# SOLUTION MANUAL ENGLISH UNIT PROBLEMS CHAPTER 8



# CHAPTER 8

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This problem set compared to the fifth edition chapter 8 set.

New	5th	SI	New	5th	SI	New	5th	SI
140	new	7	154	90	42	168	new	92
141	new	10	155	91	45	169	104	93
142	new	11	156	92	49mod	170	105	95
143	new	14	157	93	50	171	106	102
144	new	15	158	94	59	172	107	104
145	new	16	159	96	65	173	108	105
146	new	20	160	97	63	174	110	110
147	83	21	161	98	77	175	111	119
148	84	27	162	100	72	176	109	113
149	85	32	163	101	73	177	112	115
150	86	34	164	102	78	178	new	120
151	87	35	165	new	82	179	new	124
152	88	36	166	103	86	180	95	132
153	89	39	167	new	91	181	99	130
						182	113	137

### **Concept Problems**

#### 8.140E

Water at 20 psia, 240 F receives 40 Btu/lbm in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 8.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller.

The constant T (isothermal) process therefore changes s the most.

#### 8.141E

Saturated water vapor at 20 psia is compressed to 60 psia in a reversible adiabatic process. Find the change in v and T.

Process adiabatic: dq = 0Process reversible:  $ds_{gen} = 0$ Change in s:  $ds = dq/T + ds_{gen} = 0 + 0 = 0$  thus s is constant

Table F.7.2:  $T_1 = 227.96$  F,  $v_1 = 20.091$  ft<sup>3</sup>/lbm,  $s_1 = 1.732$  Btu/lbm R Table F.7.2 at 60 psia and  $s = s_1 = 1.732$  Btu/lbm R 1.732 - 1.7134

$$T = 400 + 40 \frac{1.732}{1.736 - 1.7134} = 400 + 40 \times 0.823 = 432.9 F$$

 $v = 8.353 + (8.775 - 8.353) \times 0.823 = 8.700 \text{ ft}^3/\text{lbm}$ 

## 8.142E

A computer chip dissipates 2 Btu of electric work over time and rejects that as heat transfer from its 125 F surface to 70 F air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 125 F, we assume chip state is constant.

Energy:  $U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = W_{\text{electrical in}} - Q_{\text{out }1}$ 

Entropy:

$$S_2 - S_1 = 0 = -\frac{Q_{out 1}}{T_{surf}} + {}_1S_2 gen1$$

$${}_{1}S_{2 \text{ gen1}} = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{W_{\text{electrical in}}}{T_{\text{surf}}} = \frac{2 \text{ Btu}}{(125 + 459.7) \text{ R}} = 0.0034 \text{ Btu/R}$$

C.V.2 From chip surface at 125 F to air at 70 F, assume constant state.

Energy: 
$$U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = Q_{out 1} - Q_{out 2}$$

Entropy:

 $S_2 - S_1 = 0 = \frac{Q_{out1}}{T_{surf}} - \frac{Q_{out2}}{T_{air}} + {}_1S_2 gen2$ 

$${}_{1}S_{2 \text{ gen2}} = \frac{Q_{\text{out2}}}{T_{\text{air}}} - \frac{Q_{\text{out1}}}{T_{\text{surf}}} = \frac{2 \text{ Btu}}{529.7 \text{ R}} - \frac{2 \text{ Btu}}{584.7 \text{ R}} = 0.000 \text{ 36 Btu/R}$$



# 8.143E

Two 10 lbm blocks of steel, one at 400 F the other at 70 F, come in thermal contact. Find the final temperature and the total entropy generation in the process?

C.V. Both blocks, no external heat transfer, C from table F.2.

Energy Eq.: 
$$U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0$$
  
=  $m_AC(T_2 - T_{A1}) + m_BC(T_2 - T_{B1})$ 

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = 235 F$$

Entropy Eq.: 
$$S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_2 gen$$
  
 ${}_1S_2 gen = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}}$   
 $= 10 \times 0.11 \ln \frac{235 + 459.7}{400 + 459.7} + 10 \times 0.11 \ln \frac{235 + 459.7}{529.7}$   
 $= -0.2344 + 0.2983 = 0.0639 Btu/R$ 



#### 8.144E

One lbm of air at 540 R is mixed with one lbm air at 720 R in a process at a constant 15 psia and Q = 0. Find the final T and the entropy generation in the process.

C.V. All the air.

Energy Eq.:  $U_2 - U_1 = 0 - W$ Entropy Eq.:  $S_2 - S_1 = 0 + {}_1S_2$  gen Process Eq.: P = C;  $W = P(V_2 - V_1)$ 



Substitute W into energy Eq.

$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$\begin{split} H_2 - H_1 &= m_A (h_2 - h_1)_A + m_B (h_2 - h_1)_B \\ &= m_A C_p (T_2 - T_{A1}) + m_B C_p (T_2 - T_{B1}) = 0 \end{split}$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = 630 R$$

Entropy change is from Eq. 8.25 with no change in P and Table F.4 for  $C_p$ 

$${}_{1}S_{2 \text{ gen}} = S_{2} - S_{1} = m_{A}C_{p} \ln \frac{T_{2}}{T_{A1}} + m_{B}C_{p} \ln \frac{T_{2}}{T_{B1}}$$
$$= 1 \times 0.24 \ln \frac{630}{540} + 1 \times 0.24 \ln \frac{630}{720}$$
$$= 0.037 - 0.032 = 0.005 \text{ Btu/R}$$

Remark: If you check the volume does not change and there is no work.

#### 8.145E

One lbm of air at 15 psia is mixed with one lbm air at 30 psia, both at 540 R, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.

## C.V. All the air.

Energy Eq.:  $U_2 - U_1 = 0 - 0$ Entropy Eq.:  $S_2 - S_1 = 0 + {}_1S_2$  gen Process Eqs.: V = C; W = 0, Q = 0States A1, B1:  $u_{A1} = u_{B1}$  $V_A = m_A RT_1 / P_{A1}$ ;  $V_B = m_B RT_1 / P_{B1}$ 



 $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \implies u_2 = (u_{A1} + u_{B1})/2 = u_{A1}$ State 2:  $T_2 = T_1 = 540 \text{ R} \text{ (from } u_2\text{)}; \quad m_2 = m_A + m_B = 2 \text{ kg};$ 

$$V_2 = m_2 RT_1/P_2 = V_A + V_B = m_A RT_1/P_{A1} + m_B RT_1/P_{B1}$$

Divide with  $m_A RT_1$  and get

$$2/P_2 = 1/P_{A1} + 1/P_{B1} = \frac{1}{15} + \frac{1}{30} = 0.1 \text{ psia}^{-1} \implies P_2 = 20 \text{ psia}^{-1}$$

Entropy change from Eq. 8.25 with the same T, so only P changes

$${}_{1}S_{2 \text{ gen}} = S_{2} - S_{1} = -m_{A}R \ln \frac{P_{2}}{P_{A1}} - m_{B}R \ln \frac{P_{2}}{P_{B1}}$$
$$= -1 \times 53.34 \left[ \ln \frac{20}{15} + \ln \frac{20}{30} \right]$$
$$= -53.34 \left( 0.2877 - 0.4055 \right) = 6.283 \text{ lbf-ft/R} = 0.0081 \text{ Btu/R}$$

#### 8.146E

A window receives 600 Btu/h of heat transfer at the inside surface of 70 F and transmits the 600 Btu/h from its outside surface at 36 F continuing to ambient air at 23 F. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.



Window only: 
$$\dot{S}_{gen win} = \dot{S}_{win} - \dot{S}_{inside} = 1.21 - 1.133 = 0.077 \text{ Btu/h-R}$$

If you want to include the generation in the outside air boundary layer where T changes from 36 F to the ambient 23 F then it is

$$\dot{S}_{gen tot} = \dot{S}_{amb} - \dot{S}_{inside} = 1.243 - 1.133 = 0.11 \text{ Btu/h-R}$$

## **Entropy**, Clausius

### 8.147E

Consider the steam power plant in Problem 7.100E and show that this cycle satisfies the inequality of Clausius.

