/kg

State 2a: $h_{air}(T_{2a} + 20) = 531.28$ kJ/kg

Air temperature should be 253.9°C at the point where the water is at state 2a.

C.V. Section 2a-3, i-a

$$\dot{m}_{H_2O}(h_3 - h_{2a}) = \dot{m}_{air}(h_i - h_a)$$

$$\dot{m}_{H_2O} = 175 \frac{903.16 - 531.28}{3456.5 - 1008.42} = \mathbf{26.584\ kg/s}$$

Take C.V. Total: $\dot{m}_{H_2O}(h_3 - h_2) = \dot{m}_{air}(h_i - h_e)$

$\Rightarrow h_e = h_i - \dot{m}_{H_2O}(h_3 - h_2)/\dot{m}_{air}$

$= 903.6 - 26.584(3456.5 - 194.85)/175 = 408.13$ kJ/kg

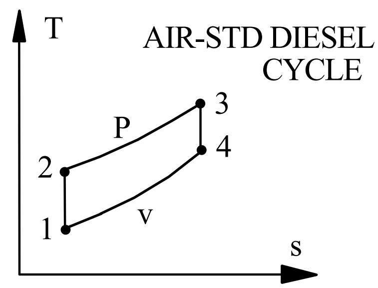
$\Rightarrow T_e = 406.7$ K = **133.6 °C**, $T_e > T_2 = 46.5$ °C OK.

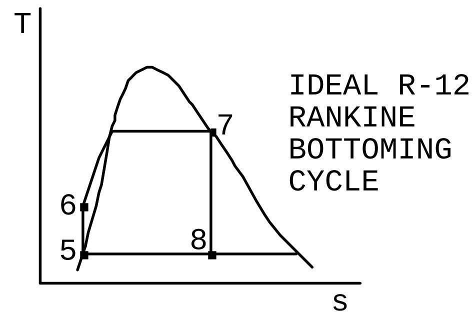
11.142

A simple Rankine cycle with R-22 as the working fluid is to be used as a bottoming cycle for an electrical generating facility driven by the exhaust gas from a Diesel engine as the high temperature energy source in the R-22 boiler. Diesel inlet conditions are 100 kPa, 20°C, the compression ratio is 20, and the maximum temperature in the cycle is 2800°C. Saturated vapor R-22 leaves the bottoming cycle boiler at 110°C, and the condenser temperature is 30°C. The power output of the Diesel engine is 1 MW. Assuming ideal cycles throughout, determine

a. The flow rate required in the diesel engine.

b. The power output of the bottoming cycle, assuming that the diesel exhaust is cooled to 200°C in the R-22 boiler.





Diesel cycle information given means:

Inlet state: $P_1 = 100$ kPa, $T_1 = 20\ ^oC$,

Compression ratio: $v_1/v_2 = 20$,

High temperature: $T_3 = 2800^oC$, Power output: $\dot{W}_{DIESEL} = 1.0$ MW

Rankine cycle information given means:

Boiler exit state: $T_7 = 110\ ^oC$, $x_7 = 1.0$

Condenser temperature: $T_5 = T_8 = 30^oC$

a) Consider the Diesel cycle

$$T_2 = T_1(v_1/v_2)^{k-1} = 293.2(20)^{0.4} = 971.8\ K$$

$$P_2 = P_1(v_1/v_2)^k = 100(20)^{1.4} = 6629\ kPa$$

$$q_H = C_{P0}(T_3 - T_2) = 1.004(3073.2 - 971.8) = 2109.8\ kJ/kg$$

$$v_1 = \frac{0.287 \times 293.2}{100} = 0.8415, \quad v_2 = \frac{0.8415}{20} = 0.04208$$

$$v_3 = v_2(T_3/T_2) = 0.04208(3073.2/971.8) = 0.13307$$

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = 3073.2\left(\frac{0.133\ 07}{0.8415}\right)^{0.4} = 1469.6\ K$$

$q_L = 0.717(293.2 - 1469.6) = -843.5$ kJ/kg

$w_{NET} = 2109.8 - 843.5 = 1266.3$ kJ/kg

$\dot{m}_{AIR} = \dot{W}_{NET}/w_{NET} = 1000/1266.3 =$ **0.79 kg/s**

b) Consider the Rankine cycle

$s_8 = s_7 = 0.60758 = 0.2399 + x_8 \times 0.4454, \quad x_8 = 0.8255$

$h_8 = 64.59 + 0.8255 \times 135.03 = 176.1$ kJ/kg

$w_T = h_7 - h_8 = 198.0 - 176.1 = 21.9$ kJ/kg

$-w_P = v_5(P_6 - P_5) = 0.000774(3978.5 - 744.9) = 2.50$

$h_6 = h_5 - w_P = 64.6 + 2.5 = 67.1$ kJ/kg

$q_H = h_7 - h_6 = 198.0 - 67.1 = 130.9$ kJ/kg

Connecting the two cycles.

$\dot{Q}_H$ available from Diesel exhaust cooled to 200 ^{o}C:

$\dot{Q}_H = 0.79 \times 0.717(1469.6 - 473.2) = 564$ kW

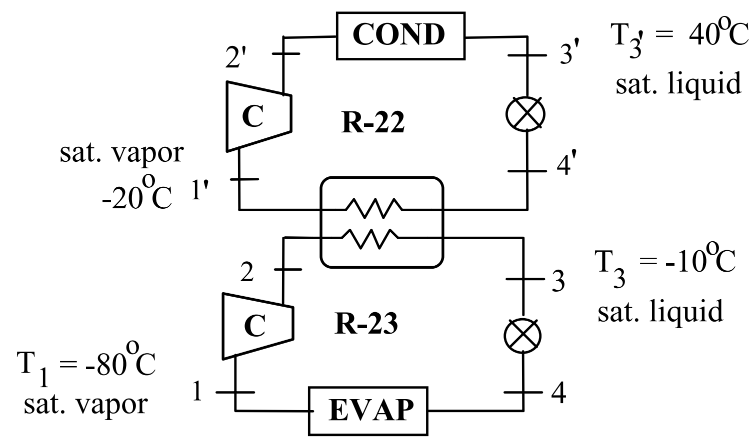
$\Rightarrow \dot{m}_{R\text{-}12} = \dot{Q}_H/q_H = 564/130.9 = 4.309$ kg/s

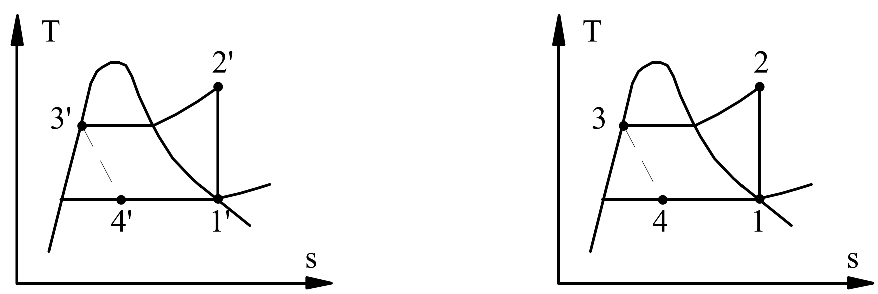
$\dot{W}_{R\text{-}12} = 4.309(21.9 - 2.5) =$ **83.6 kW**

Comment: The heat exchange process between the two cycles is not realistic. The exhaust must be expanded down to 100 kPa from state 4 and then flow at constant P through a heat exchanger.

11.143

A cascade system is composed of two ideal refrigeration cycles, as shown in Fig. 11.41. The high-temperature cycle uses R-22. Saturated liquid leaves the condenser at 40°C, and saturated vapor leaves the heat exchanger at –20°C. The low-temperature cycle uses a different refrigerant, R-23. Saturated vapor leaves the evaporator at –80°C, h = 330 kJ/kg, and saturated liquid leaves the heat exchanger at –10°C, h = 185 kJ/kg. R-23 out of the compressor has h = 405 kJ/kg. Calculate the ratio of the mass flow rates through the two cycles and the coefficient of performance of the system.





	T,°C	P	h	s		T,°C	P	h	s
1′	-20	0.245	242.1	0.9593	1	-80	0.12	330	1.76
2′	71	1.534	289.0	0.9593	2	50	1.90	405	1.76
3′	40	1.534	94.3		3	-10	1.90	185	
4′	-20		94.3		4	-80	0.12	185	

$$\dot{m}/\dot{m}' = \frac{h_1' - h_4'}{h_2 - h_3} = \frac{242.1 - 94.3}{405 - 185} = \mathbf{0.672}$$

$$q_L = h_1 - h_4 = 330 - 185 = 145 \text{ kJ/kg}$$

$$-\dot{W}_{TOT}/\dot{m} = (h_2 - h_1) + (\dot{m}'/\dot{m})(h_2' - h_1')$$

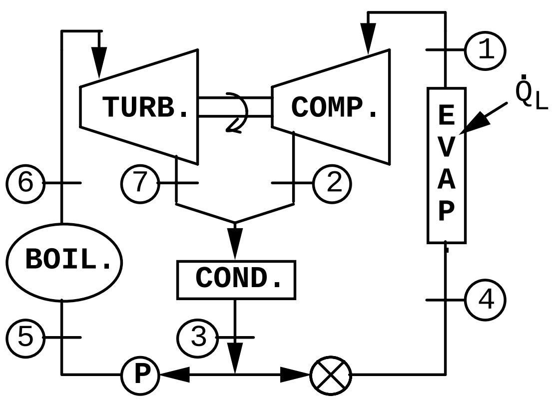
$$= (405 - 330) + (1/0.672)(289 - 242.1) = 144.8 \text{ kJ/kg}$$

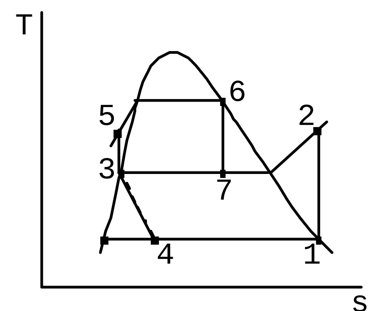
$$\beta = Q_L/-\dot{W}_{TOT} = 145/144.8 = \mathbf{1.0}$$

11.144

Consider an ideal dual-loop heat-powered refrigeration cycle using R-12 as the working fluid, as shown in Fig. P11.87. Saturated vapor at 105°C leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at −15°C leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the condenser at 45°C is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to that through the refrigeration loop. Find also the performance of the cycle, in terms of the ratio $\dot{Q}_L / \dot{Q}_H$.

Solution:





T_1 = -15 °C sat. vap.

Table B.3.1 $T_6 = 105^oC$ sat. vapor => $P_5 = P_6 = 3.6509$ MPa

Table B.3.1 $T_3 = 45^oC$ sat. liquid => $P_2 = P_3 = P_7 = 1.0843$ MPa

$h_1 = 180.97; \quad h_3 = h_4 = 79.71; \quad h_6 = 206.57$

C.V. Turbine

$s_7 = s_6 = 0.6325 = 0.2877 + x_7 \times 0.3934; \qquad x_7 = 0.8765$

$h_7 = 79.71 + 0.8765 \times 125.16 = 189.41$

C.V. Compressor (computer tables are used for this due to value of P)

$s_2 = s_1 = 0.7051,\ P_2$ => $T_2 = 54.7^oC, \quad h_2 = 212.6$ kJ/kg

CV: turbine + compressor

Continuity Eq.: $\dot{m}_1 = \dot{m}_2, \quad \dot{m}_6 = \dot{m}_7$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_7 h_7$

$\dot{m}_6/\dot{m}_1 = (212.6 - 180.97)/(206.57 - 189.41) = \mathbf{1.843}$

CV: pump

$$w_P = v_3(P_5 - P_3) = 0.000811(3651 - 1084) = 2.082 \text{ kJ/kg}$$

$$h_5 = h_3 + w_P = 81.79 \text{ kJ/kg}$$

CV: evaporator $\Rightarrow \quad \dot{Q}_L = \dot{m}_1(h_1 - h_4)$

CV: boiler $\Rightarrow \quad \dot{Q}_H = \dot{m}_6(h_6 - h_5)$

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{m}_1(h_1 - h_4)}{\dot{m}_6(h_6 - h_5)} = \frac{180.97 - 79.71}{1.843(206.57 - 81.79)} = \mathbf{0.44}$$

11.145

For a cryogenic experiment heat should be removed from a space at 75 K to a reservoir at 180 K. A heat pump is designed to use nitrogen and methane in a cascade arrangement (see Fig. 11.41), where the high temperature of the nitrogen condensation is at 10 K higher than the low-temperature evaporation of the methane. The two other phase changes take place at the listed reservoir temperatures. Find the saturation temperatures in the heat exchanger between the two cycles that gives the best coefficient of performance for the overall system.

