SOLUTION MANUAL
ENGLISH UNIT PROBLEMS
CHAPTER 7

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FUNDAMENTALS

of

Thermodynamics

Sixth Edition

CHAPTER 7

This problem set compared to the fifth edition chapter 7 set and the current chapter 7 SI problem set.

New	5th	SI	New	5th	SI	New	5th	SI
92	new	2	101	55	40	110	70	63
93	new	3	102	56	44	111	59	80
94	new	5	103	58	47	112	61	75
95	new	7	104	60	48	113	66	73
96	new	15	105	63	51	114	62	61
97	54	20	106	64	60	115	67	84
98	new	22	107	65	72	116	71	87
99	new	30	108	68	-	117	72	91
100	57	26	109	69	62	118	73	79mod

Concept Problems

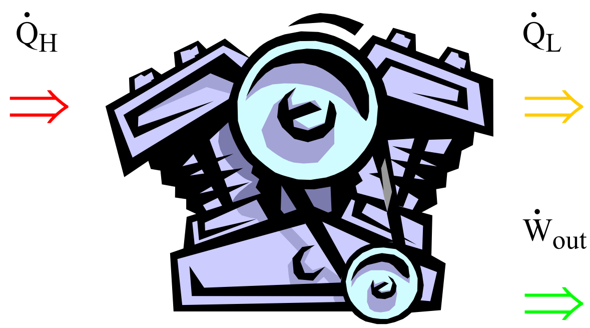
7.92E

A gasoline engine produces 20 hp using 35 Btu/s of heat transfer from burning fuel. What is its thermal efficiency and how much power is rejected to the ambient?

Conversion Table A.1: 20 hp = 20 × 2544.4/3600 Btu/s = 14.14 Btu/s

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \frac{14.14}{35} = \mathbf{0.40}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 35 - 14.14 = \mathbf{20.9\ Btu/s}$



7.93E

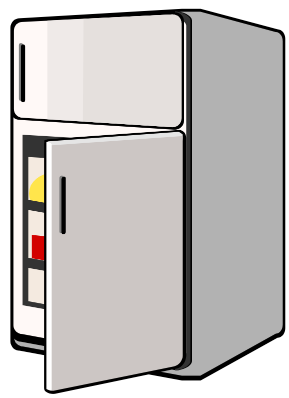
A refrigerator removes 1.5 Btu from the cold space using 1 Btu work input. How much energy goes into the kitchen and what is its coefficient of performance?

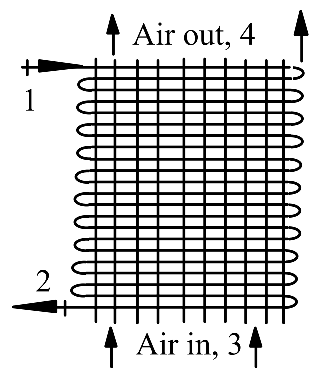
C.V. Refrigerator. The energy Q_H goes into the kitchen air.

Energy Eq.: $Q_H = W + Q_L = 1 + 1.5 = \mathbf{2.5\ btu}$

COP: $\beta = \dfrac{Q_L}{W} = 1.5 / 1 = \mathbf{1.5}$

The back side of the refrigerator has a black grille that heats the kitchen air. Other models have that at the bottom with a fan to drive the air over it.





7.94E

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 0.75 Btu/s of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

Cooling capacity: $\dot{Q}_L = \beta\ \dot{W} = 1.75 \times 0.75 = \mathbf{1.313\ Btu/s}$

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 0.75$ Btu/s, that is heating it.

7.95E

A car engine takes atmospheric air in at 70 F, no fuel, and exhausts the air at 0 F producing work in the process. What do the first and the second laws say about that?

Energy Eq.: $W = Q_H - Q_L$ = change in energy of air. **OK**

2nd law: Exchange energy with only one reservoir. **NOT OK**.

This is a violation of the statement of Kelvin-Planck.

Remark: You cannot create and maintain your own energy reservoir.

7.96E

A large stationary diesel engine produces 20 000 hp with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 1400 R and the intake is 520 R. How large a mass flow rate is that if that accounts for half the $\dot{Q}_L$? Can the exhaust flow energy be used?