Solution:

Show Clausius:  $\int \frac{dQ}{T} \leq 0$ 

For this problem we have two heat transfer terms:

Boiler:	1000 Btu/s at 1200 F = 1660 R	
Condenser:	580 Btu/s at 100 F = 560 R	
$\int \frac{dQ}{T} = \frac{Q_H}{T_H}.$	$\frac{Q_{\rm L}}{T_{\rm L}} = \frac{1000}{1660} - \frac{580}{560}$	
= 0.6	024 - 1.0357 = -0.433 Btu/s R < 0	OK

# 8.148E

Find the missing properties and give the phase of the substance

a. H <sub>2</sub> O $s = 1.75$ Btu/lbm R, $P = 4$ lbf/in. <sup>2</sup> $h = ?$ $T = ?$ $x = ?$
b. H <sub>2</sub> O $u = 1350$ Btu/lbm, $P = 1500$ lbf/in. <sup>2</sup> $T = ? x = ? s = ?$
c. R-22 $T = 30$ F, $P = 60$ lbf/in. <sup>2</sup> $s = ? x = ?$
d. R-134a $T = 10$ F, $x = 0.45$ $v = ? s = ?$
e. NH <sub>3</sub> $T = 60$ F, $s = 1.35$ Btu/lbm R $u = ? x = ?$
a) Table F.7.1: $s < s_g$ so 2 phase $T = T_{sat}(P) = 152.93 F$
$x = (s - s_f)/s_{fg} = (1.75 - 0.2198)/1.6426 = 0.9316$
$h = 120.9 + 0.9316 \times 1006.4 = 1058.5 Btu/lbm$
b) Table F.7.2, $x =$ undefined, $T = 1020$ F, $s = 1.6083$ Btu/lbm R
c) Table F.9.1, x = undefined, $s_g(P) = 0.2234$ Btu/lbm R, $T_{sat} = 22.03$ F
s = 0.2234 + (30 - 22.03) (0.2295 - 0.2234) / (40 - 22.03)
= 0.2261 Btu/lbm R
d) Table F.10.1 $v = v_f + xv_{fg} = 0.01202 + 0.45 \times 1.7162 = 0.7843 \text{ ft}^3/\text{lbm},$
$s = s_f + xs_{fg} = 0.2244 + 0.45 \times 0.1896 = 0.3097 \text{ Btu/lbm R}$
e) Table F.8.1: $s > s_g$ so superheated vapor Table F.8.2: $x =$ undefined
$P = 40 + (50-40) \times (1.35-1.3665) / (1.3372-1.3665) = 45.6 \text{ psia}$
Interpolate to get $v = 6.995 \text{ft}^3/\text{lbm}$ , $h = 641.0 \text{ Btu/lbm}$
$u = h - Pv = 641.0 - 45.6 \times 6.995 \times \frac{144}{778} = 581.96 \text{ Btu/lbm}$
P b c, e d a T b c, e d a c, e d a c, e s

### **Reversible Processes**

#### 8.149E

In a Carnot engine with water as the working fluid, the high temperature is 450 F and as  $Q_L$  is received, the water changes from saturated liquid to saturated vapor. The water pressure at the low temperature is 14.7 lbf/in.<sup>2</sup>. Find T<sub>L</sub>, cycle thermal efficiency, heat added per pound-mass, and entropy, *s*, at the beginning of the heat rejection process.



$$\eta_{\text{cycle}} = 1 - T_{\text{L}}/T_{\text{H}} = 1 - \frac{212 + 459.67}{450 + 459.67} = 0.262$$
  
Table F.8.1:  $s_3 = s_2 = s_{\text{g}}(T_{\text{H}}) = 1.4806$  Btu/lbm R

### 8.150E

Consider a Carnot-cycle heat pump with R-22 as the working fluid. Heat is rejected from the R-22 at 100 F, during which process the R-22 changes from saturated vapor to saturated liquid. The heat is transferred to the R-22 at 30 F. a. Show the cycle on a T-s diagram.

- b. Find the quality of the R-22 at the beginning and end of the isothermal heat addition process at 30 F.
- c. Determine the coefficient of performance for the cycle.



Table F.9.1  
b) State 3 is saturated liquid  
$$s_4 = s_3 = 0.0794$$
 Btu/lbm R  
 $= 0.0407 + x_4(0.1811)$   
 $x_4 = 0.214$   
State 2 is saturated vapor  
 $s_1 = s_2 = 0.2096$  Btu/lbm R  
 $= 0.0407 + x_1(0.1811)$   
 $x_1 = 0.9326$ 

c) 
$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{559.67}{100 - 30} = 7.995$$

## 8.151E

Do Problem 8.150 using refrigerant R-134a instead of R-22.



c) 
$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{559.67}{100 - 30} = 7.995$$

## 8.152E

Water at 30 lbf/in.<sup>2</sup>, x = 1.0 is compressed in a piston/cylinder to 140 lbf/in.<sup>2</sup>, 600 F in a reversible process. Find the sign for the work and the sign for the heat transfer.

Solution:



## 8.153E

Two pound-mass of ammonia in a piston/cylinder at 120 F, 150 lbf/in.<sup>2</sup> is expanded in a reversible adiabatic process to 15 lbf/in.<sup>2</sup>. Find the work and heat transfer for this process.

Control mass Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.8.3:  $m(s_2 - s_1) = \int_{-1}^{2} dQ/T + {}_1S_{2,gen}$ Process:  ${}_1Q_2 = \emptyset$ ,  ${}_1S_{2,gen} = \emptyset \implies s_2 = s_1$ State 1: T, P Table F.8.2,  $u_1 = 596.6$  Btu/lbm,  $s_1 = 1.2504$  Btu/lbm R State 2: P<sub>2</sub>,  $s_2 \implies 2$  phase Table F.8.1 (sat. vapor F.8.2 also) Interpolate:  $s_{g2} = 1.3921$  Btu/lbm R,  $s_f = 0.0315$  Btu/lbm R  $x_2 = (1.2504 - 0.0315)/1.3606 = 0.896$ ,  $u_2 = 13.36 + 0.896 \times 539.35 = 496.6$  Btu/lbm  ${}_1W_2 = m(u_1 - u_2) = 2 \times (596.6 - 496.6) = 100$  Btu



#### 8.154E

A cylinder fitted with a piston contains ammonia at 120 F, 20% quality with a volume of 60 in.<sup>3</sup>. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.



State 2: Saturated vapor,  $v_2 = 1.045 \text{ ft}^3/\text{lbm}$ ,  $s_2 = 1.140 \text{ Btu/lbm R}$ Process: T = constant, since two-phase then P = constant

$${}_{1}W_{2} = \frac{286.5 \times 144}{778} \times 0.15 \times (1.045 - 0.2318) = 6.47$$
 Btu  
 ${}_{1}Q_{2} = 579.7 \times 0.15(1.1400 - 0.5137) = 54.46$  Btu

- or -  $h_1 = 178.79 + 0.2 \times 453.84 = 269.56$  Btu/lbm;  $h_2 = 632.63$  Btu/lbm  ${}_1Q_2 = m(h_2 - h_1) = 0.15(632.63 - 269.56) =$ **54.46** Btu

### 8.155E

One pound-mass of water at 600 F expands against a piston in a cylinder until it reaches ambient pressure, 14.7 lbf/in.<sup>2</sup>, at which point the water has a quality of 90%. It may be assumed that the expansion is reversible and adiabatic.

- a. What was the initial pressure in the cylinder?
- b. How much work is done by the water?