The nitrogen cycle is the bottom cycle and the methane cycle is the top cycle. Both std. refrigeration cycles.

$T_{Hm} = 180 \text{ K} = T_{3m}$, $T_{LN} = 75 \text{ K} = T_{4N} = T_{1N}$

$T_{Lm} = T_{4m} = T_{1m} = T_{3N} - 10$, Trial and error on T_{3N} or T_{Lm}.

For each cycle we have,

$-w_C = h_2 - h_1$, $s_2 = s_1$, $-q_H = h_2 - h_3$, $q_L = h_1 - h_4 = h_1 - h_3$

Nitrogen: $T_4 = T_1 = 75$ K $\Rightarrow$ $h_1 = 74.867$ kJ/kg, $s_1 = 5.4609$ kJ/kg K

N_2	T_3	h_3	P_2	h_2	$-w_c$	$-q_H$	q_L
a)	120	-17.605	2.5125	202.96	128.1	220.57	92.47
b)	115	-34.308	1.9388	188.35	113.5	222.66	109.18
c)	110	-48.446	1.4672	173.88	99.0	222.33	123.31

Methane: $T_3 = 180$ K $\Rightarrow$ $h_3 = -0.5$ kJ/kg, $P_2 = 3.28655$ MPa

CH_4	T_4	h_1	s_1	h_2	$-w_c$	$-q_H$	q_L
a)	110	221	9.548	540.3	319.3	540.8	221.5
b)	105	212.2	9.691	581.1	368.9	581.6	212.7
c)	100	202.9	9.851	629.7	426.8	630.2	203.4

The heat exchanger that connects the cycles transfers a Q

$$\dot{Q}_{Hn} = q_{Hn}\,\dot{m}_n = \dot{Q}_{Lm} = q_{Lm}\,\dot{m}_m \;\;=>\;\; \dot{m}_m/\dot{m}_n = q_{Hn}/q_{Lm}$$

The overall unit then has

$$\dot{Q}_{L\ 75\ K} = \dot{m}_n\, q_{Ln}\ ; \quad \dot{W}_{tot\ in} = -\,(\dot{m}_n w_{cn} + \dot{m}_m w_{cm})$$

$$\beta = \dot{Q}_{L\ 75\ K}/\dot{W}_{tot\ in} = q_{Ln}/[-w_{cn} - (\dot{m}_m/\dot{m}_n) w_{cm}]$$

Case	$\dot{m}_m/\dot{m}_n$	$w_{cn}+(\dot{m}_m/\dot{m}_n)w_{cm}$	β
a)	0.996	446.06	0.207
b)	1.047	499.65	0.219
c)	1.093	565.49	0.218

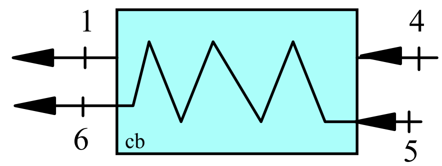
A maximum coeff. of performance is between case b) and c).

Availabilty or Exergy Concepts

11.146

Find the flows and fluxes of exergy in the condenser of Problem 11.32. Use those to determine the second law efficiency.

For this case we select $T_o = 12°C = 285$ K, the ocean water temperature.



The states properties from Tables B.1.1 and B.1.3

1: 45°C, x = 0: $h_1 = 188.42$ kJ/kg,

3: 3.0 MPa, 600°C: $s_3 = 7.5084$ kJ/kg K

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 7.5084 = 0.6386 + x_4\,(7.5261) \quad => \quad x_4 = 0.9128$$

$$=> \; h_4 = 188.42 + 0.9128\,(2394.77) = 2374.4 \text{ kJ/kg}$$

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$$\dot{Q}_L = \dot{m} q_L = 25 \times 2186 = 54.65 \text{ MW} = \dot{m}_{ocean}\, C_p\, \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L / C_p\, \Delta T = 54\,650 / (4.18 \times 3) = 4358 \text{ kg/s}$$

The net drop in exergy of the water is

$$\dot{\Phi}_{water} = \dot{m}_{water}\,[h_4 - h_1 - T_o(s_4 - s_1)]$$

$$= 25\,[\,2374.4 - 188.4 - 285\,(7.5084 - 0.6386)]$$

$$= 54\,650 - 48\,947 = \mathbf{5703\ kW}$$

The net gain in exergy of the ocean water is

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}[h_6 - h_5 - T_o(s_6 - s_5)]$$

$$= \dot{m}_{ocean}[C_p(T_6 - T_5) - T_o C_p \ln\left(\frac{T_6}{T_5}\right)\,]$$

$$= 4358\,[\,4.18(15 - 12) - 285 \times 4.18 \ln \frac{273 + 15}{273 + 12}\,]$$

$$= 54\,650 - 54\,364 = \mathbf{286\ kW}$$

The second law efficiency is

$$\eta_{II} = \dot{\Phi}_{ocean} / \dot{\Phi}_{water} = \frac{286}{5703} = \mathbf{0.05}$$

In reality all the exergy in the ocean water is destroyed as the 15°C water mixes with the ocean water at 12°C after it flows back out into the ocean and the efficiency does not have any significance. Notice the small rate of exergy relative to the large rates of energy being transferred.

11.147

Find the availability of the water at all four states in the Rankine cycle described in Problem 11.33. Assume that the high-temperature source is 500°C and the low-temperature reservoir is at 25°C. Determine the flow of availability in or out of the reservoirs per kilogram of steam flowing in the cycle. What is the overall cycle second law efficiency?

Solution:

Reference State: 100 kPa, 25°C, $s_o = 0.3674$ kJ/kg K, $h_o = 104.89$ kJ/kg

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$

$$= 191.83 - 104.89 - 298.15(0.6493 - 0.3674) = 2.89 \text{ kJ/kg}$$

$$\psi_2 = 195.35 - 104.89 - 298.15(0.6493 - 0.3674) = \psi_1 + 3.525 = 6.42 \text{ kJ/kg}$$

$$\psi_3 = 3222.3 - 104.89 - 298.15(6.8405 - 0.3674) = 1187.5 \text{ kJ/kg}$$

$$\psi_4 = \psi_3 - w_{T,s} = 131.96 \text{ kJ/kg}$$

$$\Delta\psi_H = (1 - T_o/T_H)q_H = 0.6144 \times 3027 = 1859.7 \text{ kJ/kg}$$

$$\Delta\psi_L = (1 - T_o/T_o)q_C = 0 \text{ kJ/kg}$$

$$\eta_{II} = w_{NET}/\Delta\psi_H = (1055.5 - 3.53)/1859.7 = \mathbf{0.5657}$$

Notice— $T_H > T_3$, $T_L < T_4 = T_1$ so cycle is externally irreversible. Both q_H and q_C over finite ΔT.

11.148

Find the flows of exergy into and out of the feedwater heater in Problem 11.43.

State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m^3/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5$ => $x_6 = (s_6 – s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708 \text{ kJ/kg}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.96 \text{ kJ/kg}$$

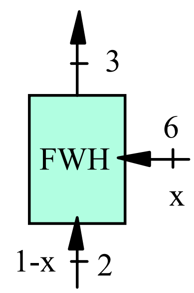
C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x)\, h_2 + x\, h_6 = 1\, h_3$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = 0.1046$$

$$\dot{m}_{extr} = x\, \dot{m}_{tot} = 0.1046 \times 5 = 0.523 \text{ kg/s}$$

$$\dot{m}_2 = (1\text{-}x)\, \dot{m}_{tot} = (1 – 0.1046)\, 5 = 4.477 \text{ kg/s}$$



Reference State: 100 kPa, 20°C, $s_o = 6.2826$ kJ/kg K, $h_o = 1516.1$ kJ/kg

$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$

$$= 299.96 - 1516.1 - 293.15(1.121 - 6.2826) = 296.21 \text{ kJ/kg}$$

$$\psi_6 = 1461.53 - 1516.1 - 293.15(4.7306 - 6.2826) = 400.17 \text{ kJ/kg}$$

$$\psi_3 = 421.48 - 1516.1 - 293.15(1.5121 - 6.2826) = 303.14 \text{ kJ/kg}$$

The rate of exergy flow is then

$$\dot{\Phi}_2 = \dot{m}_2\psi_2 = 4.477 \times 296.21 = \mathbf{1326\ kW}$$

$$\dot{\Phi}_6 = \dot{m}_6\psi_6 = 0.523 \times 400.17 = \mathbf{209.3\ kW}$$

$$\dot{\Phi}_3 = \dot{m}_3\psi_3 = 5.0 \times 303.14 = \mathbf{1516\ kW}$$

The mixing is destroying 1326 + 209 – 1516 = 19 kW of exergy

11.149

Find the availability of the water at all the states in the steam power plant described in Problem 11.57. Assume the heat source in the boiler is at 600°C and the low-temperature reservoir is at 25°C. Give the second law efficiency of all the components.

From solution to 11.21 and 11.57:

States	0	1 sat liq.	2a	3	4a (x = 0.7913)
h [kJ/kg]	104.89	191.81	195.58	2804.14	2085.24
s [kJ/kg K]	0.3674	0.6492	0.6529	6.1869	6.5847

The entropy for state 2a was done using the compressed liquid entry at 2MPa at the given h. You could interpolate in the compressed liquid tables to get at 3 MPa or use the computer tables to be more accurate.

Definition of flow exergy: $\psi = h - h_o - T_o(s - s_o)$

$\psi_1 = 191.81 - 104.89 - 298.15(0.6492 - 0.3674) = 2.90$ kJ/kg

$\psi_{2a} = 195.58 - 104.89 - 298.15(0.6529 - 0.3674) = 5.57$ kJ/kg

$\psi_3 = 2804.14 - 104.89 - 298.15(6.1869 - 0.3674) = 964.17$ kJ/kg

$\psi_{4a} = 2085.24 - 104.89 - 298.15(6.5847 - 0.3674) = 126.66$ kJ/kg

$\eta_{II\ Pump} = (\psi_{2a} - \psi_1) / w_{p\ ac} = (5.57 - 2.9) / 3.775 = \mathbf{0.707}$

$\eta_{II\ Boiler} = (\psi_3 - \psi_{2a}) / [(1- T_o/T_H)\ q_H]$

$= (964.17 - 3.18) / [0.658 \times 2608.6] = \mathbf{0.56}$

$\eta_{II\ Turbine} = w_{T\ ac} / (\psi_3 - \psi_{4a}) = 718.9 / (964.17 - 126.66) = \mathbf{0.858}$

$\eta_{II\ Cond} = \Delta\psi_{amb} / (\psi_{4a} - \psi_1) = \mathbf{0}$

Remark: Due to the interpolation the efficiency for the pump is not quite correct. It should have a second law efficiency greater than the isentropic efficiency.

11.150

Consider the Brayton cycle in Problem 11.72. Find all the flows and fluxes of exergy and find the overall cycle second-law efficiency. Assume the heat transfers are internally reversible processes, and we then neglect any external irreversibility.

Solution:

Efficiency is from Eq.11.8

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = \mathbf{0.547}$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\ 000}{0.547} = 25\ 594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\ 594 \text{ kW}/ 960 \text{ kJ/kg} = \mathbf{26.66\ kg/s}$$

Temperature after compression is

$$T_2 = T_1\ r_p^{(k-1)/k} = 290 \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 + \frac{960}{1.004} = \mathbf{1596.5\ K}$$

For the exit flow I need the exhaust temperature

$$T_4 = T_3\ r_p^{-\frac{k-1}{k}} = 1596.5 \times 16^{-0.2857} = 723 \text{ K}$$

$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H$ since the low T exergy flow out is lost

The high T exergy input from combustion is

$$\dot{\Phi}_H = \dot{m}(\psi_3 - \psi_2) = \dot{m}[h_3 - h_2 - T(s_3 - s_2)]$$

$$= 26.66\ [960 - 298 \times 1.004 \ln (\frac{1596.5}{640.35})] = \mathbf{18\ 303\ kW}$$

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H = 14\ 000 / 18\ 303 = \mathbf{0.765}$$

$$\dot{\Phi}_{flow\ in} = \dot{m}(\psi_4 - \psi_o) = \dot{m}[h_4 - h_o - T(s_4 - s_o)]$$

$$= 26.66\ [\ 1.004(17 - 25) - 298 \times 1.004 \ln (\frac{290}{298})\ ] = \mathbf{2.0\ kW}$$

$$\dot{\Phi}_{flow\ out} = \dot{m}(\psi_1 - \psi_o) = \dot{m}[h_1 - h_o - T(s_1 - s_o)]$$

$$= 26.66\ [\ 1.004(723 - 298) - 298 \times 1.004 \ln (\frac{723}{298})\ ] = \mathbf{4302\ kW}$$

11.151

For Problem 11.141, determine the change of availability of the water flow and that of the air flow. Use these to determine a second law efficiency for the boiler heat exchanger.