Power 20 000 hp = 20 000 × 2544.4 / 3600 = 14 136 Btu/s

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \dfrac{14\,136}{0.4} = 35\,339$ Btu/s

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 35\,339 - 14\,136 = 21\,203$ Btu/s

Exhaust flow: $\frac{1}{2}\dot{Q}_L = \dot{m}_{air}(h_{1400} - h_{520})$

$$\dot{m}_{air} = \frac{1}{2}\frac{\dot{Q}_L}{h_{1400} - h_{520}} = \frac{1}{2}\frac{21\,203}{343.02 - 124.38} = \mathbf{48.49\ lbm/s}$$

Heat Engines and Refrigerators

7.97E

Calculate the thermal efficiency of the steam power plant cycle described in Problem 6.167.

Solution:

From solution to problem 6.167, 168

$$\dot{W}_{NET} = 33\,000 - 400 = 32\,600 \text{ hp} = 8.3 \times 10^7 \text{ Btu/h}$$

$$\dot{Q}_{H,tot} = \dot{Q}_{econ} + \dot{Q}_{gen}$$

$$= 4.75 \times 10^7 + 2.291 \times 10^8 = 2.766 \times 10^8 \text{ Btu/h;}$$

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{8.3 \times 10^7}{2.766 \times 10^8} = \mathbf{0.30}$$

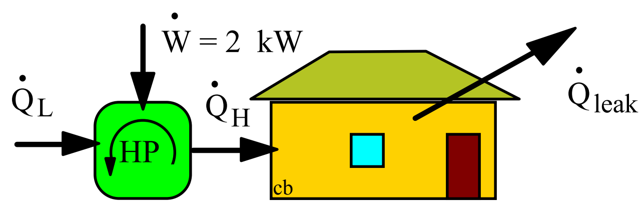
7.98E

A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 90 F, which loses energy at a rate of 10 Btu/s to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Power input: $\dot{W} = 2 \text{ kW} = 2 \times 2544.4 / 3600 = 1.414 \text{ Btu/s}$

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ Btu/s}$

Definition of COP: $\beta = \text{COP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{1.414} = \mathbf{7.07}$



7.99E

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.5 lbm liquid water at 50 F. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} = P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. F.7.1 : h_1 = 18.05 btu/lbm, Tbl. F.7.4 : h_2 = - 143.34 kJ/kg

$${}_1Q_2 = 0.5(-143.34 - 18.05) = -80.695 \text{ Btu}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -{}_1Q_2/\beta = 80.695/3.5 = \mathbf{23.06\ Btu}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

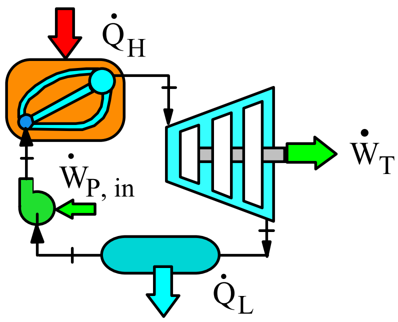
$$\Delta t = W/\dot{W} = (23.06 \times 1055)/750 = \mathbf{32.4\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

7.100E

In a steam power plant 1000 Btu/s is added at 1200 F in the boiler, 580 Btu/s is taken out at 100 F in the condenser and the pump work is 20 Btu/s. Find the plant thermal efficiency. Assume the same pump work and heat transfer to the boiler as given, how much turbine power could be produced if the plant were running in a Carnot cycle?

Solution:



CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{W}_T = 1000 + 20 - 580 = 440 \text{ Btu/s}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H} = \frac{420}{1000} = \mathbf{0.42}$$

$$\eta_{carnot} = \dot{W}_{net} / \dot{Q}_H = 1 - T_L/T_H = 1 - \frac{100 + 459.67}{1200 + 459.67} = 0.663$$

$$\dot{W}_T - \dot{W}_{P,in} = \eta_{carnot}\dot{Q}_H = 663 \text{ Btu/s} \quad \Rightarrow \quad \dot{W}_T = \mathbf{683\ \frac{Btu}{s}}$$

Carnot Cycles and Absolute T

7.101E

Calculate the thermal efficiency of a Carnot-cycle heat engine operating between reservoirs at 920 F and 110 F. Compare the result with that of Problem 7.97.

Solution:

$T_H = 920$ F , $\quad T_L = 110$ F

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{110 + 459.67}{920 + 459.67} = \mathbf{0.587} \quad \text{(about twice 7.97: 0.3)}$$

7.102E

A car engine burns 10 lbm of fuel (equivalent to addition of Q_H) at 2600 R and rejects energy to the radiator and the exhaust at an average temperature of 1300 R. If the fuel provides 17 200 Btu/lbm what is the maximum amount of work the engine can provide?