### Solution:

C.V. Water. Process: Rev., Q = 0Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -{}_1W_2$ Entropy Eq.8.3:  $m(s_2 - s_1) = \int dQ/T$ Process: Adiabatic Q = 0 and reversible  $=> s_2 = s_1$ State 2:  $P_2 = 14.7 \text{ lbf/in}^2$ ,  $x_2 = 0.90$  from Table F.7.1  $s_2 = 0.3121 + 0.9 \times 1.4446 = 1.6123 \text{ Btu/lbm R}$   $u_2 = 180.1 + 0.9 \times 897.5 = 987.9 \text{ Btu/lbm}$ State 1 Table F.7.2: at  $T_1 = 600 \text{ F}$ ,  $s_1 = s_2$ 

$$\Rightarrow$$
 P<sub>1</sub> = **335 lbf/in<sup>2</sup>** u<sub>1</sub> = 1201.2 Btu/lbm

From the energy equation

 ${}_{1}W_{2} = m(u_{1} - u_{2}) = 1(1201.2 - 987.9) = 213.3 \text{ Btu}$ 



### 8.156E

A closed tank, V = 0.35 ft<sup>3</sup>, containing 10 lbm of water initially at 77 F is heated to 350 F by a heat pump that is receiving heat from the surroundings at 77 F. Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

1:  $v_1 = V/m = 0.35/10 = 0.035 \text{ ft}^3/\text{lbm} \implies x_1 = 2.692 \times 10^{-5}$ 

 $u_1 = 45.11$  Btu/lbm,  $s_1 = 0.08779$  Btu/lbm R

Continuity eq. (same mass) and constant volume fixes v<sub>2</sub>

State 2:  $T_2$ ,  $v_2 = v_1 \implies x_2 = (0.035 - 0.01799) / 3.3279 = 0.00511$ 

 $u_2 = 321.35 + 0.00511 \times 788.45 = 325.38$  Btu/lbm

 $s_2 = 0.5033 + 0.00511 \times 1.076 = 0.5088$  Btu/lbm R

Energy eq. has zero work, thus provides heat transfer as  $_{1}Q_{2} = m(u_{2} - u_{1}) = 10(325.38 - 45.11) = 2802.7$  Btu

Entropy equation for the total control volume gives for a reversible process:

 $m(s_2 - s_1) = Q_L/T_0$   $\Rightarrow Q_L = mT_0(s_2 - s_1)$  = 10(77 + 459.67)(0.5088 - 0.08779)= 2259.4 Btu



and the energy equation for the heat pump gives

 $W_{HP} = {}_{1}Q_{2} - Q_{L} = 2802.7 - 2259.4 = 543.3 \text{ Btu}$ 

#### 8.157E

A cylinder containing R-134a at 60 F, 30 lbf/in.<sup>2</sup>, has an initial volume of 1 ft<sup>3</sup>. A piston compresses the R-134a in a reversible, isothermal process until it reaches the saturated vapor state. Calculate the required work and heat transfer to accomplish this process.

Solution:

C.V. R-134a. Continuity Eq.:  $m_2 = m_1 = m$ ; Energy Eq.:5.11  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.8.3:  $m(s_2 - s_1) = \int dQ/T + {}_1S_2 gen$ Process: T = constant, reversible so  ${}_1S_2 gen = 0$ State 1: (T, P) Table F.10.2  $u_1 = 168.41$  Btu/lbm,  $s_1 = 0.4321$  Btu/lbm R  $m = V/v_1 = 1/1.7367 = 0.5758$  lbm



As T is constant we can find Q by integration as

 $_{1}Q_{2} = \int Tds = mT(s_{2} - s_{1}) = 0.5758 \times 519.7 \times (0.4108 - 0.4321) = -6.374$  Btu The work is then from the energy equation  $_{1}W_{2} = m(u_{1} - u_{2}) + _{1}Q_{2} = 0.5758 \times (168.41 - 166.28) - 6.374 = -5.15$  Btu

#### 8.158E

A rigid, insulated vessel contains superheated vapor steam at 450 lbf/in.<sup>2</sup>, 700 F. A valve on the vessel is opened, allowing steam to escape. It may be assumed that the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only) Cont.Eq.:  $m_2 = m_1 = m$ ; Energy Eq.:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.:  $m(s_2 - s_1) = \int dQ/T + {}_1S_2 g_{en} = 0 + 0$ 



State 1: Table F.7.2  $v_1 = 1.458 \text{ ft}^3/\text{lbm}, s_1 = 1.6248 \text{ Btu/lbm R}$ State 2: Table F.7.1  $s_2 = s_1 = 1.6248 \text{ Btu/lbm R} = s_g \text{ at } P_2$   $\Rightarrow P_2 = 76.67 \text{ lbf/in}^2, v_2 = v_g = 5.703 \text{ ft}^3/\text{lbm}$  $\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{1.458}{5.703} = 0.744$ 

### **Entropy Generation**

#### 8.159E

An insulated cylinder/piston contains R-134a at 150 lbf/in.<sup>2</sup>, 120 F, with a volume of 3.5 ft<sup>3</sup>. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 15 lbf/in.<sup>2</sup>. It is claimed that the R-134a does 180 Btu of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume Q = 0.

State 1: Table F.10.2,  $v_1 = 0.3332 \text{ ft}^3/\text{lbm}$ ,  $u_1 = 175.33 \text{ Btu/lbm}$ ,

 $s_1 = 0.41586$  Btu/lbm R,  $m = V_1/v_1 = 3.5/0.3332 = 10.504$  lbm

Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = \emptyset - 180 \Rightarrow$ 

 $u_2 = u_1 - \frac{1}{1}W_2/m = 158.194$  Btu/lbm

State 2:  $P_2$ ,  $u_2 \implies$  Table F.10.2:  $T_2 = -2$  F ;  $s_2 = 0.422$  Btu/lbm R

Entropy Eq.8.14:  $m(s_2 - s_1) = \int dQ/T + {}_{1}S_{2,gen} = {}_{1}S_{2,gen}$ 

 ${}_{1}S_{2,gen} = m(s_2 - s_1) = 10.504 (0.422 - 0.41586) = 0.0645 \text{ Btu/R}$ 

This is possible since  $_1S_{2 \text{ gen}} > \emptyset$ 



#### 8.160E

A mass and atmosphere loaded piston/cylinder contains 4 lbm of water at 500 lbf/in.<sup>2</sup>, 200 F. Heat is added from a reservoir at 1200 F to the water until it reaches 1200 F. Find the work, heat transfer, and total entropy production for the system and surroundings.

Solution:

C.V. Water out to surroundings at 1200 F. This is a control mass. Energy Eq.5.11:  $U_2 - U_1 = {}_1Q_2 - {}_1W_2$ Entropy Eq.8.14:  $m(s_2 - s_1) = \int dQ/T + {}_1S_{2 \text{ gen}} = {}_1Q_2/T_{res} + {}_1S_{2 \text{ gen}}$ Process:  $P = \text{constant so} \qquad {}_1W_2 = P(V_2 - V_1) = mP(v_2 - v_1)$ State 1: Table F.7.3,  $v_1 = 0.01661 \text{ ft}^3/\text{lbm}$   $h_1 = 169.18 \text{ Btu/lbm}$ ,  $s_1 = 0.2934 \text{ Btu/lbm R}$ State 2: Table F.7.2,  $v_2 = 1.9518 \text{ ft}^3/\text{lbm}$ ,  $h_2 = 1629.8 \text{ Btu/lbm}$ ,  $s_2 = 1.8071 \text{ Btu/lbm R}$ 



Work is found from the process (area in P-V diagram)

$$_{1}W_{2} = mP(v_{2} - v_{1}) = 4 \times 500(1.9518 - 0.01661) \frac{144}{778} = 716.37 \text{ Btu}$$

The heat transfer from the energy equation is

$${}_{1}Q_{2} = U_{2} - U_{1} + {}_{1}W_{2} = m(u_{2} - u_{1}) + mP(v_{2} - v_{1}) = m(h_{2} - h_{1})$$
  
 ${}_{1}Q_{2} = 4(1629.8 - 169.18) = 5842.48 Btu$ 

Entropy generation from entropy equation (or Eq.8.18)

$${}_{1}S_{2 \text{ gen}} = m(s_{2} - s_{1}) - \frac{1Q_{2}}{T_{\text{res}}} = 4(1.8071 - 0.2934) - \frac{5842.48}{1659.67} = 2.535 \text{ Btu/R}$$

### 8.161**E**

A 1 gallon jug of milk at 75 F is placed in your refrigerator where it is cooled down to the refrigerators inside temperature of 40 F. Assume the milk has the properties of liquid water and find the entropy generated in the cooling process.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.:  $m_2 = m_1 = m$ ;

Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ 

Entropy Eq.8.14:  $m(s_2 - s_1) = \int dQ/T + {}_1S_{2 \text{ gen}}$ 

State 1: Table F.7.1:  $v_1 \cong v_f = 0.01606 \text{ ft}^3/\text{lbm}, h_1 = h_f = 43.085 \text{ Btu/lbm};$  $s_f = 0.08395 \text{ Btu/lbm R}$ 

State 2: Table F.7.1:  $h_2 = h_f = 8.01$  Btu/lbm,  $s_2 = s_f = 0.0162$  Btu/lbm R Process: P = constant = 14.7 psia  $\implies {}_1W_2 = mP(v_2 - v_1)$ 

 $V_1 = 1 \text{ Gal} = 231 \text{ in}^3 \implies m = 231 / 0.01606 \times 12^3 = 8.324 \text{ lbm}$ 

Substitute the work into the energy equation and solve for the heat transfer

 $_{1}Q_{2} = m(h_{2} - h_{1}) = 8.324 (8.01 - 43.085) = -292 Btu$ 

The entropy equation gives the generation as

 ${}_{1}S_{2 \text{ gen}} = m(s_{2} - s_{1}) - {}_{1}Q_{2}/T_{\text{refrig}}$ = 8.324 (0.0162 - 0.08395) - (-292 / 500) = - 0.564 + 0.584 = **0.02 Btu/R** 



#### 8.162E

A cylinder/piston contains water at 30 lbf/in.<sup>2</sup>, 400 F with a volume of 1 ft<sup>3</sup>. The piston is moved slowly, compressing the water to a pressure of 120 lbf/in.<sup>2</sup>. The loading on the piston is such that the product PV is a constant. Assuming that the room temperature is 70 F, show that this process does not violate the second law.