From solution to 11.141:

$\dot{m}_{H2O} = 26.584$ kg/s, $h_2 = 194.85$ kJ/kg, $s_2 = 0.6587$ kJ/kg K

$h_3 = 3456.5$ kJ/kg, $s_3 = 7.2338$, $s^\circ_{Ti} = 7.9820$, $s^\circ_{Te} = 7.1762$ kJ/kg K

$h_i = 903.16$ kJ/kg, $h_e = 408.13$ kJ/kg

$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = 1301.28$ kJ/kg

$\psi_i - \psi_e = h_i - h_e - T_0(s^\circ_{Ti} - s^\circ_{Te}) = 254.78$ kJ/kg

$$\eta_{II} = \frac{(\psi_3 - \psi_2)\dot{m}_{H2O}}{(\psi_i - \psi_e)\dot{m}_{air}} = \frac{1301.28 \times 26.584}{254.78 \times 175} = \mathbf{0.776}$$

Review Problems

11.152

A simple steam power plant is said to have the four states as listed: 1: (20°C, 100 kPa), 2: (25°C, 1 MPa), 3: (1000°C, 1 MPa), 4: (250°C, 100 kPa) with an energy source at 1100°C and it rejects energy to a 0°C ambient. Is this cycle possible? Are any of the devices impossible?

Solution:

The cycle should be like Figure 11.3 for an ideal or Fig.11.9 for an actual pump and turbine in the cycle. We look the properties up in Table B.1:

State 1: $h_1 = 83.94$, $s_1 = 0.2966$ State 2: $h_2 = 104.87$, $s_2 = 0.3673$

State 3: $h_3 = 4637.6$, $s_3 = 8.9119$ State 4: $h_4 = 2974.3$, $s_4 = 8.0332$

We may check the overall cycle performance

Boiler: $q_H = h_3 - h_2 = 4637.6 - 104.87 = 4532.7$ kJ/kg

Condenser: $q_L = h_4 - h_1 = 2974.3 - 83.94 = 2890.4$ kJ/kg

$$\eta_{cycle} = q_{net} / q_H = (q_H - q_L) / q_H = 1642.3 / 4532.7 = 0.362$$

$$\eta_{carnot} = 1 - T_L / T_H = 1 - \frac{273.15}{273.15 + 1100} = \mathbf{0.80} > \eta_{cycle} \quad \textbf{OK}$$

Check the second law for the individual devices:

C.V. Boiler plus wall to reservoir

$$s_{gen} = s_3 - s_2 - \frac{q_H}{T_{res}} = 8.9119 - 0.3673 - \frac{4532.7}{1373} = 5.24 \text{ kJ/kg K} > 0 \quad \textbf{OK}$$

C.V. Condenser plus wall to reservoir

$$s_{gen} = s_1 - s_4 + \frac{q_L}{T_{res}} = 0.2966 - 8.0332 + \frac{2890.4}{273} = 2.845 \text{ kJ/kg K} > 0 \quad \textbf{OK}$$

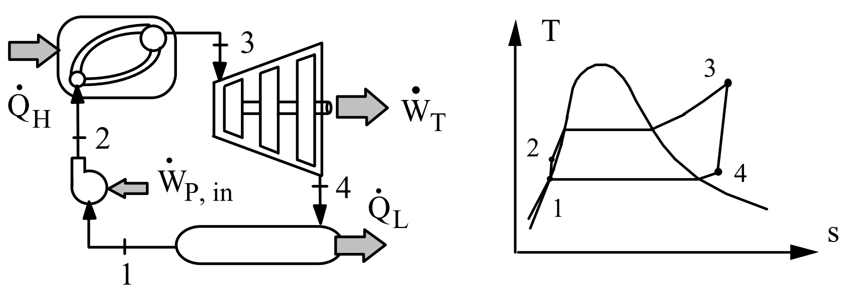
C.V. Pump: $w_p = h_2 - h_1 = 20.93$ kJ/kg ;

$$s_{gen} = s_2 - s_1 = 0.3673 - 0.2966 = 0.0707 \text{ kJ/kg K} > 0 \quad \textbf{OK}$$

C.V. Turbine: $w_T = h_3 - h_4 = 4637.6 - 2974.3 = 1663.3$ kJ/kg

$$s_{gen} = s_4 - s_3 = 8.0332 - 8.9119 = - 0.8787 \text{ kJ/kg K}$$

$$s_{gen} < 0 \qquad \textbf{NOT POSSIBLE}$$



11.153

Do Problem 11.31 with R-134a as the working fluid in the Rankine cycle.

Consider the ammonia Rankine-cycle power plant shown in Fig. P11.31, a plant that was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

a. Determine the turbine power output and the pump power input for the cycle.
b. Determine the mass flow rate of water through each heat exchanger.
c. What is the thermal efficiency of this power plant?

Solution:

a) Turbine

$s_2 = s_1 = 1.7183 = 1.0485 + x_2 \times 0.6733 \quad => \quad x_2 = 0.9948$

$h_2 = 213.58 + 0.9948 \times 190.65 = 403.24$ kJ/kg

$w_T = h_1 - h_2 = 409.84 - 403.24 = 6.6$ kJ/kg

$\dot{W}_T = \dot{m}w_T = 6600$ kW

Pump: $w_P \approx v_3(P_4 - P_3) = 0.000794(572.8 - 415.8) = 0.125$ kJ/kg

$w_P = w_P / \eta_S = 0.125 \quad => \quad \dot{W}_P = \dot{m}w_P = \mathbf{125\ kW}$

b) Consider the condenser heat transfer to the low T water

$\dot{Q}_{\text{to low T H2O}} = 1000(403.24 - 213.58) = 189\,660$ kW

$\dot{m}_{\text{low T H2O}} = \dfrac{189660}{29.38 - 20.98} = \mathbf{22\,579\ kg/s}$

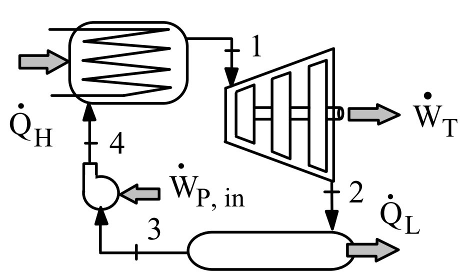
$h_4 = h_3 - w_P = 213.58 + 0.125 = 213.71$ kJ/kg

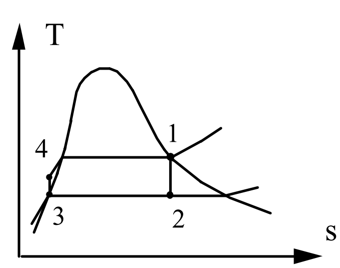
Now consider the boiler heat transfer from the high T water

$\dot{Q}_{\text{from high T H2O}} = 1000(409.84 - 213.71) = 196\,130$ kW

$\dot{m}_{\text{high T H2O}} = \dfrac{196130}{104.87 - 96.50} = \mathbf{23\,432\ kg/s}$

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{6600 - 125}{196130} = \mathbf{0.033}$

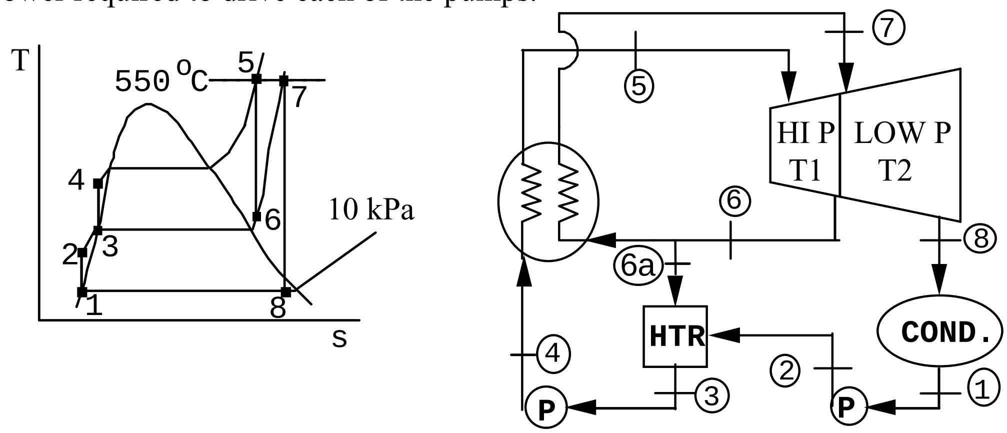




11.154

An ideal steam power plant is designed to operate on the combined reheat and regenerative cycle and to produce a net power output of 10 MW. Steam enters the high-pressure turbine at 8 MPa, 550°C, and is expanded to 0.6 MPa, at which pressure some of the steam is fed to an open feedwater heater, and the remainder is reheated to 550°C. The reheated steam is then expanded in the low-pressure turbine to 10 kPa. Determine the steam flow rate to the high-pressure turbine and the power required to drive each of the pumps.

a)



b) $-w_{P12} = 0.00101(600 - 10) = 0.6$ kJ/kg

$h_2 = h_1 - w_{P12} = 191.8 + 0.6 = 192.4$ kJ/kg

$-w_{P34} = 0.00101(8000 - 600) = 8.1$ kJ/kg

$h_4 = h_3 - w_{P34} = 670.6 + 8.1 = 678.7$; $h_5 = 3521.0$ kJ/kg,

$s_6 = s_5 = 6.8778 \Rightarrow T_6 = 182.32\ ^oC \quad h_6 = 2810.0$ kJ/kg,

$h_7 = 3591.9, \quad s_8 = s_7 = 8.1348 = 0.6493 + x_8 \times 7.5009 \Rightarrow x_8 = 0.9979$

$h_8 = 191.83 + 0.9979 \times 2392.8 = 2579.7$ kJ/kg

CV: heater

Cont: $m_{6a} + m_2 = m_3 = 1$ kg, 1st law: $m_{6a}h_6 + m_2h_2 = m_3h_3$

$$m_{6a} = \frac{670.6 - 192.4}{2810.0 - 192.4} = 0.1827, \quad m_2 = m_7 = 1 - m_{6a} = 0.8173$$

CV: turbine

$$w_T = (h_5 - h_6) + (1 - m_{6a})(h_7 - h_8)$$
$$= 3521 - 2810 + 0.8173(3591.9 - 2579.7) = 1538.2 \text{ kJ/kg}$$

CV: pumps

$$w_P = m_2w_{P12} + m_4w_{P34} = 0.8214\times(-0.6) + 1\times(-8.1) = -8.6 \text{ kJ/kg}$$

$$w_N = 1538.2 - 8.6 = 1529.6 \text{ kJ/kg } (m_5)$$

$$\dot{m}_5 = \dot{W}_N/w_N = 10000/1529.6 = \mathbf{6.53\ kg/s}$$

11.155

Steam enters the turbine of a power plant at 5 MPa and 400°C, and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20 000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant and how does this compare with a Carnot cycle.

Solution: $\dot{W}_T = 20\ 000$ kW and $\eta_{Ts} = 85$ %

State 3: $h_3 = 3195.6$ kJ/kg , $s_3 = 6.6458$ kJ/kgK

State 1: $P_1 = P_4 = 10$ kPa , sat liq , $x_1 = 0$

$T_1 = 45.8^oC$, $h_1 = h_f = 191.8$ kJ/kg , $v_1 = v_f = 0.00101$ m^3/kg

C.V Turbine : 1st Law: $q_T + h_3 = h_4 + w_T$; $q_T = 0$

$w_T = h_3 - h_4$, Assume Turbine is isentropic

$s_{4s} = s_3 = 6.6458$ kJ/kgK , $s_{4s} = s_f + x_{4s}\, s_{fg}$, solve for $x_{4s} = 0.7994$

$h_{4s} = h_f + x_{4s}h_{fg} = 1091.0$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 1091$ kJ/kg , $w_T = \eta_{Ts}w_{Ts} = 927.3$ kJ/kg

$\dot{m} = \frac{\dot{W}_T}{w_T} =$ **21.568 kg/s** , $h_4 = h_3 - w_T = 2268.3$ kJ/kg

C.V. Condenser: 1st Law : $h_4 = h_1 + q_c + w_c$; $w_c = 0$

$q_c = h_4 - h_1 = 2076.5$ kJ/kg , $\dot{Q}_c = \dot{m}\, q_c =$ **44786 kW**

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 5.04$ kJ/kg

1st Law : $h_2 = h_1 + w_p = 196.8$ kJ/kg

C.V Boiler : 1st Law : $q_B + h_2 = h_3 + w_B$; $w_B = 0$

$q_B = h_3 - h_2 = 2998.8$ kJ/kg

$w_{net} = w_T - w_P = 922.3$ kJ/kg

$\eta_{th} = w_{net} / q_B =$ **0.307**

Carnot cycle : $T_H = T_3 = 400^oC$, $T_L = T_1 = 45.8^oC$

$\eta_{th} = \frac{T_H - T_L}{T_H} =$ **0.526**

11.156

Consider an ideal combined reheat and regenerative cycle in which steam enters the high-pressure turbine at 3.0 MPa, 400°C, and is extracted to an open feedwater heater at 0.8 MPa with exit as saturated liquid. The remainder of the steam is reheated to 400°C at this pressure, 0.8 MPa, and is fed to the low-pressure turbine. The condenser pressure is 10 kPa. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

In this setup the flow is separated into fractions x and 1-x after coming out of T1. The two flows are recombined in the FWH.