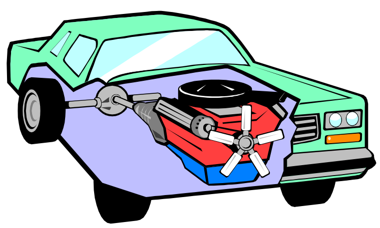
Solution:

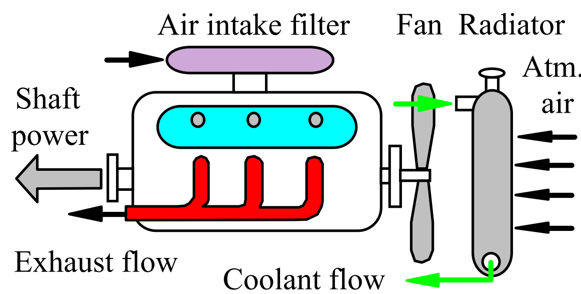
A heat engine $Q_H = m\, q_{fuel} = 10 \times 17200 = 170\ 200$ Btu

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{1300}{2600} = 0.5$$

$$W = \eta\, Q_H = 0.5 \times 170\ 200 = \mathbf{85\ 100\ Btu}$$





7.103E

An air-conditioner provides 1 lbm/s of air at 60 F cooled from outside atmospheric air at 95 F. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\, \Delta h \cong \dot{m}\, C_p\, \Delta T = 1 \times 0.24 \times (95 - 60) = 8.4 \text{ Btu/s}$$

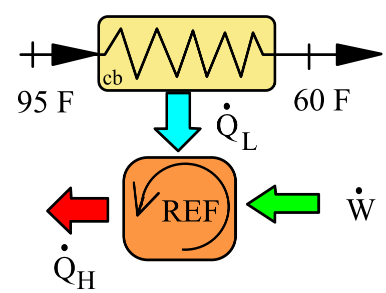
Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{8.4}{14.8} = \mathbf{0.57\ Btu/s}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 40$ F and $T_H = 110$ F are more likely; secondly

$$\beta < \beta_{carnot}$$



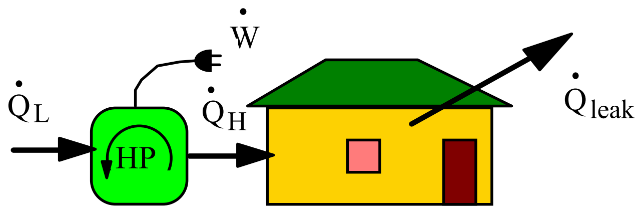
7.104E

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 68 F at all times. When the ambient temperature outside drops to 15 F, the rate at which heat is lost from the house is estimated to be 80000 Btu/h. What is the minimum electrical power required to drive the heat pump?

Solution:

Minimum power if we assume a Carnot cycle

$\dot{Q}_H = \dot{Q}_{leak} = 80\ 000$ Btu/h



$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{527.7}{53} = 9.957$$

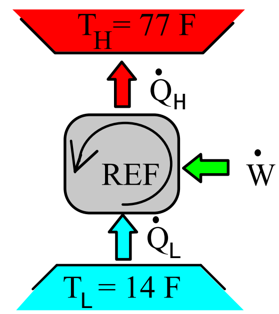
$$\Rightarrow \dot{W}_{IN} = 80\ 000 / 9.957 = 8035 \text{ Btu/h} = \mathbf{2.355\ kW}$$

7.105E

An inventor has developed a refrigeration unit that maintains the cold space at 14 F, while operating in a 77 F room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

Assume Carnot cycle then



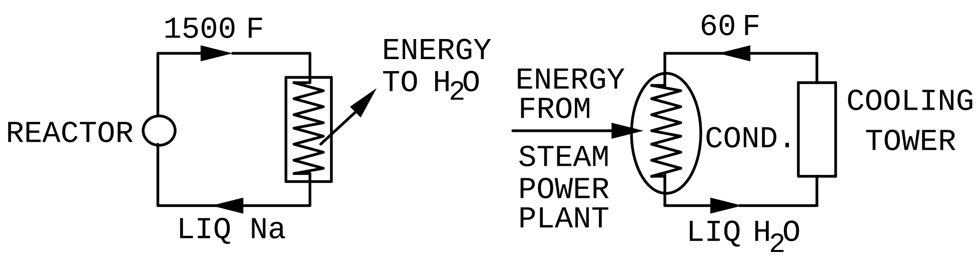
$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{14 + 459.67}{77 - 14} = 7.5$$