Solution: C.V.: Water + cylinder out to room at 70 F Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.8.14:  $m(s_2 - s_1) = {}_1Q_2 / T_{room} + {}_1S_2 gen$ Process: PV = constant = Pmv  $\Rightarrow v_2 = P_1v_1/P_2$   ${}_1w_2 = \int Pdv = P_1v_1 \ln(v_2/v_1)$ State 1: Table B.1.3,  $v_1 = 16.891 \text{ ft}^3/\text{lbm}$ ,  $u_1 = 1144 \text{ Btu/lbm}$ ,  $s_1 = 1.7936 \text{ Btu/lbm R}$ State 2:  $P_2$ ,  $v_2 = P_1v_1/P_2 = 30 \times 16.891/120 = 4.223 \text{ ft}^3/\text{lbm}$ Table F.7.3:  $T_2 = 425.4 \text{ F}$ ,  $u_2 = 1144.4 \text{ Btu/lbm}$ ,  $s_2 = 1.6445 \text{ Btu/lbmR}$   ${}_1w_2 = 30 \times 16.891 \times \frac{144}{778} \ln(\frac{4.223}{16.891}) = -130.0 \text{ Btu}$  ${}_1q_2 = u_2 - u_1 + {}_1w_2 = 1144.4 - 1144 - 130 = -129.6 \text{ Btu/lbm}$ 

 $_{1}s_{2,gen} = s_2 - s_1 - \frac{192}{T_{room}} = 1.6445 - 1.7936 + \frac{129.6}{529.67}$ = 0.0956 Btu/lbm R > Ø satisfy 2<sup>nd</sup> law.

#### 8.163E

One pound mass of ammonia (NH<sub>3</sub>) is contained in a linear spring-loaded piston/cylinder as saturated liquid at 0 F. Heat is added from a reservoir at 225 F until a final condition of 125 lbf/in.<sup>2</sup>, 160 F is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH<sub>3</sub> out to the reservoir. Continuity Eq.:  $m_2 = m_1 = m$ Energy Eq.5.11:  $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.8.14:  $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{res} + {}_1S_{2,gen}$ Process: P = A + BV linear in V =>  ${}_1W_2 = \int P \, dV = \frac{1}{2} (P_1 + P_2)(V_2 - V_1) = \frac{1}{2} (P_1 + P_2)m(v_2 - v_1)$ State 1: Table F.8.1  $P_1 = 30.4 \text{ psia},$   $v_1 = 0.0242 \text{ ft}^3/\text{lbm}$   $u_1 = 42.5 \text{ Btu/lbm},$  $s_1 = 0.0967 \text{ Btu/lbm R}$ 

State 2: Table F.8.2 sup. vap.

 $v_{2} = 2.9574 \text{ ft}^{3}/\text{lbm}, \quad u_{2} = 686.9 - 125 \times 2.9574 \times 144/778 = 618.5 \text{ Btu/lbm},$   $s_{2} = 1.3178 \text{ Btu/lbm R}$   $1W_{2} = \frac{1}{2} (30.4 + 125)1(2.9574 - 0.0242) \times 144/778 = 42.2 \text{ Btu}$   $1Q_{2} = m(u_{2} - u_{1}) + 1W_{2} = 1(618.5 - 42.5) + 42.2 = 618.2 \text{ Btu}$  $S_{gen} = m(s_{2} - s_{1}) - 1Q_{2}/T_{res} = 1(1.3178 - 0.0967) - \frac{618.2}{684.7} = 0.318 \text{ Btu/R}$ 

## Entropy of a Liquid or Solid

#### 8.164E

A foundry form box with 50 lbm of 400 F hot sand is dumped into a bucket with 2  $ft^3$  water at 60 F. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the process.

C.V. Sand and water, P = const.

Energy Eq.: 
$$m_{sand}(u_2 - u_1)_{sand} + m_{H_2O}(u_2 - u_1)_{H_2O} = -P(V_2 - V_1)$$
  
 $\Rightarrow m_{sand}\Delta h_{sand} + m_{H_2O}\Delta h_{H_2O} = \emptyset, \quad m_{H_2O} = \frac{2}{0.016035} = 124.73 \text{ lbm}$   
 $50 \times 0.19(T_2 - 400) + 124.73 \times 1.0(T_2 - 60) = \emptyset, \quad T_2 = 84 \text{ F}$   
 $\Delta S = 50 \times 0.19 \times \ln\left(\frac{544}{860}\right) + 124.73 \times 1.0 \times \ln\left(\frac{544}{520}\right) = 1.293 \text{ Btu/R}$ 



### 8.165E

Four pounds of liquid lead at 900 F are poured into a form. It then cools at constant pressure down to room temperature at 68 F as heat is transferred to the room. The melting point of lead is 620 F and the enthalpy change between the phases  $h_{if}$  is 10.6 Btu/lbm. The specific heats are in Table F.2 and F.3. Calculate the net entropy change for this process.

Solution:

C.V. Lead, constant pressure process

 $m_{Pb}(u_2 - u_1)_{Pb} = {}_1Q_2 - P(V_2 - V_1)$ 

We need to find changes in enthalpy (u + Pv) for each phase separately and then add the enthalpy change for the phase change.

 $C_{lig} = 0.038$  Btu/lbm R,  $C_{sol} = 0.031$  Btu/lbm R

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid

Cooling of the solid to the final temperature

$${}_{1}Q_{2} = m_{Pb}(h_{2} - h_{1}) = m_{Pb}(h_{2} - h_{620,sol} - h_{if} + h_{620,f} - h_{900})$$
  
= 4 × [0.031 × (68 - 620) - 10.6 + 0.038 × (620 - 900)]  
= -68.45 - 42.4 - 42.56 = -153.4 Btu

 $\Delta S_{CV} = m_{Pb} [C_{p \text{ sol}} \ln(T_2/1079.7) - (h_{if}/1079.7) + C_{P \text{ lig}} \ln(1079.7/T_1)]$ 

$$= 4 \times \left[ 0.031 \ln \frac{527.7}{1079.7} - \frac{10.6}{1079.7} + 0.038 \ln \frac{1079.6}{1359.7} \right] = -0.163 \text{ Btu/R}$$

$$\Delta S_{SUR} = -1Q_2/T_0 = 153.4/527.6 = 0.2908 \text{ Btu/R}$$

The net entropy change from Eq.8.18 is equivalent to total entropy generation

 $\Delta S_{net} = \Delta S_{CV} + \Delta S_{SUR} = 0.1277 \text{ Btu/R}$ 



#### 8.166E

A hollow steel sphere with a 2-ft inside diameter and a 0.1-in. thick wall contains water at 300 lbf/in.<sup>2</sup>, 500 F. The system (steel plus water) cools to the ambient temperature, 90 F. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water. This is a control mass.

```
Energy Eq.: U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_{H_2O}(u_2 - u_1) + {}_{m_{steel}}(u_2 - u_1)

Process: V = constant \implies {}_1W_2 = 0

V_{steel} = \frac{\pi}{6} [2.0083^3 - 2^3] = 0.0526 \text{ ft}^3

m_{steel} = (\rho V)_{steel} = 490 \times 0.0526 = 25.763 \text{ lbm}

V_{H_2O} = \pi/6 \times 2^3 = 4.189 \text{ ft}^3, m_{H_2O} = V/v = 2.372 \text{ lbm}

v_2 = v_1 = 1.7662 = 0.016099 + x_2 \times 467.7 \implies x_2 = 3.74 \times 10^{-3}

u_2 = 61.745 \text{ Btu/lbm}, s_2 = 0.1187 \text{ Btu/lbm R}

1Q_2 = \Delta U_{steel} + \Delta U_{H_2O} = (mC)_{steel}(T_2 - T_1) + m_{H_2O}(u_2 - u_1)

= 25.763 \times 0.107(90-500) + 2.372(61.74 - 1159.5)

= -1130 - 2603.9 = -3734 \text{ Btu}

\Delta S_{SYS} = \Delta S_{STEEL} + \Delta S_{H_2O} = 25.763 \times 0.107 \times \ln(550/960)

+ 2.372(0.1187 - 1.5701) = -4.979 \text{ Btu/R}

\Delta S_{SUR} = -Q_{12}/T_{SUR} = 3734/549.67 = 6.793 \text{ Btu/R}
```