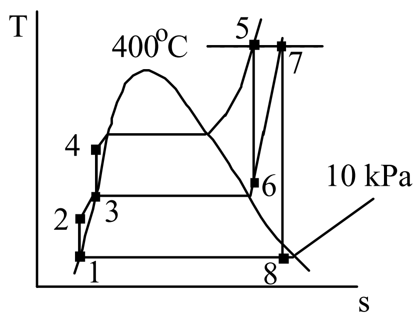
C.V. T1 $\quad s_6 = s_5 = 6.9211$ kJ/kg K $\Rightarrow$ $h_6 = 2891.6$ kJ/kg

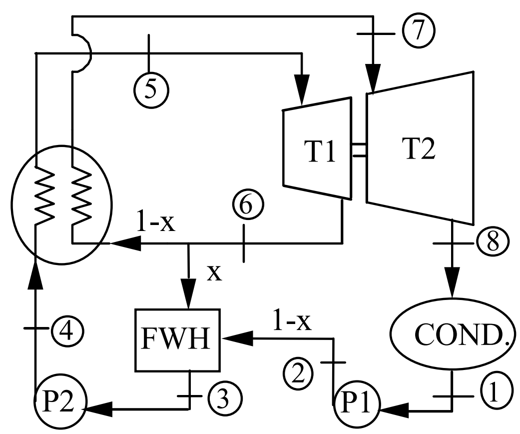
$$w_{T1} = h_5 - h_6 = 3230.82 - 2891.6 = 339.22 \text{ kJ/kg}$$

C.V. Pump 1:

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$\Rightarrow \; h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$





C.V. FWH, $\quad h_3 = h_f = 721.1$

Energy equation per unit mass flow exit at 3:

$$x\, h_6 + (1 - x)\, h_2 = h_3 \quad \Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V. Pump 2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$\Rightarrow \; h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

C.V. Boiler/steam generator including reheater.

Total flow from 4 to 5 only fraction 1-x from 6 to 7

$$q_H = h_5 - h_4 + (1 - x)(h_7 - h_6) = 2507.3 + 301.95 = 2809.3 \text{ kJ/kg}$$

C.V. Turbine 2

$s_8 = s_7 = 7.5715$ kJ/kg K => $x_8 = (7.5715 - 0.6492)/7.501 = 0.92285$

$h_8 = h_f + x_8\, h_{fg} = 191.81 + 0.92285 \times 2392.82 = 2400.0$ kJ/kg

$w_{T2} = h_7 - h_8 = 3267.07 - 2400.02 = 867.05$ kJ/kg

Sum the work terms to get net work. Total flow through T1 only fraction 1-x through T2 and P1 and after FWH we have the total flow through P2.

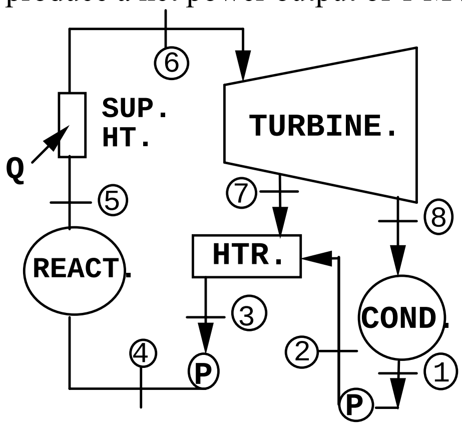
$w_{net} = w_{T1} + (1 - x)\, w_{T2} - (1 - x)\, w_{P1} - w_{P2}$

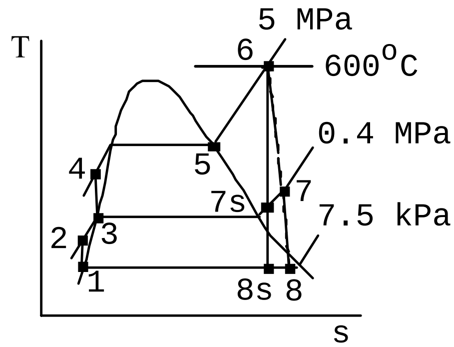
$= 339.2 + 697.3 - 0.64 – 2.45 =$ **1033.41 kJ/kg**

$\eta_{cycle} = w_{net} / q_H = 1033.41 / 2809.3 =$ **0.368**

11.157

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this heat exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 0.4 MPa. The isentropic turbine efficiency is 87%, and the condenser pressure is 7.5 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1 MW.





$\dot{W}_{NET} = 1$ MW , $\eta_{ST} = 0.87$

$-w_{P12} = 0.001008(400 - 7.5) = 0.4$ kJ/kg

$h_2 = h_1 - w_{P12} = 168.8 + 0.4 = 169.2$ kJ/kg

$-w_{P34} = 0.001084(5000 - 400) = 5.0$ kJ/kg

$h_4 = h_3 - w_{P34} = 604.7 + 5.0 = 609.7$ kJ/kg

$s_{7S} = s_6 = 7.2589$, P_7=0.4 MPa => $T_{7S} = 221.2$ °C, $h_{7S} = 2904.5$ kJ/kg

$h_6 - h_7 = \eta_{ST}(h_6 - h_{7S}) \Rightarrow 3666.5 - h_7 = 0.87(3666.5$

$- 2904.5) = 662.9 \Rightarrow h_7 = 3003.6$ kJ/kg

$s_{8S} = s_6 = 7.2589 = 0.5764 + x_{8S} \times 7.6750$; $x_{8S} = 0.8707$

$h_{8S} = 168.8 + 0.8707 \times 2406.0 = 2263.7$ kJ/kg

$h_6 - h_8 = \eta_{ST}(h_6 - h_{8S}) \Rightarrow 3666.5 - h_8 = 0.87(3666.5$

$- 2263.7) = 1220.4 \Rightarrow h_8 = 2446.1$ kJ/kg

CV: heater

cont: $m_2 + m_7 = m_3 = 1.0$ kg, Energy Eq.: $m_2h_2 + m_7h_7 = m_3h_3$

$m_7 = (604.7\text{-}169.2)/(3003.6\text{-}169.2) = 0.1536$

CV: turbine

$$w_T = (h_6 - h_7) + (1 - m_7)(h_7 - h_8)$$
$$= 3666.5\text{-}3003.6 + 0.8464(3003.6\text{-}2446.1) = 1134.8 \text{ kJ/kg}$$

CV: pumps

$$w_P = m_1 w_{P12} + m_3 w_{P34} = 0.8464(-0.4) + 1(-5.0) = -5.3 \text{ kJ/kg}$$

$$w_{NET} = 1134.8 - 5.3 = 1129.5 \quad => \quad \dot{m} = 1000/1129.5 = 0.885 \text{ kg/s}$$

CV: reactor

$$\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 0.885(2794.3 - 609.7) = \mathbf{1933\ kW}$$

CV: superheater

$$\dot{Q}_{SUP} = 0.885(h_6 - h_5) = 0.885(3666.5 - 2794.3) = \mathbf{746\ kW}$$

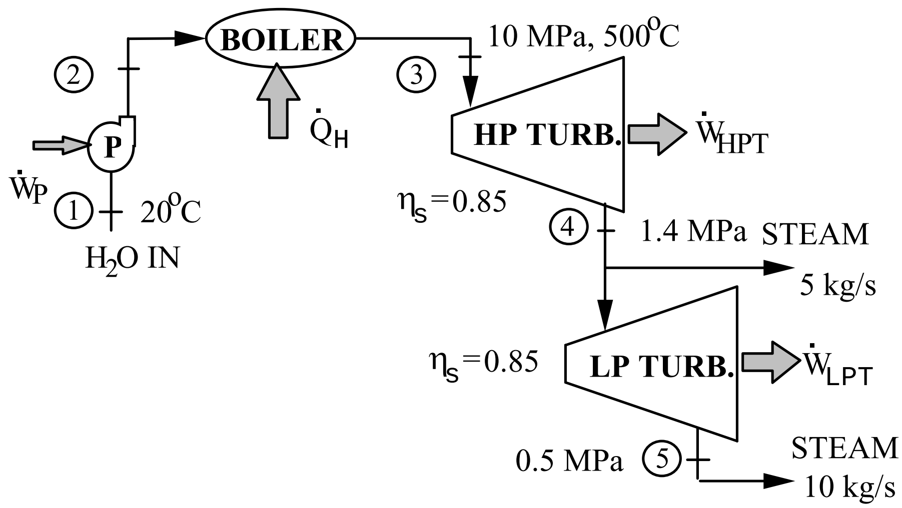
11.158

An industrial application has the following steam requirement: one 10-kg/s stream at a pressure of 0.5 MPa and one 5-kg/s stream at 1.4 MPa (both saturated or slightly superheated vapor). It is obtained by cogeneration, whereby a high-pressure boiler supplies steam at 10 MPa, 500°C to a turbine. The required amount is withdrawn at 1.4 MPa, and the remainder is expanded in the low-pressure end of the turbine to 0.5 MPa providing the second required steam flow. Assuming both turbine sections have an isentropic efficiency of 85%, determine the following.

a. The power output of the turbine and the heat transfer rate in the boiler.

b. Compute the rates needed were the steam generated in a low-pressure boiler without cogeneration. Assume that for each, 20°C liquid water is pumped to the required pressure and fed to a boiler.

Solution:



a) **With cogeneration**

high-pressure turbine, first the ideal then the actual.

$s_{4S} = s_3 = 6.5966$ kJ/kg K $\Rightarrow$ $T_{4S} = 219.9$ °C, $h_{4S} = 2852.6$ kJ/kg

$w_{S\ HPT} = h_3 - h_{4S} = 3373.7 - 2852.6 = 521.1$ kJ/kg

actual turbine from Eq.9.27

$w_{HPT} = \eta_S w_{S\ HPT} = 0.85 \times 521.1 = 442.9$ kJ/kg

$h_4 = h_3 - w = 3373.7 - 442.9 = 2930.8$ kJ/kg

$\Rightarrow$ $T_4 = 251.6$°C, $s_4 = 6.7533$ kJ/kg K

low-pressure turbine first the ideal then the actual

$s_{5S} = s_4 = 6.7533 = 1.8607 + x_{5S} \times 4.9606$, $x_{5S} = 0.9863$

$h_{5S} = 640.23 + 0.9863 \times 2108.5 = 2719.8$ kJ/kg

$w_{S\ LPT} = h_4 - h_{5S} = 2930.8 - 2719.8 = 211.0$ kJ/kg

actual turbine from Eq.9.27

$w_{LPT} = \eta_S w_{S\,LPT} = 0.85 \times 211.0 = 179.4$ kJ/kg

$h_5 = h_4 - w = 2930.8 - 179.4 = 2751.4 > h_G$ **OK**

$\dot{W}_{TURB} = 15 \times 442.9 + 10 \times 179.4 =$ **8438 kW**

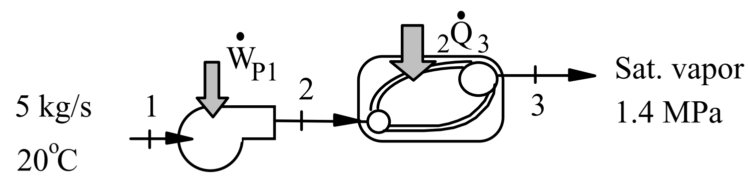
$\dot{W}_P = 15[0.001002(10000 - 2.3)] = 150.3$ kW

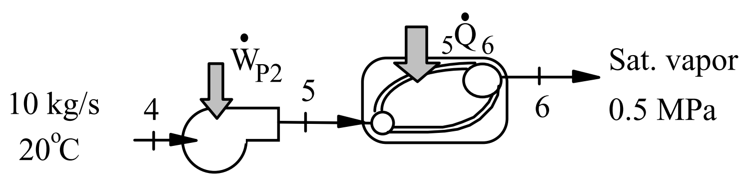
$h_2 = h_1 + w_P = 83.96 + 10.02 = 94.0$ kJ/kg

$\dot{Q}_H = \dot{m}_1(h_3 - h_2) = 15(3373.7 - 94.0) =$ **49196 kW**

b) **Without cogeneration**

This is to be compared to the amount of heat required to supply 5 kg/s of 1.4 MPa sat. vap. plus 10 kg/s of 0.5 MPa sat. vap. from 20°C water.