$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

7.106E

Liquid sodium leaves a nuclear reactor at 1500 F and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 60 F. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



T_H = 1500 F = 1960 R, T_L = 60 F = 520 R

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1960 - 520}{19860} = 0.735$$

It might be misleading to use 1500 F as the value for T_H, since there is not a supply of energy available at a constant temperature of 1500 F (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 1500 F.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 60 F, and leaves the condenser at some higher temperature.

⇒ The water does not provide for condensing steam at a constant temperature of 60 F.

7.107E

A house is heated by an electric heat pump using the outside as the low-temperature reservoir. For several different winter outdoor temperatures, estimate the percent savings in electricity if the house is kept at 68 F instead of 75 F. Assume that the house is losing energy to the outside directly proportional to the temperature difference as $\dot{Q}_{loss} = K(T_H - T_L)$.

Solution:

Heat Pump $\dot{Q}_{LOSS} \propto (T_H - T_L)$

Max Perf. $\dfrac{\dot{Q}_H}{\dot{W}_{in}} = \dfrac{T_H}{T_H - T_L} = \dfrac{K(T_H - T_L)}{\dot{W}_{in}}, \qquad \dot{W}_{in} = \dfrac{K(T_H - T_L)^2}{T_H}$

A: T_{H_A} = 75 F = 534.7 R B: T_{H_B} = 68 F = 527.7 R

T_L, F	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
-10	13.512	11.529	14.7 %
10	7.902	6.375	19.3 %
30	3.787	2.736	27.8 %
50	1.169	0.614	47.5 %

7.108E

Refrigerant-22 at 180 F, x = 0.1 flowing at 4 lbm/s is brought to saturated vapor in a constant-pressure heat exchanger. The energy is supplied by a heat pump with a low temperature of 50 F. Find the required power input to the heat pump.

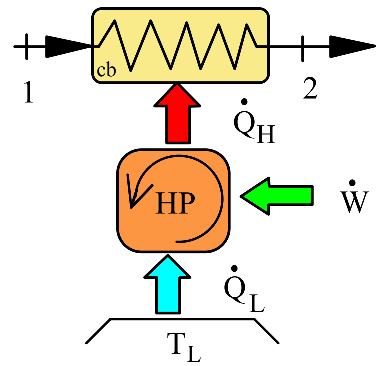
Solution:

C.V. Heat exchanger

$\dot{m}_1 = \dot{m}_2$;

$\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Assume a Carnot heat pump, T_H = 640 R, T_L = 510 R



$$\beta' = \frac{\dot{Q}_H}{\dot{W}} = \frac{T_H}{T_H - T_L} = 4.923$$

Table F.9.1:

$$h_1 = h_f + x_1 h_{fg} = 68.5 + 0.1 \times 41.57 = 72.66 \text{ Btu/lbm},$$

$$h_2 = h_g = 110.07 \text{ Btu/lbm}$$

Energy equation for line 1-2:

$$\dot{Q}_H = \dot{m}_{R\text{-}12}(h_2 - h_1) = 4\ (110.07 - 72.66) = 149.64 \text{ Btu/s}$$

$$\dot{W} = \frac{\dot{Q}_H}{\beta'} = \frac{149.64}{4.923} = \mathbf{30.4\ Btu/s}$$

7.109E

A heat engine has a solar collector receiving 600 Btu/h per square foot inside which a transfer media is heated to 800 R. The collected energy powers a heat engine which rejects heat at 100 F. If the heat engine should deliver 8500 Btu/h what is the minimum size (area) solar collector?