#### **Entropy of Ideal Gases**

#### 8.167E

Oxygen gas in a piston/cylinder at 500 R and 1 atm with a volume of 1  $\text{ft}^3$  is compressed in a reversible adiabatic process to a final temperature of 1000 R. Find the final pressure and volume using constant heat capacity from Table F.4.

Solution:

C.V. Air. Assume a reversible, adiabatic process.					
Energy Eq.5.11:	u <sub>2</sub> -	$u_1 = 0 - 1$	1w2;		
Entropy Eq.8.14:	s <sub>2</sub> -	$s_1 = \int dq/r$	$\Gamma + {}_{1}s_{2 \text{ gen}} = 0$	0	
Process:	Adiabatic	$_{1}q_{2} = 0$	Reversible	$_1$ s <sub>2 gen</sub> = 0	
Properties:	Table F.4:	k = 1.3	393		

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.8.32

$$P_2 = P_1(T_2 / T_1)^{\frac{k}{k-1}} = 14.7 (1000/500)^{3.5445} = 171.5 \text{ psia}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.8.33

$$V_2 = V_1 (T_2 / T_1)^{\frac{1}{1-k}} = 1 \times \left(\frac{1000}{500}\right)^{\frac{1}{1-1.393}} = 0.171 \text{ ft}^3$$



#### 8.168E

Oxygen gas in a piston/cylinder at 500 R and 1 atm with a volume of 1  $\text{ft}^3$  is compressed in a reversible adiabatic process to a final temperature of 1000 R. Find the final pressure and volume using Table F.6.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.5.11:  $u_2 - u_1 = 0 - {}_1w_2$ ; Entropy Eq.8.14:  $s_2 - s_1 = \int dq/T + {}_1s_2 {}_{gen} = \emptyset$ Process: Adiabatic  ${}_1q_2 = 0$  Reversible  ${}_1s_2 {}_{gen} = 0$ 

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.8.28

$$s_{T2}^{\circ} - s_{T1}^{\circ} = R \ln \frac{P_2}{P_1}$$

Properties: Table F.6:  $s_{T1}^{\circ} = 48.4185/31.999 = 1.5131$  Btu/lbm R,

$$s_{T2}^{\circ} = 53.475/31.999 = 1.6711$$
 Btu/lbm R  
 $\frac{P_2}{P_1} = \exp\left[(s_{T2}^{\circ} - s_{T1}^{\circ})/R\right] = \exp\left(\frac{1.6711 - 1.5131}{48.28/778}\right) = 12.757$ 

$$P_2 = 14.7 \times 12.757 = 187.5$$
 psia

Ideal gas law:  $P_1V_1 = mRT_1$  and  $P_2V_2 = mRT_2$ Take the ratio of these so mR drops out to give

$$V_2 = V_1 \times (T_2 / T_1) \times (P_1 / P_2) = 1 \times (\frac{1000}{500}) \times (\frac{14.7}{187.5}) = 0.157 \text{ ft}^3$$



#### 8.169E

A handheld pump for a bicycle has a volume of 2 in.<sup>3</sup> when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so an air pressure of 45 lbf/in.<sup>2</sup> is obtained. The outside atmosphere is at  $P_0$ ,  $T_0$ . Consider two cases: (1) it is done quickly (~1 s), and (2) it is done slowly (~1 h).

a. State assumptions about the process for each case.

b. Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

State 1:  $P_0, T_0$  State 2: 45 lbf/in.<sup>2</sup>, ?

One piece of information must resolve the ? for a state 2 property. Case I) Quickly means no time for heat transfer

Q = 0, so a reversible adiabatic compression.

$$u_2 - u_1 = -1 w_2$$
;  $s_2 - s_1 = \int dq/T + 1 s_2 gen = \emptyset$ 

With constant s and constant heat capacity we use Eq.8.32

$$T_2 = T_1 (P_2 / P_1)^{\frac{k-1}{k}} = 536.7 \left(\frac{45}{14.696}\right)^{\frac{0.4}{1.4}} = 738.9 R$$

Use ideal gas law PV = mRT at both states so ratio gives

$$=$$
 V<sub>2</sub> = P<sub>1</sub>V<sub>1</sub>T<sub>2</sub>/T<sub>1</sub>P<sub>2</sub> = **0.899 in<sup>3</sup>**

Case II) Slowly, time for heat transfer so  $T = constant = T_0$ .

The process is then a reversible isothermal compression.

$$T_2 = T_0 = 536.7 \text{ R}$$
 =>  $V_2 = V_1 P_1 / P_2 = 0.653 \text{ in}^3$ 



#### 8.170E

A piston/cylinder contains air at 2500 R, 2200 lbf/in.<sup>2</sup>, with  $V_1 = 1$  in.<sup>3</sup>,  $A_{cyl} = 1$  in.<sup>2</sup> as shown in Fig. P8.95. The piston is released and just before the piston exits the end of the cylinder the pressure inside is 30 lbf/in.<sup>2</sup>. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, Q = 0. Continuity Eq.:  $m_2 = m_1 = m$ , Energy Eq.5.11:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -{}_1W_2$ Entropy Eq.8.14:  $m(s_2 - s_1) = \int dQ/T + {}_1S_{2 \text{ gen}} = 0 + {}_1S_{2 \text{ gen}}$ State 1:  $(T_1, P_1)$  State 2:  $(P_2, ?)$ 

So one piece of information is needed for the ?, assume reversible process.  $_1S_{2 \text{ gen}} = 0 \implies s_2 - s_1 = 0$ 

State 1: Table F.5:  $u_1 = 474.33$  Btu/lbm,  $s_{T1}^\circ = 2.03391$  Btu/lbm R

m = P<sub>1</sub>V<sub>1</sub>/RT<sub>1</sub> = 
$$\frac{2200 \times 1.0}{53.34 \times 2500 \times 12}$$
 = 1.375 × 10<sup>-3</sup> lbm

State 2: P<sub>2</sub> and from Entropy eq.:  $s_2 = s_1$  so from Eq.8.28

$$s_{T2}^{\circ} = s_{T1}^{\circ} + R \ln \frac{P_2}{P_1} = 2.03391 + \frac{53.34}{778} \ln(\frac{30}{2200}) = 1.73944 \text{ Btu/lbm R}$$

Now interpolate in Table F.5 to get T<sub>2</sub>

$$T_{2} = 840 + 40 (1.73944 - 1.73463)/(1.74653 - 1.73463) = 816.2 \text{ R}$$

$$u_{2} = 137.099 + (144.114 - 137.099) \ 0.404 = 139.93 \text{ Btu/lbm}$$

$$V_{2} = V_{1} \frac{T_{2} P_{1}}{T_{1} P_{2}} = \frac{1 \times 816.2 \times 2200}{2500 \times 30} = 23.94 \text{ in}^{3}$$

$$\Rightarrow L_{2} = V_{2} / A_{cyl} = 23.94/1 = 23.94 \text{ in}$$

$${}_{1}W_{2} = m(u_{1} - u_{2}) = 1.375 \times 10^{-3}(474.33 - 139.93) = 0.46 \text{ Btu}$$

#### 8.171E

A 25-ft<sup>3</sup> insulated, rigid tank contains air at 110 lbf/in.<sup>2</sup>, 75 F. A valve on the tank is opened, and the pressure inside quickly drops to 15 lbf/in.<sup>2</sup>, at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

C.V.: Air remaining inside tank, m<sub>2</sub>.