Pump 1 and boiler 1

$w_P = 0.001002(1400 - 2.3) = 14.0$ kJ/kg,

$h_2 = h_1 + w_P = 83.96 + 14.0 = 85.4$ kJ/kg

${}_2\dot{Q}_3 = \dot{m}_1(h_3 - h_2) = 5(2790.0 - 85.4) = 13\,523$ kW

$\dot{W}_{P1} = 5 \times 14.0 = 7$ kW

Pump 2 and boiler 2

$h_5 = h_4 + w_{P2} = 83.96 + 0.001002(500 - 2.3) = 84.5$ kJ/kg

${}_5\dot{Q}_6 = \dot{m}_4(h_6 - h_5) = 10(2748.7 - 84.5) = 26\,642$ kW

$\dot{W}_{P2} = 10 \times 0.5 = 5$ kW

Total $\dot{Q}_H = 13523 + 26642 =$ **40 165 kW**

11.159

Repeat Problem 11.75, but assume that the compressor has an efficiency of 82%, that both turbines have efficiencies of 87%, and that the regenerator efficiency is 70%.

a) From solution 11.54: $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8$ K

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$$

$$-w_C = -w_{SC}/\eta_{SC} = 201.6/0.82 = 245.8 \text{ kJ/kg} = w_{T1}$$

$$= C_{P0}(T_4 - T_5) = 1.004(1600 - T_5) \quad \Rightarrow \quad T_5 = 1355.2 \text{ K}$$

$$w_{ST1} = w_{T1}/\eta_{ST1} = 245.8/0.87 = 282.5 \text{ kJ/kg}$$

$$= C_{P0}(T_4 - T_{5S}) = 1.004(1600 - T_{5S}) \quad \Rightarrow \quad T_{5S} = 1318.6 \text{ K}$$

$$s_{5S} = s_4 \Rightarrow P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 600(\frac{1318.6}{1600})^{3.5} = \mathbf{304.9\ kPa}$$

b) $P_6 = 100$ kPa, $s_{6S} = s_5$

$$T_{6S} = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1355.2\left(\frac{100}{304.9}\right)^{0.286} = 985.2\text{K}$$

$$w_{ST2} = C_{P0}(T_5 - T_{6S}) = 1.004(1355.2 - 985.2) = 371.5 \text{ kJ/kg}$$

$$w_{T2} = \eta_{ST2} \times w_{ST2} = 0.87 \times 371.5 = \mathbf{323.2\ kJ/kg}$$

$$323.2 = C_{P0}(T_5 - T_6) = 1.004(1355.2 - T_6) \Rightarrow T_6 = 1033.3\text{K}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 150/323.2 = \mathbf{0.464\ kg/s}$$

c) $w_C = 245.8 = C_{P0}(T_2 - T_1) = 1.004(T_2 - 300) \Rightarrow T_2 = 544.8$ K

$$\eta_{REG} = \frac{h_3 - h_2}{h_6 - h_2} = \frac{T_3 - T_2}{T_6 - T_2} = \frac{T_3 - 544.8}{1033.3 - 544.8} = 0.7$$

$$\Rightarrow T_3 = 886.8 \text{ K}$$

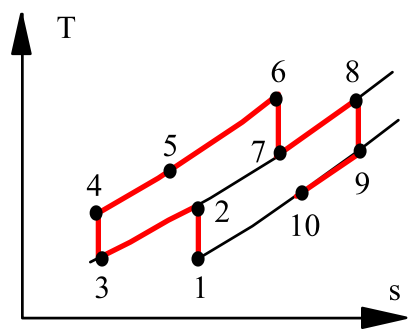
$$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 886.8) = 716 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 323.2/716 = \mathbf{0.451}$$

11.160

Consider a gas turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance of the first compressor is 100 kPa, the temperature entering each compressor is 20°C, and the temperature entering each turbine is 1100°C. A regenerator is also incorporated into the cycle and it has an efficiency of 70%. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

See Fig.11.23 for the configuration.



$P_2/P_1 = P_4/P_3 = P_6/P_7 = P_8/P_9 = 8.0$

$P_1 = 100$ kPa

$T_1 = T_3 = 20\ ^oC, \quad T_6 = T_8 = 1100\ ^oC$

Assume constant specific heat

$s_2 = s_1$ and $s_4 = s_3$

$$T_4 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.15(8)^{0.286} = 531 \text{ K}$$

Total $-w_C = 2 \times (-w_{12}) = 2C_{P0}(T_2 - T_1) = 2 \times 1.004(531 - 293.15) =$ **477.6 kJ/kg**

Also $s_6 = s_7$ and $s_8 = s_9$: $\Rightarrow \quad T_7 = T_9 = T_6\left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 1373.15\left(\frac{1}{8}\right)^{0.286} = 758 \text{ K}$

Total $w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 1.004(1373.15 - 758) = 1235.2$ kJ/kg

$w_{NET} = 1235.2 - 477.6 = 757.6$ kJ/kg

Ideal regenerator: $T_5 = T_9, \ T_{10} = T_4$ so the actual one has

$$\eta_{REG} = \frac{h_5 - h_4}{h_9 - h_4} = \frac{T_5 - T_4}{T_9 - T_4} = \frac{T_5 - 531}{758 - 531} = 0.7 \qquad \Rightarrow T_5 = 689.9 \text{ K}$$

$\Rightarrow q_H = (h_6 - h_5) + (h_8 - h_7) = C_{P0}(T_6 - T_5) + C_{P0}(T_8 - T_7)$

$= 1.004(1373.15 – 689.9) + 1.004\ (1373.15 – 758) = 1303.6$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 757.6/1303.6 =$ **0.581**

11.161

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 5 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. The maximum cycle temperature is 1500 K, and the cycle has a single turbine stage with an isentropic efficiency of 86%. The cycle also includes a regenerator with an efficiency of 80%. Calculate the temperature at the exit of each compressor stage, the second-law efficiency of the turbine and the cycle thermal efficiency.

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K State 7: $P_7 = P_o = 100$ kPa

State 3: $T_3 = 330$ K; State 6: $T_6 = 1500$ K, $P_6 = P_4$

$P_2 = 5\ P_1 = 500$ kPa; $P_4 = 5\ P_3 = 2500$ kPa

Ideal compression $T_{2s} = T_1\ (P_2/P_1)^{(k-1)/k} = 475.4$ K

1st Law: $q + h_i = h_e + w;\ q = 0 \ \Rightarrow\ w_{c1} = h_1 - h_2 = C_P(T_1 - T_2)$

$w_{c1\,s} = C_P(T_1 - T_{2s}) = -176.0$ kJ/kg, $w_{c1} = w_{c1\,s}/\ \eta = -214.6$

$T_2 = T_1 - w_{c1}/C_P =$ **513.9 K**

$T_{4s} = T_3\ (P_4/P_3)^{(k-1)/k} = 475.4$ K

$w_{c2\,s} = C_P(T_3 - T_{4s}) = -193.6$ kJ/kg; $w_{c2} = -236.1$ kJ/kg

$T_4 = T_3 - w_{c2}\ /\ C_P =$ **565.2 K**

Ideal Turbine (reversible and adiabatic)

$T_{7s} = T_6(P_7/P_6)^{(k-1)/k} = 597.4$ K $\Rightarrow$ $w_{Ts} = C_P(T_6 - T_{7s}) = 905.8$ kJ/kg

1st Law Turbine: $q + h_6 = h_7 + w;\ \ q = 0$

$w_T = h_6 - h_7 = C_P(T_6 - T_7) = \eta_{Ts}\ w_{Ts} = 0.86 \times 905.8 = 779.0$ kJ/kg

$T_7 = T_6 - w_T/\ C_P = 1500 - 779/1.004 = 723.7$ K

$$s_6 - s_7 = C_P \ln \frac{T_6}{T_7} - R \ln \frac{P_6}{P_7} = -0.1925 \text{ kJ/kg K}$$

$$\psi_6 - \psi_7 = (h_6 - h_7) - T_o(s_6 - s_7) = 779.0 - 298.15(-0.1925) = 836.8 \text{ kJ/kg}$$

$$\eta_{2nd\ Law} = \frac{w_T}{\psi_6 - \psi_7} = 779.0\ /\ 836.8 = \mathbf{0.931}$$

d) $\eta_{th} = q_H\ /\ w_{net}$; $w_{net} = w_T + w_{c1} + w_{c2} = 328.3$ kJ/kg

1st Law Combustor: $q + h_i = h_e + w;\ \ w = 0$

$q_c = h_6 - h_5 = C_P(T_6 - T_5)$

Regenerator: $\eta_{reg} = \dfrac{T_5 - T_4}{T_7 - T_4} = 0.8$ -> $T_5 = 692.1$ K

$q_H = q_c = 810.7$ kJ/kg; $\eta_{th} =$ **0.405**

11.162

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.

Use table A.7 and interpolation.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.28

$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$

$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.83521 + 0.287 \ln 9 = 7.4658$

This becomes trial and error so estimate first at 680 K and use A.7.1.

$LHS_{680} = 7.7090 - 0.287 \ln(680/290) = 7.4644$ (too low)

$LHS_{700} = 7.7401 - 0.287 \ln(700/290) = 7.4872$ (too high)

Interpolate to get: $T_2 = 681.23$ K, $u_2 = 497.9$ kJ/kg

$P_2 = P_1 (T_2/T_1) (v_1/v_2) = 90 (681.23 / 290) \times 9 = 1902.7$ kPa

${}_1w_2 = u_1 - u_2 = 207.2 - 497.9 = -290.7$ kJ/kg

Combustion 2 to 3: constant volume $v_3 = v_2$

$q_H = u_3 - u_2 = 1486.3 - 497.9 = 988.4$ kJ/kg

$P_3 = P_2(T_3/T_2) = 1902.7 (1800/681.2) = 5028$ kPa

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.28 as before

$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 8.8352 + 0.287 \ln(1/9) = 8.2046$

This becomes trial and error so estimate first at 850 K and use A.7.1.

$LHS_{850} = 7.7090 - 0.287 \ln(850/1800) = 8.1674$ (too low)

$LHS_{900} = 7.7401 - 0.287 \ln(900/1800) = 8.2147$ (too high)

Interpolation $\Rightarrow T_4 = 889.3$ K, $u_4 = 666$ kJ/kg

$P_4 = P_3(T_4/T_3)(v_3/v_4) = 5028 (889.3/1800) (1/9) =$ **276 kPa**

${}_3w_4 = u_3 - u_4 = 1486.3 - 666.0 = 820.3$ kJ/kg

Net work and overall efficiency

$w_{NET} = {}_3w_4 + {}_1w_2 = 820.3 - 290.7 =$ **529.6 kJ/kg**

$\eta = w_{NET}/q_H = 529.6/988.4 =$ **0.536**

11.163

The effect of a number of open feedwater heaters on the thermal efficiency of an ideal cycle is to be studied. Steam leaves the steam generator at 20 MPa, 600°C, and the cycle has a condenser pressure of 10 kPa. Determine the thermal efficiency for each of the following cases. **A:** No feedwater heater. **B**: One feedwater heater operating at 1 MPa. **C:** Two feedwater heaters, one operating at 3 MPa and the other at 0.2 MPa.