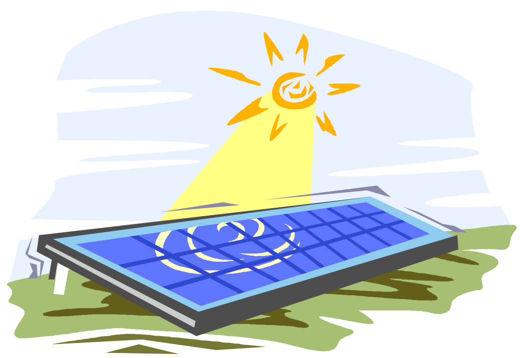
Solution:

$T_H = 800$ R $\qquad\qquad T_L = 100 + 459.67 = 560$ R

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{560}{800} = 0.30$$

$$\dot{W} = \eta\, \dot{Q}_H \quad => \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{8500}{0.30} = 28\ 333 \text{ Btu/h}$$

$$\dot{Q}_H = 600\ A \quad => \quad A = \frac{\dot{Q}_H}{600} = \mathbf{47\ ft^2}$$



7.110E

Six-hundred pound-mass per hour of water runs through a heat exchanger, entering as saturated liquid at 250 F and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 60 F. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger

$\dot{m}_1 = \dot{m}_2$; $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

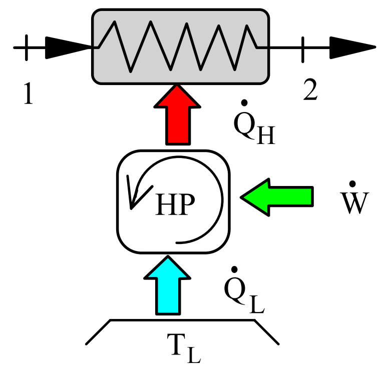


Table F.7.1: $h_1 = 218.58$ Btu/lbm

$h_2 = 1164.19$ Btu/lbm

$$\dot{Q}_H = \frac{600}{3600}(1164.19 - 218.58) = 157.6 \text{ Btu/s}$$

Assume a Carnot heat pump, $T_H = 250$ F = 710 R.

$$\beta = \dot{Q}_H/\dot{W} = \frac{T_H}{T_H - T_L} = \frac{710}{190} = 3.737$$

$$\dot{W} = \dot{Q}_H/\beta = 157.6/3.737 = \mathbf{42.2\ Btu/s}$$

Finite ΔT Heat Transfer

7.111E

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance that is one third of the theoretical maximum and it is mechanically pulled by the engine. How much fuel energy should you spend extra to remove 1 Btu at 60 F when the ambient is at 95 F?

Solution:

Air conditioner

$$\beta = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

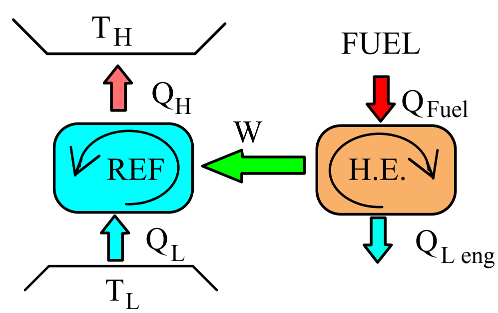
$$\beta_{actual} = \beta / 3 = 4.93$$

$$W = Q_L / \beta = 1 / 4.93 = 0.203 \text{ Btu}$$

Work from engine

$$W = \eta_{eng} \, Q_{fuel} = 0.203 \text{ Btu}$$

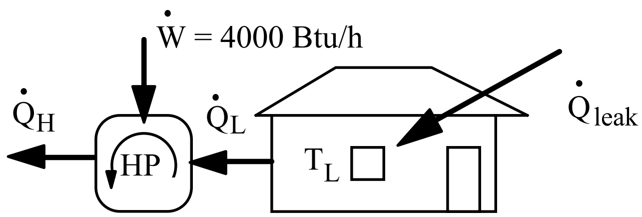
$$Q_{fuel} = W / \eta_{eng} = \frac{0.203}{0.35} = \mathbf{0.58\ Btu}$$



7.112E

A heat pump cools a house at 70 F with a maximum of 4000 Btu/h power input. The house gains 2000 Btu/h per degree temperature difference to the ambient and the heat pump coefficient of performance is 60% of the theoretical maximum. Find the maximum outside temperature for which the heat pump provides sufficient cooling.

Solution:



Here:
$T_L = T_{house}$
$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\dot{Q}_{leak} = 2000\,(T_{amb} - T_{house}) = \dot{Q}_L$$

which must be removed by the heat pump.