Cont.Eq.:  $m_2 = m$ ; Energy Eq.:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ Entropy Eq.:  $m(s_2 - s_1) = \int dQ/T + {}_1S_2 g_{en} = 0 + 0$ 



$$s_2 = s_1 \rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 535 (15/110)^{0.286} = 302.6 \text{ R}$$
  

$$m_1 = P_1 V/RT_1 = 110 \times 144 \times 25 / (53.34 \times 535) = 13.88 \text{ lbm}$$
  

$$m_2 = P_2 V/RT_2 = 15 \times 144 \times 25 / (53.34 \times 302.6) = 3.35 \text{ lbm}$$
  

$$m_e = m_1 - m_2 = 10.53 \text{ lbm}$$

#### 8.172E

A rigid container with volume 7  $\text{ft}^3$  is divided into two equal volumes by a partition. Both sides contain nitrogen, one side is at 300 lbf/in.<sup>2</sup>, 400 F, and the other at 30 lbf/in.<sup>2</sup>, 200 F. The partition ruptures, and the nitrogen comes to a uniform state at 160 F. Assume the temperature of the surroundings is 68 F, determine the work done and the net entropy change for the process.

Solution:

C.V.: A + B Control mass no change in volume  $\Rightarrow {}_{1}W_{2} = 0$   $m_{A1} = P_{A1}V_{A1}/RT_{A1} = 300 \times 144 \times 3.5 / (55.15 \times 859.7) = 3.189 \text{ lbm}$   $m_{B1} = P_{B1}V_{B1}/RT_{B1} = 30 \times 144 \times 3.5 / (55.15 \times 659.7) = 0.416 \text{ lbm}$   $P_{2} = m_{TOT}RT_{2}/V_{TOT} = 3.605 \times 55.15 \times 619.7/(144 \times 7) = 122.2 \text{ lbf/in}^{2}$   $\Delta S_{SYST} = 3.189 [0.249 \ln \frac{619.7}{859.7} - \frac{55.15}{778} \ln \frac{122.2}{300}]$   $+ 0.416 [0.249 \ln \frac{619.7}{659.7} - \frac{55.15}{778} \ln \frac{122.2}{30}]$  = -0.0569 - 0.0479 = -0.1048 Btu/R  $1Q_{2} = m_{A1}(u_{2} - u_{1}) + m_{B1}(u_{2} - u_{1})$   $= 3.189 \times 0.178(160 - 400) + 0.416 \times 0.178(160 - 200) = -139.2 \text{ Btu}$  $\Delta S_{SURR} = -1Q_{2}/T_{0} = 139.2 / 527.7 = +0.2638 \text{ Btu/R}$ 

#### 8.173E

Nitrogen at 90 lbf/in.<sup>2</sup>, 260 F is in a 20 ft<sup>3</sup> insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 20 ft<sup>3</sup>. The valve is opened and the nitrogen fills both tanks. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

C.V. Both tanks + pipe + valve. Insulated : Q = 0, Rigid: W = 0  $m(u_2 - u_1) = 0 - 0 \implies u_2 = u_1 = u_{a1}$ Entropy Eq.:  $m(s_2 - s_1) = \int dQ/T + {}_1S_{2 \text{ gen}} = {}_1S_{2 \text{ gen}}$ State 1:  $P_1$ ,  $T_1$ ,  $V_a \implies$  Ideal gas  $m = PV/RT = (90 \times 20 \times 144)/(55.15 \times 720) = 6.528 \text{ lbm}$ 2:  $V_2 = V_a + V_b$ ; uniform final state  $v_2 = V_2 / m$ ;  $u_2 = u_{a1}$ 



Ideal gas u (T) => 
$$u_2 = u_{a1}$$
 =>  $T_2 = T_{a1}$  = **720 R**  
 $P_2 = mR T_2 / V_2 = (V_1 / V_2) P_1 = \frac{1}{2} \times 90 = 45 \text{ lbf/in.}^2$   
 $S_{gen} = m(s_2 - s_1) = m (s_{T2}^o - s_{T1}^o - R \ln (P_2 / P_1))$   
 $= m (0 - R \ln (P_2 / P_1) = -6.528 \times 55.15 \times (1/778) \ln \frac{1}{2} = 0.32 \text{ Btu/R}$ 

Irreversible due to unrestrained expansion in valve  $P \downarrow$  but no work out.

If not a uniform final state then flow until  $P_{2b} = P_{2a}$  and value is closed. Assume no Q between A and B

 $m_{a2} + m_{b2} = m_{a1}$ ;  $m_{a2}v_{a2} + m_{b2}v_{b2} = m_{a1}v_{a1}$  $m_{a2} s_{a2} + m_{b2} s_{b2} - m_{a1} s_{a1} = 0 + {}_{1}S_{2 \text{ gen}}$ Now we must assume  $m_{a2}$  went through rev adiabatic expansion 1)  $V_2 = m_{a2} v_{a2} + m_{b2} v_{b2}$ ; 2)  $P_{b2} = P_{a2}$ ; 3)  $s_{a2} = s_{a1}$ ; 4) Energy eqs. 4 Eqs 4 unknowns : P<sub>2</sub>,  $T_{a2}$ ,  $T_{b2}$ ,  $x = m_{a2} / m_{a1}$  $V_2 / m_{a1} = x v_{a2} + (1 - x) v_{b2} = x \times (R T_{a2} / P_2) + (1 - x) (R T_{b2} / P_2)$  $m_{a2}(u_{a2} - u_{a1}) + m_{b2}(u_{b2} - u_{a1}) = 0$  $x C_v (T_{a2} - T_{a1}) + (1 - x) (T_{b2} - T_{a1}) C_v = 0$  $x T_{a2} + (1 - x)T_{b2} = T_{a1}$  $P_2 V_2 / m_{a1} R = x T_{a2} + (1 - x) T_{b2} = T_{a1}$  $P_2 = m_{a1} R T_{a1} / V_2 = m_{a1} R T_{a1} / 2V_{a1} = \frac{1}{2} P_{a1} = 45 \text{ lbf/in.}^2$  $s_{a2} = s_{a1} \implies T_{a2} = T_{a1} (P_2 / P_{a1})^{k-1/k} = 720 \times (1/2)^{0.2857} = 590.6 \text{ R}$ Now we have final state in A  $v_{a2} = R T_{a2} / P_2 = 5.0265$ ;  $m_{a2} = V_a / v_{a2} = 3.979$  lbm

 $x = m_{a2} / m_{a1} = 0.6095$  $m_{b2} = m_{a1} - m_{a2} = 2.549$  lbm Substitute into energy equation

 $T_{h2} = (T_{a1} - x T_{a2}) / (1 - x) = 922 R$  $_{1}S_{2 \text{ gen}} = m_{b2} (s_{b2} - s_{a1}) = m_{b2} (C_{p} \ln (T_{b2} / T_{a1}) - R \ln (P_{2} / P_{a1}))$  $= 2.549 [0.249 \ln (922/720) - (55.15/778) \ln (1/2)]$ = 0.2822 Btu/R

#### **Polytropic Processes**

#### 8.174E

Helium in a piston/cylinder at 20°C, 100 kPa is brought to 400 K in a reversible polytropic process with exponent n = 1.25. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

#### Solution:

C.V. Helium, control mass.  $C_v = 0.744$  Btu/lbm R, R = 386 ft lbf/ lbm R Process  $Pv^n = C$  & Pv = RT  $=> Tv^{n-1} = C$   $T_1 = 70 + 460 = 530$  R,  $T_2 = 720$  R  $T_1v^{n-1} = T_2v^{n-1} => v_2/v_1 = (T_1/T_2)^{1/n-1} = 0.2936$   $P_2/P_1 = (v_1/v_2)^n = 4.63 => P_2 = 69.4$  lbf/in.<sup>2</sup>  $_1w_2 = \int P dv = \int C v^{-n} dv = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1)$   $= \frac{386}{778 \times (-0.25)} (720 - 530) = -377$  Btu/lbm  $_1q_2 = u_2 - u_1 + _1w_2 = C_v (T_2 - T_1) + _1w_2$ = 0.744(720 - 530) + (-377) = -235.6 Btu/lbm

#### 8.175E

A cylinder/piston contains air at ambient conditions, 14.7 lbf/in.<sup>2</sup> and 70 F with a volume of 10 ft<sup>3</sup>. The air is compressed to 100 lbf/in.<sup>2</sup> in a reversible polytropic process with exponent, n = 1.2, after which it is expanded back to 14.7 lbf/in.<sup>2</sup> in a reversible adiabatic process.