a) no feed water heater

$w_P = \int_1^2 vdP$

$\approx 0.00101(20000 - 10)$

$= 20.2$ kJ/kg

$h_2 = h_1 + w_P = 191.8 + 20.2 = 212.0$

$s_4 = s_3 = 6.5048$

$= 0.6493 + x_4 \times 7.5009$

$x_4 = 0.78064$

$h_4 = 191.83 + 0.780\,64 \times 2392.8$

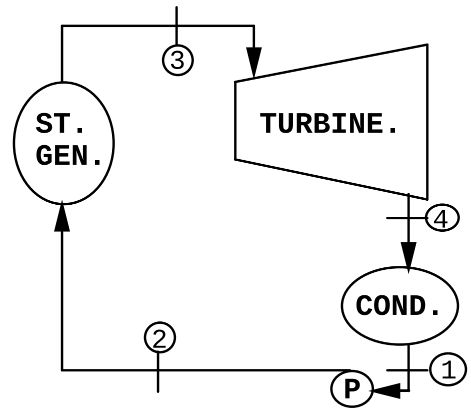
$= 2059.7$

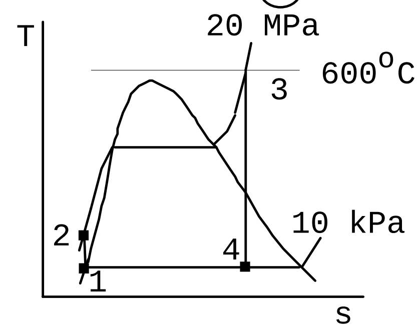
$w_T = h_3 - h_4 = 3537.6 - 2059.7$

$= 1477.9$ kJ/kg

$w_N = w_T - w_P = 1477.9 - 20.2 = 1457.7$

$q_H = h_3 - h_2 = 3537.6 - 212.0 = 3325.6$





$$\eta_{TH} = \frac{w_N}{q_H} = \frac{1457.7}{3325.6} = \mathbf{0.438}$$

b) one feedwater heater

$w_{P12} = 0.00101(1000 - 10)$

$= 1.0$ kJ/kg

$h_2 = h_1 + w_{P12} = 191.8 + 1.0 = 192.8$

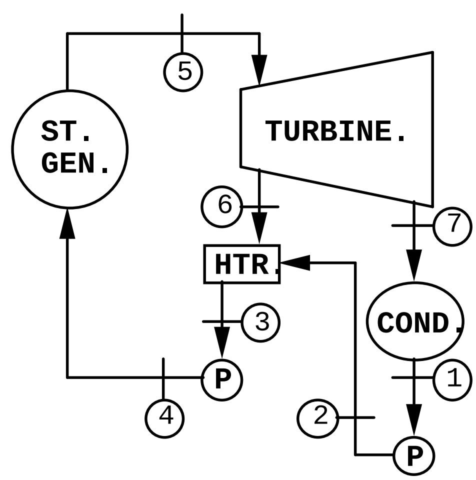
$w_{P34} = 0.001127\ (20000 - 1000)$

$= 21.4$ kJ/kg

$h_4 = h_3 + w_{P34} = 762.8 + 21.4 = 784.2$

$s_6 = s_5 = 6.5048$

$= 2.1387 + x_6 \times 4.4478$



$x_6 = 0.9816$

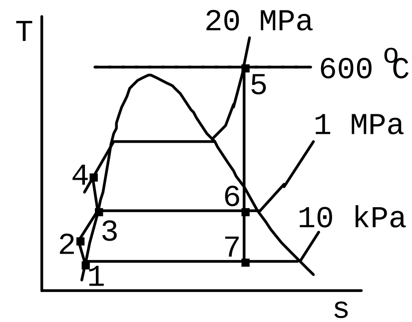
$h_6 = 762.8 + 0.9816 \times 2015.3 = 2741.1$

CV: heater

const: $m_3 = m_6 + m_2 = 1.0$ kg

1st law: $m_6h_6 + m_2h_2 = m_3h_3$

$$m_6 = \frac{762.8 - 192.8}{2741.1 - 192.8} = 0.2237$$



$m_2 = 0.7763, \quad h_7 = 2059.7$ ($= h_4$ of part a))

CV: turbine $\quad w_T = (h_5 - h_6) + m_2(h_6 - h_7)$

$= (3537.6 - 2741.1) + 0.7763(2741.1 - 2059.7) = 1325.5$ kJ/kg

CV: pumps

$w_P = m_1w_{P12} + m_3w_{P34} = 0.7763(1.0) + 1(21.4) = 22.2$ kJ/kg

$w_N = 1325.5 - 22.2 = 1303.3$ kJ/kg

CV: steam generator

$q_H = h_5 - h_4 = 3537.6 - 784.2 = 2753.4$ kJ/kg

$\eta_{TH} = w_N/q_H = 1303.3/2753.4 =$ **0.473**

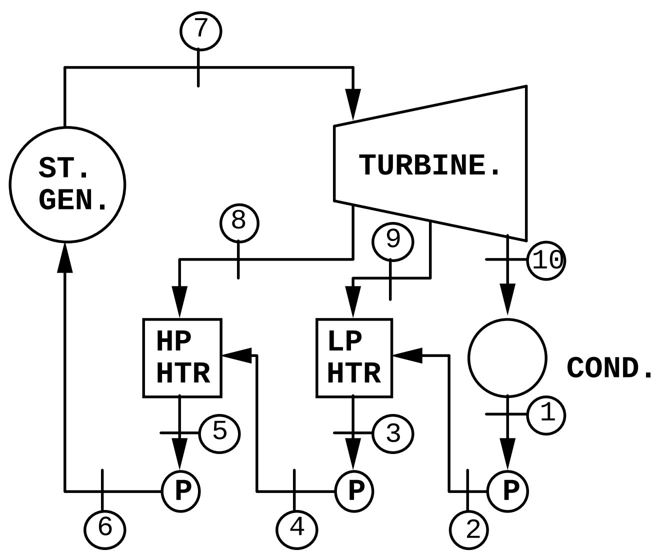
c) two feedwater heaters

$w_{P12} = 0.00101 \times (200 - 10) = 0.2$ kJ/kg

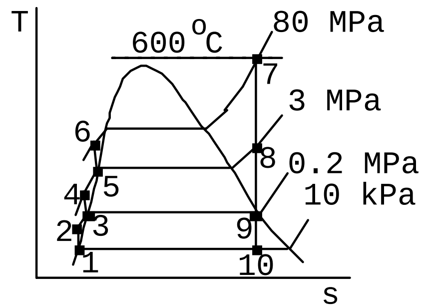
$h_2 = w_{P12} + h_1 = 191.8 + 0.2 = 192.0$

$w_{P34} = 0.001061 \times (3000 - 200) = 3.0$ kJ/kg

$h_4 = h_3 + w_{P34} = 504.7 + 3.0 = 507.7$



$w_{P56} = 0.001217(20000 - 3000)$
$= 20.7$ kJ/kg
$h_6 = h_5 + w_{P56} = 1008.4 + 20.7 = 1029.1$
$s_8 = s_7 = 6.5048$ at $P_8 = 3$ MPa $\Big\}$ $T_8 = 293.2\ ^oC$, $h_8 = 2974.8$
$s_9 = s_8 = 6.5048 = 1.5301 + x_9 \times 5.5970$



$x_9 = 0.8888 \;\Rightarrow\; h_9 = 504.7 + 0.888 \times 2201.9 = 2461.8$ kJ/kg

CV: high pressure heater

cont: $m_5 = m_4 + m_8 = 1.0$ kg ; 1st law: $m_5h_5 = m_4h_4 + m_8h_8$

$$m_8 = \frac{1008.4 - 507.7}{2974.8 - 507.7} = 0.2030 \qquad m_4 = 0.7970$$

CV: low pressure heater

cont: $m_9 + m_2 = m_3 = m_4$; 1st law: $m_9h_9 + m_2h_2 = m_3h_3$

$$m_9 = \frac{0.7970(504.7 - 192.0)}{2461.8 - 192.0} = 0.1098$$

$m_2 = 0.7970 - 0.1098 = 0.6872$

CV: turbine

$$w_T = (h_7 - h_8) + (1 - m_8)(h_8 - h_9) + (1 - m_8 - m_9)(h_9 - h_{10})$$
$$= (3537.6 - 2974.8) + 0.797(2974.8 - 2461.8)$$
$$+ 0.6872(2461.8 - 2059.7) = 1248.0 \text{ kJ/kg}$$

CV: pumps

$$w_P = m_1w_{P12} + m_3w_{P34} + m_5w_{P56}$$
$$= 0.6872(0.2) + 0.797(3.0) + 1(20.7) = 23.2 \text{ kJ/kg}$$

$w_N = 1248.0 - 23.2 = 1224.8$ kJ/kg

CV: steam generator

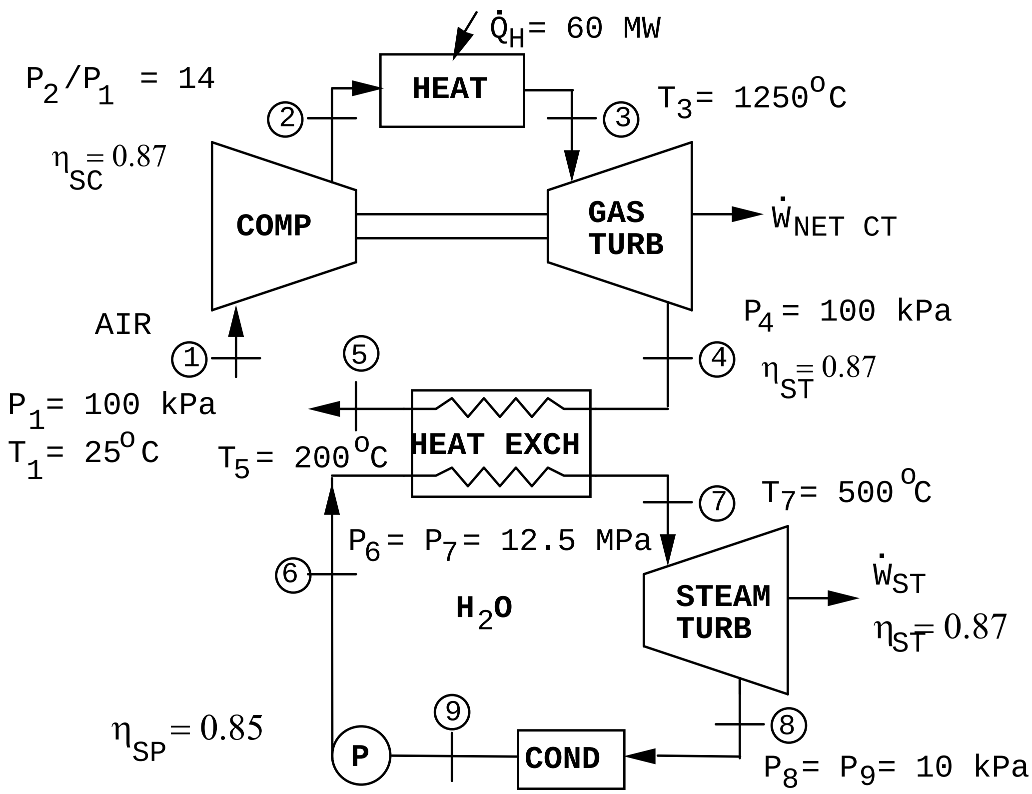
$q_H = h_7 - h_6 = 3537.6 - 1029.1 = 2508.5$ kJ/kg

$\eta_{TH} = w_N/q_H = 1224.8/2508.5 =$ **0.488**

11.164

The power plant shown in Fig. 11.40 combines a gas-turbine cycle and a steam-turbine cycle. The following data are known for the gas-turbine cycle. Air enters the compressor at 100 kPa, 25°C, the compressor pressure ratio is 14, and the isentropic compressor efficiency is 87%; the heater input rate is 60 MW; the turbine inlet temperature is 1250°C, the exhaust pressure is 100 kPa, and the isentropic turbine efficiency is 87%; the cycle exhaust temperature from the heat exchanger is 200°C. The following data are known for the steam-turbine cycle. The pump inlet state is saturated liquid at 10 kPa, the pump exit pressure is 12.5 MPa, and the isentropic pump efficiency is 85%; turbine inlet temperature is 500°C and the isentropic turbine efficiency is 87%. Determine

a. The mass flow rate of air in the gas-turbine cycle.
b. The mass flow rate of water in the steam cycle.
c. The overall thermal efficiency of the combined cycle.



a) From Air Tables, A.7: $P_{r1} = 1.0913$, $h_1 = 298.66$, $h_5 = 475.84$ kJ/kg

$s_2 = s_1 \Rightarrow P_{r2S} = P_{r1}(P_2/P_1) = 1.0913 \times 14 = 15.2782$

$T_{2S} = 629$ K, $h_{2S} = 634.48$

$w_{SC} = h_1 - h_{2S} = 298.66 - 634.48 = -335.82$ kJ/kg

$w_C = w_{SC}/\eta_{SC} = -335.82/0.87 = -386 = h_1 - h_2 \quad \Rightarrow \quad h_2 = 684.66$ kJ/kg

At $T_3 = 1523.2$ K: $P_{r3} = 515.493$, $h_3 = 1663.91$ kJ/kg

$$\dot{m}_{AIR} = \dot{Q}_H/(h_3 - h_2) = \frac{60\ 000}{1663.91 - 684.66} = \mathbf{61.27\ kg/s}$$

b) $P_{r4S} = P_{r3}(P_4/P_3) = 515.493(1/14) = 36.8209$

$\Rightarrow\ T_{4S} = 791\text{ K},\quad h_{4S} = 812.68\text{ kJ/kg}$

$w_{ST} = h_3 - h_{4S} = 1663.91 - 812.68 = 851.23\text{ kJ/kg}$

$w_T = \eta_{ST} \times w_{ST} = 0.87 \times 851.23 = 740.57 = h_3 - h_4 \quad \Rightarrow \quad h_4 = 923.34\text{ kJ/kg}$