$\beta' = \dot{Q}_H / \dot{W} = 1 + \dot{Q}_L / \dot{W} = 0.6\,\beta'_{carnot} = 0.6\,T_{amb} / (T_{amb} - T_{house})$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})$:

$$(T_{amb} - T_{house}) + 2000\,(T_{amb} - T_{house})^2 / \dot{W} = 0.6\,T_{amb}$$

Since $T_{house} = 529.7$ R and $\dot{W} = 4000$ Btu/h it follows

$$T_{amb}^2 - 1058.6\,T_{amb} + 279522.7 = 0$$

Solving => $T_{amb} = \mathbf{554.5\ R = 94.8\ F}$

7.113E

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures estimate the percent savings in electricity if the house is kept at 77 F instead of 68 F. Assume that the house is gaining energy from the outside directly proportional to the temperature difference.

Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

Max Perf. $\dfrac{\dot{Q}_L}{\dot{W}_{in}} = \dfrac{T_L}{T_H - T_L} = \dfrac{K(T_H - T_L)}{\dot{W}_{in}}, \quad \dot{W}_{in} = \dfrac{K(T_H - T_L)^2}{T_L}$

A: T_{L_A} = 68 F = 527.7 R   B: T_{L_B} = 77 F = 536.7 R

T_H, F	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
115	4.186	2.691	35.7 %
105	2.594	1.461	43.7 %
95	1.381	0.604	56.3 %

7.114E

A thermal storage is made with a rock (granite) bed of 70 ft^3 which is heated to 720 R using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 520 R. The rock bed therefore cools down and as it reaches 520 R the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

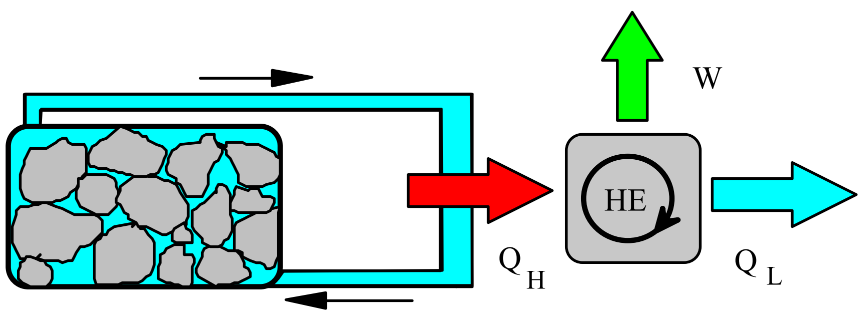
$$u_2 - u_1 = q = C\,\Delta T = 0.21\ (720 - 520) = 42 \text{ Btu/lbm}$$

$$m = \rho V = 172 \times 70 = 12040 \text{ lbm}; \quad Q = mq = \mathbf{505\ 680\ Btu}$$

To get the efficiency assume a Carnot cycle device

$\eta = 1 - T_o / T_H = 1 - 520/720 = \mathbf{0.28}$ at the beginning of process

$\eta = 1 - T_o / T_H = 1 - 520/520 = \mathbf{0}$ at the end of process

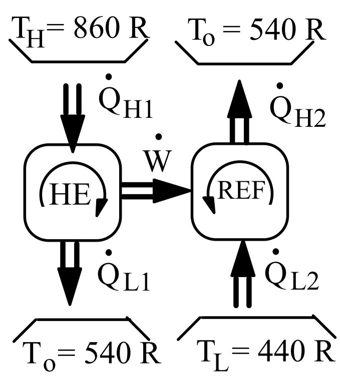


Review Problems

7.115E

We wish to produce refrigeration at −20 F. A reservoir is available at 400 F and the ambient temperature is 80 F, as shown in Fig. P7.84. Thus, work can be done by a cyclic heat engine operating between the 400 F reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 400 F reservoir to the heat transferred from the −20 F reservoir, assuming all processes are reversible.

Solution: Equate the work from the heat engine to the refrigerator.



$$W = Q_{H1}\left(\frac{T_H - T_O}{T_H}\right)$$

also

$$W = Q_{L2}\left(\frac{T_O - T_L}{T_L}\right)$$

$$\frac{Q_H}{Q_L} = \left(\frac{T_O - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_O}\right) = \frac{100}{440} \times \frac{860}{320} = 0.611$$