- a. Show the two processes in P-v and T-s diagrams.
- b. Determine the final temperature and the net work.
- c. What is the potential refrigeration capacity (in British thermal units) of the air at the final state?



b)  $m = P_1 V_1 / RT_1 = 14.7 \times 144 \times 10/(53.34 \times 529.7) = 0.7492 \text{ lbm}$ 

$$T_{2} = T_{1}(P_{2}/P_{1})^{\frac{n-1}{n}} = 529.7 \left(\frac{100}{14.7}\right)^{0.167} = 729.6 \text{ R}$$

$${}_{1}w_{2} = \int_{1}^{2} Pdv = \frac{P_{2}v_{2} - P_{1}v_{1}}{1 - n} = \frac{R(T_{2} - T_{1})}{1 - n} = \frac{53.34(729.6 - 529.7)}{778(1 - 1.20)}$$

$$= -68.5 \text{ Btu/lbm}$$

$$T_{3} = T_{2}(P_{3}/P_{2})^{\frac{k-1}{k}} = 729.7 \left(\frac{14.7}{100}\right)^{0.286} = 421.6 \text{ R}$$

$${}_{2}w_{3} = C_{V0}(T_{2} - T_{3}) = 0.171(729.6 - 421.6) = +52.7 \text{ Btu/lbm}$$

$$w_{\text{NET}} = 0.7492(-68.5 + 52.7) = -11.8$$
 Btu

c) Refrigeration: warm to  $T_0$  at const P,

$$Q_{31} = mC_{P0}(T_1 - T_3) = 0.7492 \times 0.24(529.7 - 421.6) = 19.4 Btu$$

#### 8.176E

A cylinder/piston contains carbon dioxide at 150 lbf/in.<sup>2</sup>, 600 F with a volume of 7 ft<sup>3</sup>. The total external force acting on the piston is proportional to  $V^3$ . This system is allowed to cool to room temperature, 70 F. What is the total entropy generation for the process?

State 1: 
$$P_1 = 150 \text{ lbf/in}^2$$
,  $T_1 = 600 \text{ F} = 1060 \text{ R}$ ,  $V_1 = 7 \text{ ft}^3$  Ideal gas  
 $m = \frac{P_1 V_1}{RT_1} = \frac{150 \times 144 \times 7}{35.10 \times 1060} = 4.064 \text{ lbm}$ 

Process:  $P = CV^3$  or  $PV^{-3} = const.$  polytropic with n = -3.

$$P_{2} = P_{1}(T_{2}/T_{1})^{\frac{n}{n-1}} = 150 \left(\frac{530}{1060}\right)^{0.75} = 89.2 \text{ lbf/in}^{2}$$

$$\& V_{2} = V_{1}(T_{1}/T_{2})^{\frac{1}{n-1}} = V_{1} \times \frac{P_{1}}{P_{2}} \times \frac{T_{2}}{T_{1}} = 7 \times \frac{150}{89.2} \times \frac{530}{1060} = 5.886$$

$${}_{1}W_{2} = \int PdV = \frac{P_{2}V_{2} - P_{1}V_{1}}{1 - n} = \frac{(89.2 \times 5.886 - 150 \times 7)}{1 + 3} \times \frac{144}{778} = -24.3 \text{ Btu}$$

$${}_{1}Q_{2} = 4.064 \times 0.158 \times (530 - 1060) - 24.3 = -346.6 \text{ Btu}$$

$$\Delta S_{\text{SYST}} = 4.064 \times \left[ 0.203 \times \ln\left(\frac{530}{1060}\right) - \frac{35.10}{778} \ln\left(\frac{89.2}{150}\right) \right] = -0.4765 \text{ Btu/R}$$

$$\Delta S_{\text{SURR}} = -1Q_{2}/T_{\text{SURR}} = 364.6 / 530 = +0.6879 \text{ Btu/R}$$

$$\Delta S_{\text{NET}} = +0.2114 \text{ Btu/R}$$



#### **8.177€**

A cylinder/piston contains 4 ft<sup>3</sup> of air at 16 lbf/in.<sup>2</sup>, 77 F. The air is compressed in a reversible polytropic process to a final state of 120 lbf/in.<sup>2</sup>, 400 F. Assume the heat transfer is with the ambient at 77 F and determine the polytropic exponent n and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

Solution:

$$\begin{split} \mathbf{m} &= (P_1 V_1)/(RT_1) = (16 \times 4 \times 144)/(53.34 \times 537) = 0.322 \text{ lbm} \\ T_2/T_1 &= (P_2/P_1)^{\frac{\mathbf{n} \cdot 1}{\mathbf{n}}} \implies \frac{\mathbf{n} \cdot 1}{\mathbf{n}} = \ln(T_2 / T_1) / \ln(P_2 / P_1) = 0.2337 \\ \mathbf{n} &= \mathbf{1.305}, \quad V_2 = V_1(P_1/P_2)^{1/\mathbf{n}} = 4 \times (16/20)1/1.305 = \mathbf{0.854} \text{ ft}^3 \\ \mathbf{1} W_2 &= \int P dV = \frac{P_2 V_2 - P_1 V_1}{1 - \mathbf{n}} \\ &= [(120 \times 0.854 - 16 \times 4) (144 / 778)] / (1 - 1.305) = -23.35 \text{ Btu} / \text{ lbm} \\ \mathbf{1} Q_2 &= \mathbf{m}(\mathbf{u}_2 - \mathbf{u}_1) + \mathbf{1} W_2 = \mathbf{m} C_V (T_2 - T_1) + \mathbf{1} W_2 \\ &= 0.322 \times 0.171 \times (400 - 77) - 23.35 = -5.56 \text{ Btu} / \text{ lbm} \\ \mathbf{s}_2 - \mathbf{s}_1 &= C_p \ln(T_2 / T_1) - R \ln(P_2 / P_1) \\ &= 0.24 \ln (860/537) - (53.34/778) \ln (120/16) = -0.0251 \text{ Btu/lbm R} \\ \mathbf{1} S_{2 \text{ gen}} &= \mathbf{m}(\mathbf{s}_2 - \mathbf{s}_1) - \mathbf{1} Q_2 / T_0 \\ &= 0.322 \times (-0.0251) + (5.56/537) = \mathbf{0.00226} \text{ Btu/R} \end{split}$$

## **Rates or Fluxes of Entropy**

#### 8.178E

A reversible heat pump uses 1 kW of power input to heat a 78 F room, drawing energy from the outside at 60 F. Assume every process is reversible what are the total rates of entropy into the heat pump from the outside and from the heat pump to the room?

Solution:

C.V.TOT.  
Energy Eq.: 
$$\dot{Q}_L + \dot{W} = \dot{Q}_H$$
  
Entropy Eq.:  $\dot{Q}_L - \dot{Q}_H = 0 \Rightarrow \dot{Q}_L = \dot{Q}_H \frac{T_L}{T_H}$   
 $\dot{Q}_L = \dot{Q}_L \frac{T_L}{T_H}$   
 $\dot{Q}_L \frac{T_L}{T_H}$   
 $\dot$ 

Since the process was assumed reversible the two fluxes are the same.

#### 8.179E

A farmer runs a heat pump using 2.5 hp of power input. It keeps a chicken hatchery at a constant 86 F while the room loses 20 Btu/s to the colder outside ambient at 50 F. What is the rate of entropy generated in the heat pump? What is the rate of entropy generated in the heat pump? What is

Solution:

C.V. Hatchery, steady state.