Steam cycle: $-w_{SP} \approx 0.00101(12500 - 10) = 12.615\text{ kJ/kg}$

$-w_P = - w_{SP}/\eta_{SP} = 12.615/0.85 = 14.84\text{ kJ/kg}$

$h_6 = h_9 - w_P = 191.83 + 14.84 = 206.67\text{ kJ/kg}$

At 12.5 MPa, 500 ^{o}C: $h_7 = 3341.7\text{ kJ/kg},\quad s_7 = 6.4617\text{ kJ/kg K}$

$$\dot{m}_{H_2O} = \dot{m}_{AIR}\frac{h_4 - h_5}{h_7 - h_6} = 61.27\,\frac{923.34 - 475.84}{3341.7 - 206.67} = \mathbf{8.746\ kg/s}$$

c) $s_{8S} = s_7 = 6.4617 = 0.6492 + x_{8S} \times 7.501,\quad x_{8S} = 0.7749$

$h_{8S} = 191.81 + 0.7749 \times 2392.8 = 2046.0\text{ kJ/kg}$

$w_{ST} = h_7 - h_{8S} = 3341.7 - 2046.0 = 1295.7\text{ kJ/kg}$

$w_T = \eta_{ST} \times w_{ST} = 0.87 \times 1295.7 = 1127.3\text{ kJ/kg}$

$$\dot{W}_{NET} = \left[\dot{m}(w_T+w_C)\right]_{AIR} + \left[\dot{m}(w_T+w_P)\right]_{H_2O}$$

$$= 61.27(740.57 - 386.0) + 8.746(1127.3 - 14.84)$$

$$= 21725 + 9730 = 31455\text{ kW} = 31.455\text{ MW}$$

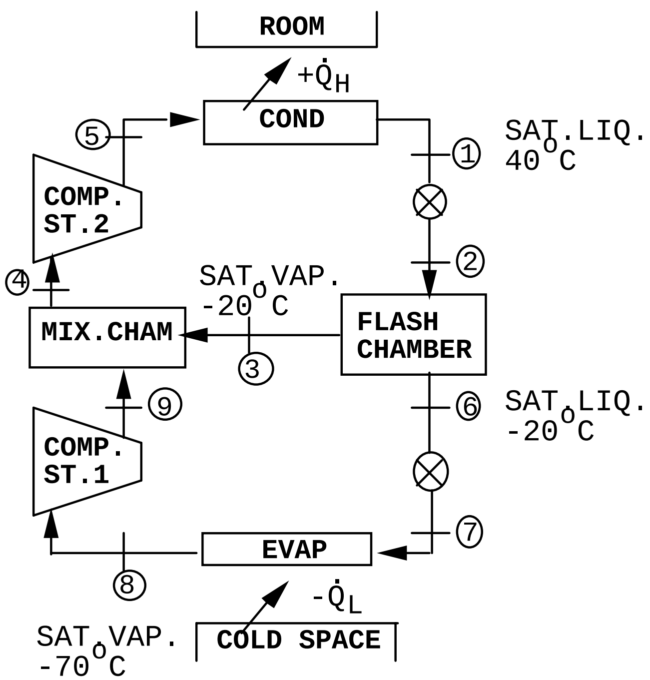
$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 31.455/60 = \mathbf{0.524}$

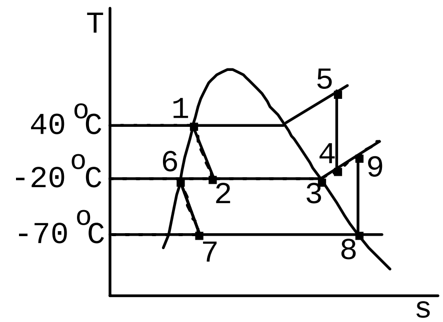
11.165

One means of improving the performance of a refrigeration system that operates over a wide temperature range is to use a two-stage compressor. Consider an ideal refrigeration system of this type that uses R-12 as the working fluid, as shown in Fig. P11.165. Saturated liquid leaves the condenser at 40°C and is throttled to −20°C. The liquid and vapor at this temperature are separated, and the liquid is throttled to the evaporator temperature, −70°C. Vapor leaving the evaporator is compressed to the saturation pressure corresponding to −20°C, after which it is mixed with the vapor leaving the flash chamber. It may be assumed that both the flash chamber and the mixing chamber are well insulated to prevent heat transfer from the ambient. Vapor leaving the mixing chamber is compressed in the second stage of the compressor to the saturation pressure corresponding to the condenser temperature, 40°C. Determine

a. The coefficient of performance of the system.

b. The coefficient of performance of a simple ideal refrigeration cycle operating over the same condenser and evaporator ranges as those of the two-stage compressor unit studied in this problem.





R-12 refrigerator with 2-stage compression

CV: expansion valve, upper loop

$h_2 = h_1 = 74.527 = 17.8 + x_2 \times 160.81; \quad x_2 = 0.353$

$m_3 = x_2 m_2 = x_2 m_1 = 0.353$ kg (for m_1=1 kg)

$m_6 = m_1 - m_3 = 0.647$ kg

CV: expansion valve, lower loop

$h_7 = h_6 = 17.8 = -26.1 + x_7 \times 181.64, \quad x_7 = 0.242$

$Q_L = m_3(h_8 - h_7) = 0.647(155.536 - 17.8)$

$q_L = 89.1$ kJ/kg-m_1

CV: 1st stage compressor

$s_8 = s_9 = 0.7744, \; P_9 = P_{SAT\,-20\,^oC} = 0.1509$ MPa

$\Rightarrow \; T_9 = 9\,^oC, \quad h_9 = 196.3$ kJ/kg

CV: mixing chamber (assume constant pressure mixing)

1st law: $m_6h_9 + m_3h_3 = m_1h_4$

or $h_4 = 0.647 \times 196.3 + 0.353 \times 178.61 = 190.06$ kJ/kg

$h_4, P_4 = 0.1509$ MPa $\Rightarrow \; T_4 = -1.0\,^oC, \; s_4 = 0.7515$ kJ/kg K

CV: 2nd stage compressor $P_4 = 0.1509 \text{ MPa} = P_9 = P_3$

$P_5 = P_{sat\,40^oC} = 0.9607$ MPa, $s_5 = s_4 \;\Rightarrow\; T_5 = 70^oC$, $h_5 = 225.8$ kJ/kg

CV: condenser

1st law: $-q_H = h_1 - h_5 = 74.527 - 225.8 = -151.27$ kJ/kg

$\beta_{2\text{ stage}} = q_L/(q_H - q_L) = 89.1/(151.27 - 89.1) =$ **1.433**

b) 1 stage compression

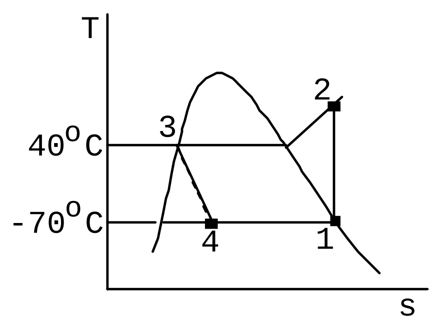
$h_3 = h_4 = 74.53$ kJ/kg

$h_1 = 155.54$ kJ/kg

$q_L = h_1 - h_4 = 81.0$ kJ/kg

$s_1 = s_2 = 0.7744$, $P_2 = 0.9607$ MPa

$\Rightarrow \; T_2 = 80.9\,^oC, h_2 = 234.0$



$q_H = h_2 - h_3 = 234.0 - 74.53 = 159.47$ kJ/kg

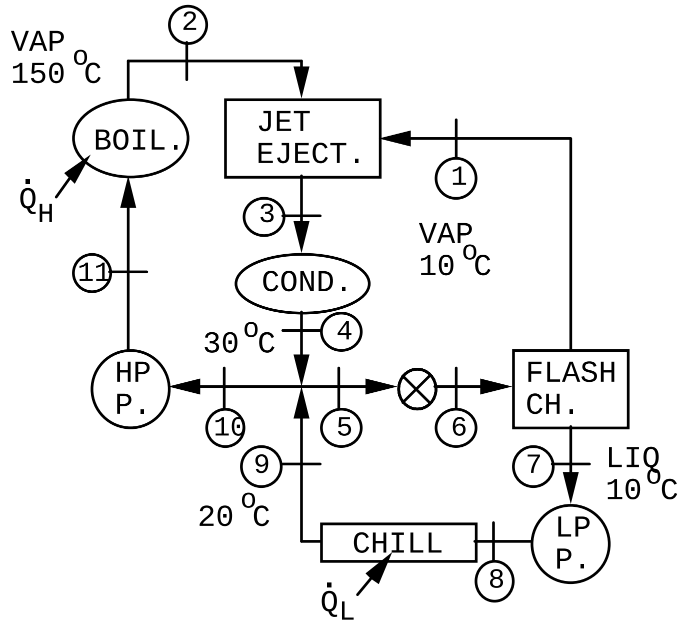
$\beta_{1\text{ stage}} = q_L/(q_H - q_L) = 81.0/(159.47 - 81.0) =$ **1.032**

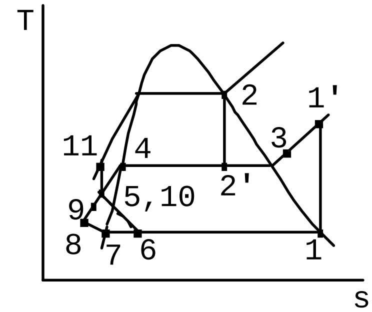
11.166

A jet ejector, a device with no moving parts, functions as the equivalent of a coupled turbine-compressor unit (see Problems 9.82 and 9.90). Thus, the turbine-compressor in the dual-loop cycle of Fig. P11.109 could be replaced by a jet ejector. The primary stream of the jet ejector enters from the boiler, the secondary stream enters from the evaporator, and the discharge flows to the condenser. Alternatively, a jet ejector may be used with water as the working fluid. The purpose of the device is to chill water, usually for an air-conditioning system. In this application the physical setup is as shown in Fig. P11.116. Using the data given on the diagram, evaluate the performance of this cycle in terms of the ratio Q_L/Q_H.

a. Assume an ideal cycle.

b. Assume an ejector efficiency of 20% (see Problem 9.90).





$T_1 = T_7 = 10\ ^oC$

$T_2 = 150\ ^oC$

$T_4 = 30\ ^oC$

$T_9 = 20\ ^oC$

Assume $T_5 = T_{10}$

(from mixing streams 4 & 9).

$P_3 = P_4 = P_5 = P_8 = P_9 = P_{10} = P_{G\ 30\ ^oC} = 4.246$ kPa

$P_{11} = P_2 = P_{G\ 150^oC} = 475.8$ kPa, $\quad P_1 = P_6 = P_7 = P_{G\ 10^oC} = 1.2276$ kPa

Cont: $\dot m_1 + \dot m_9 = \dot m_5 + \dot m_{10}, \quad \dot m_5 = \dot m_6 = \dot m_7 + \dot m_1$

$\dot m_7 = \dot m_8 = \dot m_9, \quad \dot m_{10} = \dot m_{11} = \dot m_2, \quad \dot m_3 = \dot m_4$

a) $\dot m_1 + \dot m_2 = \dot m_3$; ideal jet ejector

$s_1' = s_1$ & $s_2' = s_2$ (1' & 2' at $P_3 = P_4$)

then, $\dot m_1(h_1' - h_1) = \dot m_2(h_2 - h_2')$

From $s'_2 = s_2 = 0.4369 + x'_2 \times 8.0164;\quad x'_2 = 0.7985$

$h'_2 = 125.79 + 0.7985 \times 2430.5 = 2066.5$ kJ/kg

From $s'_1 = s_1 = 8.9008 \;\Rightarrow\; T'_1 = 112\ °C,\; h'_1 = 2710.4$ kJ/kg

$$\Rightarrow\; \dot{m}_1/\dot{m}_2 = \frac{2746.5 - 2066.5}{2710.4 - 2519.8} = 3.5677$$

Also $h_4 = 125.79$ kJ/kg, $h_7 = 42.01$ kJ/kg, $h_9 = 83.96$ kJ/kg

Mixing of streams 4 & 9 $\Rightarrow$ 5 & 10:

$$(\dot{m}_1 + \dot{m}_2)h_4 + \dot{m}_7 h_9 = (\dot{m}_7 + \dot{m}_1 + \dot{m}_2)h_{5\,=\,10}$$

Flash chamber (since $h_6 = h_5$) : $(\dot{m}_7 + \dot{m}_1)h_{5\,=\,10} = \dot{m}_1 h_1 + \dot{m}_7 h_1$

$\Rightarrow$ using the primary stream $\dot{m}_2 = 1$ kg/s:

$4.5677 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 4.5677)h_5$

& $(\dot{m}_7 + 3.5677)h_5 = 3.5677 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 202.627$ & $h_5 = 84.88$ kJ/kg