7.116E

Air in a rigid 40 ft^3 box is at 540 R, 30 $lbf/in.^2$. It is heated to 1100 R by heat transfer from a reversible heat pump that receives energy from the ambient at 540 R besides the work input. Use constant specific heat at 540 R. Since the coefficient of performance changes write $dQ = m_{air} C_v\, dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

$$\text{COP:}\quad \beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1V_1 / RT_1 = (30 \times 40 \times 144) / (540 \times 53.34) = 6.0 \text{ lbm}$$

$$dQ_H = m_{air} C_v\, dT_H = \beta'\, dW \;\cong \frac{T_H}{T_H - T_L}\; dW$$

$$=> \qquad dW = m_{air} C_v \left[\frac{T_H}{T_H - T_L}\right] dT_H$$

$${}_1W_2 = \int m_{air} C_v \left(1 - \frac{T_L}{T} \right) dT = m_{air} C_v \int \left(1 - \frac{T_L}{T} \right) dT$$

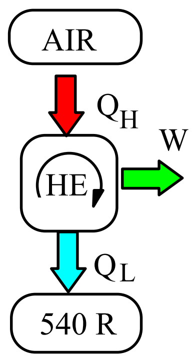
$$= m_{air} C_v \left[T_2 - T_1 - T_L \ln \frac{T_2}{T_1}\right]$$

$$= 6.0 \times 0.171 \left[1100 - 540 - 540 \ln \left(\frac{1100}{540}\right)\right] = \mathbf{180.4\ Btu}$$

7.117E

A 350-ft^3 tank of air at 80 lbf/in.^{2}, 1080 R acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 540 R. A temperature difference of 45 F between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 700 R and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:



$$T_H = T_{air} - 45 , \quad T_L = 540 \text{ R}$$

$$m_{air} = \frac{P_1 V}{RT_1} = \frac{80 \times 350 \times 144}{53.34 \times 1080} = 69.991$$

lbm

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 45}\right) dQ_H$$

$$dQ_H = -m_{air} du = -m_{air} C_v dT_{air}$$

$$W = \int dW = -m_{air} C_v \int \left[1 - \frac{T_L}{T_a - 45} \right] dT_a = -m_{air} C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 45}{T_{a1} - 45} \right]$$

$$= -69.991 \times 0.171 \times \left[700 - 1080 - 540 \ln \frac{655}{1035} \right] = \mathbf{1591\ Btu}$$

Ideal Gas Garnot Cycle

7.118E

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 7.24. The high and low temperatures are 1200 R and 600 R respectively. The heat added at the high temperature is 100 Btu/lbm and the lowest pressure in the cycle is 10 lbf/in.^{2}. Find the specific volume and pressure at all 4 states in the cycle assuming constant specific heats at 80 F.

Solution:

q_H = 100 Btu/lbm $\quad$ T_H = 1200 R

T_L = 600 R $\quad$ P_3 = 10 lbf/in.^{2}

C_v = 0.171 Btu/lbm R ; $\quad$ R = 53.34 ft-lbf/lbm-R

The states as shown in figure 7.21

1: 1200 R , $\quad$ 2: 1200 R, $\quad$ 3: 10 psi, 600 R $\quad$ 4: 600 R

$v_3 = RT_3 / P_3 = 53.34 \times 600 /(10 \times 144) = 22.225$ ft^3/lbm

$2 \rightarrow 3$ Eq.7.11 & C_v = constant

$=> C_v \ln (T_L / T_H) + R \ln (v_3/v_2) = 0$

$=> \ln (v_3/v_2) = - (C_v / R) \ln (T_L / T_H)$

$= - (0.171/53.34) \ln (600/1200) = 1.7288$

$=> v_2 = v_3 / \exp (1.7288) = 22.225/5.6339 = 3.9449$ ft^3/lbm

$1 \rightarrow 2$ $q_H = RT_H \ln (v_2 / v_1)$

$\ln (v_2 / v_1) = q_H /RT_H = 100 \times 778/(53.34 \times 1200) = 1.21547$

$v_1 = v_2 / \exp (1.21547) = 1.1699$ ft^3/lbm

$v_4 = v_1 \times v_3 / v_2 = 1.1699 \times 22.225/3.9449 = 6.591$ ft^3/lbm

$P_1 = RT_1 / v_1 = 53.34 \times 1200/(1.1699 \times 144) = 379.9$ psia

$P_2 = RT_2 / v_2 = 53.34 \times 1200/(3.9449 \times 144) = 112.7$ psia

$P_4 = RT_4 / v_4 = 53.34 \times 600/(6.591 \times 144) = 33.7$ psia