Power:  $\dot{W} = 2.5 \text{ hp} = 2.5 2544.4 / 3600 = 1.767 \text{ Btu/s}$ 

To have steady state at 30°C for the hatchery

Energy Eq.:  $0 = \dot{Q}_{H} - \dot{Q}_{Loss} \implies \dot{Q}_{H} = \dot{Q}_{Loss} = 20 \text{ Btu/s}$ 

C.V. Heat pump, steady state

Energy eq.:  $0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \implies \dot{Q}_L = \dot{Q}_H - \dot{W} = 18.233 \text{ Btu/s}$ Entropy Eq.:  $0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen HP}}$  $\dot{Q}_H = \dot{Q}_H - \dot{W} = 18.233 \text{ Btu}$ 

$$\dot{S}_{\text{gen HP}} = \frac{Q_{\text{H}}}{T_{\text{H}}} - \frac{Q_{\text{L}}}{T_{\text{L}}} = \frac{20}{545.7} - \frac{18.233}{509.7} = 0.000 \ 878 \ \frac{\text{Btu}}{\text{s R}}$$

C.V. From hatchery at 86 F to the ambient 50 F. This is typically the walls and the outer thin boundary layer of air. Through this goes  $\dot{Q}_{Loss}$ .

Entropy Eq.: 
$$0 = \frac{\dot{Q}_{Loss}}{T_{H}} - \frac{\dot{Q}_{Loss}}{T_{amb}} + \dot{S}_{gen walls}$$
$$\dot{S}_{gen walls} = \frac{\dot{Q}_{Loss}}{T_{amb}} - \frac{\dot{Q}_{Loss}}{T_{H}} = \frac{20}{509.7} - \frac{20}{545.7} = 0.00259 \frac{Btu}{s R}$$



#### **Review Problems**

#### 8.180E

A cylinder/piston contains 5 lbm of water at 80 lbf/in.<sup>2</sup>, 1000 F. The piston has cross-sectional area of 1 ft<sup>2</sup> and is restrained by a linear spring with spring constant 60 lbf/in. The setup is allowed to cool down to room temperature due to heat transfer to the room at 70 F. Calculate the total (water and surroundings) change in entropy for the process.

State 1: Table F.7.2  $v_1 = 10.831 \text{ ft}^3/\text{lbm}, u_1 = 1372.3 \text{ btu/lbm},$  $s_1 = 1.9453 \text{ Btu/lbm R}$ 

State 2: T<sub>2</sub> & on line in P-v diagram.

 $P = P_{1} + (k_{s}/A_{cyl}^{2})(V - V_{1})$ Assume state 2 is two-phase,  $=> P_{2} = P_{sat}(T_{2}) = 0.3632 \text{ lbf/in}^{2}$   $v_{2} = v_{1} + (P_{2} - P_{1})A_{cyl}^{2}/\text{mk}_{s} = 10.831 + (0.3632 - 80)1 \times 12/5 \times 60$   $= 7.6455 \text{ ft}^{3}/\text{lbm} = v_{f} + x_{2}v_{fg} = 0.01605 + x_{2} 867.579$   $x_{2} = 0.008793, u_{2} = 38.1 + 0.008793 \times 995.64 = 46.85 \text{ btu/lbm},$   $s_{2} = 0.0746 + 0.008793 \times 1.9896 = 0.0921 \text{ Btu/lbm R}$   $1W_{2} = \frac{1}{2} (P_{1} + P_{2})m(v_{2} - v_{1})$   $= \frac{5}{2}(80 + 0.3632)(7.6455 - 10.831)\frac{144}{778} = -118.46 \text{ Btu}$   $1Q_{2} = m(u_{2} - u_{1}) + 1W_{2} = 5(46.85 - 1372.3) - 118.46 = -6746 \text{ Btu}$   $\Delta S_{tot} = S_{gen tot} = m(s_{2} - s_{1}) - 1Q_{2}/T_{room}$  = 5(0.0921 - 1.9453) + 6746/529.67 = 3.47 Btu/R

#### 8.181E

Water in a piston/cylinder is at 150 lbf/in.<sup>2</sup>, 900 F, as shown in Fig. P8.130. There are two stops, a lower one at which  $V_{min} = 35$  ft<sup>3</sup> and an upper one at  $V_{max} = 105$  ft<sup>3</sup>. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 75 lbf/in.<sup>2</sup>. This setup is now cooled to 210 F by rejecting heat to the surroundings at 70 F. Find the total entropy generated in the process.

#### C.V. Water.

State 1: Table F.7.2  $v_1 = 5.3529 \text{ ft}^3/\text{lbm}, u_1 = 1330.2 \text{ btu/lbm},$  $s_1 = 1.8381 \text{ Btu/lbm}$  $m = V/v_1 = 105/5.353 = 19.615 \text{ lbm}$ 



State 2: 210 F and on line in P-v diagram.

Notice the following:  $v_g(P_{float}) = 5.818 \text{ ft}^3/\text{lbm}$ ,  $v_{bot} = V_{min}/m = 1.7843$   $T_{sat}(P_{float}) = 307.6 \text{ F}$ ,  $T_2 < T_{sat}(P_{float}) \implies V_2 = V_{min}$ State 2: 210 F,  $v_2 = v_{bot} \implies x_2 = (1.7843 - 0.0167)/27.796 = 0.06359$   $u_2 = 178.1 + 0.06359 \times 898.9 = 235.26 \text{ btu/lbm}$ ,  $s_2 = 0.3091 + 0.06359 \times 1.4507 = 0.4014 \text{ btu/lbm R}$   $1W_2 = \int PdV = P_{float}(V_2 - V_1) = 75(35 - 105) \frac{144}{778} = -971.72 \text{ Btu}$   $1Q_2 = m(u_2 - u_1) + 1W_2 = 19.615(235.26 - 1330.2) - 971.72 = -22449 \text{ Btu}$ Take C.V. total out to where we have 70 F:  $m(s_2 - s_1) = 1Q_2/T_0 + S_{gen} \implies$   $S_{gen} = m(s_2 - s_1) - 1Q_2/T_0 = 19.615(0.4014 - 1.8381) + \frac{22449}{529.67}$  $= 14.20 \text{ Btu/R} (= \Delta S_{water} + \Delta S_{sur})$ 

### 8.182E

A cylinder with a linear spring-loaded piston contains carbon dioxide gas at 300  $lbf/in.^2$  with a volume of 2 ft<sup>3</sup>. The device is of aluminum and has a mass of 8 lbm. Everything (Al and gas) is initially at 400 F. By heat transfer the whole system cools to the ambient temperature of 77 F, at which point the gas pressure is 220  $lbf/in.^2$ . Find the total entropy generation for the process.

Solution:

$$\begin{split} &\text{CO}_2: \quad m = P_1 V_1 / RT_1 = 300 \times 2 \times 144 / (35.10 \times 860) = 2.862 \text{ lbm} \\ &V_2 = V_1 (P_1 / P_2) \ (T_2 / T_1) = 2(300 / 220) (537 / 860) = 1.703 \text{ ft}^3 \\ &1W_2 \ \text{CO}_2 = \int P dV = 0.5 (P_1 + P_2) \ (V_2 - V_1) \\ &= [(300 + 220) / 2] \ (1.703 - 2) \ \frac{144}{778} = -14.29 \text{ Btu} \\ &1Q_2 \ \text{CO}_2 = mC_{V0} (T_2 - T_1) + 1W_2 = 0.156 \times 2.862 (77 - 400) - 14.29 = -158.5 \text{ Btu} \\ &1Q_2 \ \text{Al} = mC \ (T_2 - T_1) = 8 \times 0.21 (77 - 400) = -542.6 \text{ Btu} \\ &System: \ \text{CO}_2 + \text{Al} \\ &1Q_2 = -542.6 - 158.5 = -701.14 \text{ Btu} \\ &\Delta S_{\text{SYST}} = m_{\text{CO}_2} (s_2 - s_1)_{\text{CO}_2} + m_{\text{AL}} (s_2 - s_1)_{\text{AL}} \\ &= 2.862 [0.201 \text{ ln} \ (537 / 860) = -0.23086 - 0.79117 = -1.022 \text{ Btu} / R \\ &\Delta S_{\text{SURR}} = -(1Q_2 / T_0) = + \ \frac{701.14}{537} = 1.3057 \ \text{Btu} / R \\ &\Delta S_{\text{NET}} = 1.3057 - 1.022 = +0.2837 \ \text{Btu} / R \end{split}$$