LP pump: $-w_{LP\,P} = 0.0010(4.246 - 1.2276) = 0.003$ kJ/kg

$h_8 = h_7 - w_{LP\,P} = 42.01 + 0.003 = 42.01$ kJ/kg

Chiller: $\dot{Q}_L = \dot{m}_7(h_9 - h_8) = 202.627(83.96 - 42.01) = 8500$ kW (for $\dot{m}_2 = 1$)

HP pump: $-w_{HP\,P} = 0.001002(475.8 - 4.246) = 0.47$ kJ/kg

$h_{11} = h_{10} - w_{HP\,P} = 84.88 + 0.47 = 85.35$ kJ/kg

Boiler: $\dot{Q}_{11} = \dot{m}_{11}(h_2 - h_{11}) = 1(2746.5 - 85.35) = 2661.1$ kW

$\Rightarrow\; \dot{Q}_L/\dot{Q}_H = 8500/2661.1 =$ **3.194**

b) Jet eject. eff. $= (\dot{m}_1/\dot{m}_2)_{ACT}/(\dot{m}_1/\dot{m}_2)_{IDEAL} = 0.20$

$\Rightarrow\; (\dot{m}_1/\dot{m}_2)_{ACT} = 0.2 \times 3.5677 = 0.7135$

using $\dot{m}_2 = 1$ kg/s: $1.7135 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 1.7135)h_5$

& $(\dot{m}_7 + 0.7135)h_5 = 0.7135 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 39.762$ & $h_5 = h_{10} = 85.69$ kJ/kg

Then, $\dot{Q}_L = 39.762(83.96 - 42.01) = 1668$ kW

$h_{11} = 85.69 + 0.47 = 86.16$ kJ/kg

$\dot{Q}_H = 1(2746.5 - 86.16) = 2660.3$ kW

& $\dot{Q}_L / \dot{Q}_H = 1668/2660.3 =$ **0.627**

Problems solved using Table A.7.2

11.79

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P11.79, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, *x*, bypasses the burner and the rest (1 – *x*) goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction *x*. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1;\;\; s_2 = s_1$

$P_{r2} = P_{r1}(P_2/P_1) = 0.9899(450/100) = 4.4545,\;\; T_2 = 445\text{ K}$

$h_2 = 446.74, \quad -w_C = 446.74 - 290.43 = 156.3\text{ kJ/kg}$

C.V.Burner: $h_3 = h_2 + q_H = 446.74 + 1200 = 1646.74\text{ kJ/kg}$

$\Rightarrow\; T_3 = 1509\text{ K}$

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22\text{ kJ/kg}$

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1646.74 - 1046.22}{1646.74 - 446.74} = \mathbf{0.50}$$

$\dot{W}_{T1} = \dot{W}_{C,in} \;\Rightarrow\; \dot{w}_{T1} = -w_C = 156.3 = h_3 - h_4$

$h_4 = 1046.22 - 156.3 = 889.9 \;\Rightarrow\; T_4 = \mathbf{861\text{ K}}$

$P_4 = (P_{r4}/P_{rMIX})P_{MIX} = (51/91.65) \times 450 = \mathbf{250.4\text{ kPa}}$

$s_4 = s_5 \;\Rightarrow\; P_{r5} = P_{r4}(P_5/P_4) = 51(100/250.4) = 20.367$

$h_5 = 688.2 \quad T_5 = 676\text{ K}$

$w_{T2} = h_4 - h_5 = 889.9 - 688.2 = \mathbf{201.7\text{ kJ/kg}}$

11.81

Repeat Problem 11.77 when the intercooler brings the air to T_3 = 320 K. The corrected formula for the optimal pressure is $P_2 = [\ P_1P_4\ (T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 9.184, where *n* is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so n/(n - 1) = 1.4/0.4 = 3.5

$$P_2 = 400\sqrt{(320/290)^{3.5}} = 475.2 \text{ kPa}$$

C.V. C1: $s_2 = s_1 \Rightarrow P_{r2} = P_{r1}(P_2/P_1) = 0.9899(475.2/100)$

$= 4.704 \Rightarrow T_2 = 452$ K, $h_2 = 453.75$

$-w_{C1} = h_2 - h_1 = 453.75 - 290.43 =$ **163.3 kJ/kg**

C.V. Cooler: $q_{OUT} = h_2 - h_3 = 453.75 - 320.576 =$ **133.2 kJ/kg**

C.V. C2: $s_4 = s_3 \Rightarrow P_{r4} = P_{r3}(P_4/P_3) = 1.3972(1600/475.2) = 4.704$

$\Rightarrow T_4 = T_2 = 452$ K, $h_4 = 453.75$

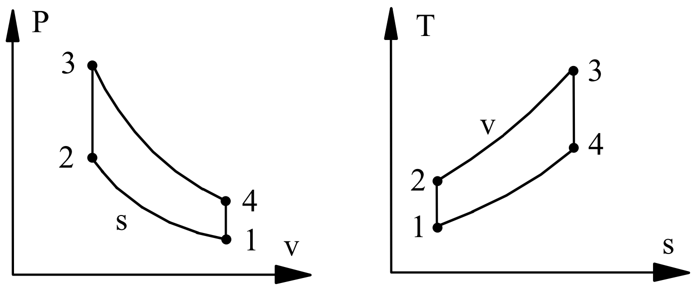
$-w_{C2} = h_4 - h_3 = 453.75 - 320.576 =$ **133.2 kJ/kg**

11.93

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Solve the problem with variable heat capacity, use A.7.1 and A.7.2.



Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From A.7.2

$$v_{r2} = \frac{v_{r1}}{8} = \frac{179.49}{8} = 22.436,$$

$$T_2 = 673 \text{ K}, \quad u_2 = 491.5 \text{ kJ/kg}, \quad P_{r2} = 20$$

$$P_2 = P_1 \times \frac{P_{r2}}{P_{r1}} = 20 \times \frac{95}{1.1146} = 1705 \text{ kPa}$$

Compression 2 to 3:

$$u_3 = u_2 + q_H = 491.5 + 1300 = 1791.5 \text{ kJ/kg}$$

$$T_3 = \mathbf{2118\ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1705 \times \frac{2118}{673} = \mathbf{5366\ kPa}$$

11.94

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.

Use table A.7 and interpolation.

Compression 1 to 2: $s_2 = s_1 \Rightarrow v_{r2} = v_{r1}(v_2/v_1)$

$v_{r2} = 196.37/9 = 21.819 \Rightarrow T_2 \cong 680$ K, $P_{r2} \cong 20.784$, $u_2 = 496.94$

$P_2 = P_1(P_{r2}/P_{r1}) = 90\ (20.784 / 0.995) = 1880$ kPa

${}_1w_2 = u_1 - u_2 = 207.19 - 496.94 = -289.75$ kJ/kg

Combustion 2 to 3:

$q_H = u_3 - u_2 = 1486.33 - 496.94 = 989.39$ kJ/kg

$P_3 = P_2(T_3/T_2) = 1880\ (1800 / 680) = 4976$ kPa

Expansion 3 to 4:

$s_4 = s_3 \Rightarrow v_{r4} = v_{r3} \times 9 = 1.143 \times 9 = 10.278$

$\Rightarrow T_4 = 889$ K, $P_{r4} = 57.773$, $u_4 = 665.8$ kJ/kg

$P_4 = P_3(P_{r4}/P_{r3}) = 4976\ (57.773 / 1051) =$ **273.5 kPa**

${}_3w_4 = u_3 - u_4 = 1486.33 - 665.8 = 820.5$ kJ/kg

$w_{NET} = {}_3w_4 + {}_1w_2 = 820.5 - 289.75 =$ **530.8 kJ/kg**

$\eta = w_{NET}/q_H = 530.8/989.39 =$ **0.536**

11.100

Answer the same three questions for the previous problem, but use variable heat capacities (use table A.7).

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1 and Table A.7.2

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \quad v_{r2} = 8.2166$$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From the vr function

$$v_1/v_2 = v_{r1}/v_{r2} = 195.36/8.2166 = \mathbf{23.78}$$

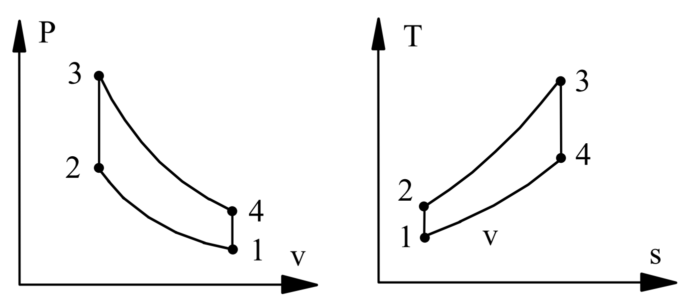
Comment: This is much too high for an actual Otto cycle.

$$-{}_1w_2 = u_2 - u_1 = 725.7 - 207.2 = \mathbf{518.5\ kJ/kg}$$

Highest pressure is after combustion

$$P_3 = P_2T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 23.78 = \mathbf{15\ 129\ kPa}$$



11.103

Repeat Problem 11.95, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (and the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $v_{r1} = 210.44$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From definition of the v_r function

$v_{r2} = v_{r1} (v_2/v_1) = 210.4 (1/7) = 30.063$

Interpolate to get: $T_2 = 603.9$ K, $u_2 = 438.1$ kJ/kg

=> $-{}_1w_2 = u_2 - u_1 = 235.8$ kJ/kg,

$u_3 = 438.1 + 1800 = 2238.1$ => $T_3 =$ **2573.4 K** , $v_{r3} = 0.34118$

$P_3 = 90 \times 7 \times 2573.4 / 283.2 =$ **5725 kPa**

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From the v_r function as before

$v_{r4} = v_{r3} (v_4/v_3) = 0.34118 (7) = 2.3883$

Interpolation $\Rightarrow$ $T_4 = 1435.4$ K, $u_4 = 1145.8$ kJ/kg

${}_3w_4 = u_3 - u_4 = 2238.1 - 1145.8 = 1092.3$ kJ/kg

Net work, efficiency and mep

➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1092.3 - 235.8 = 856.5$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 856.5 / 1800 =$ **0.476**

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029\ m^3/kg$

$v_2 = (1/7)\ v_1 = 0.1290\ m^3/kg$

$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 856.5 / (0.9029 - 0.129) =$ **1107 kPa**

11.110

Do problem 11.106, but use the properties from A.7 and not the cold air properties.

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1$ => From definition of the P_r function

$$P_{r2} = P_{r1} (P_2/P_1) = 0.9899 (6000/95) = 62.52$$

A.7.1 => $T_2 = 907$ K; $h_2 = 941.0$ kJ/kg;

$$h_3 = 2755.8 \;; \quad v_{r3} = 0.43338$$

$q_H = h_3 - h_2 = 2755.8 – 941.0 = 1814.8$ kJ/kg

$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907) \times (6000/ 95) = 20.19$

Expansion process

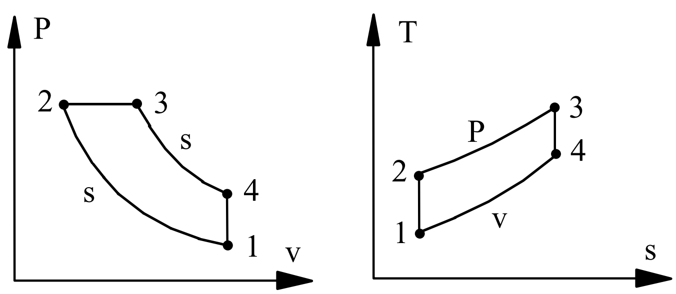
$v_{r4} = v_{r3} (v_4 / v_3) = v_{r3} (v_1 / v_3) = v_{r3} (v_1 / v_2) \times (T_2/T_3)$

$= v_{r3}\, CR \times (T_2/T_3) = 0.43338 \times 20.19 \times (907/2400) = 3.30675$

Linear interpolation $T_4 = 1294.8$ K, $u_4 = 1018.1$ kJ/kg

$q_L = u_4 - u_1 = 1018.1 – 207.2 = 810.9$ kJ/kg

$\eta = 1 – (q_L/ q_H) = 1 – (810.9/1814.8) = \mathbf{0.553}$



11.118

Do the previous problem 11.117 using values from Table A.7.1. and A.7.2

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 26.8°C$ and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

From A.7.1: $u_3 = 674.82$ kJ/kg, $v_{r3} = 9.9169$

$u_4 = 214.36$ kJ/kg, $v_{r4} = 179.49$

Energy equation with q = 0

$$_3w_4 = u_3 - u_4 = 674.82 - 214.36 = \mathbf{460.5\ kJ/kg}$$

Entropy equation, constant s expressed with the v_r function

$$v_4/v_3 = v_{r4}/v_{r3} = 179.49 / 9.9169 = \mathbf{18.1}$$