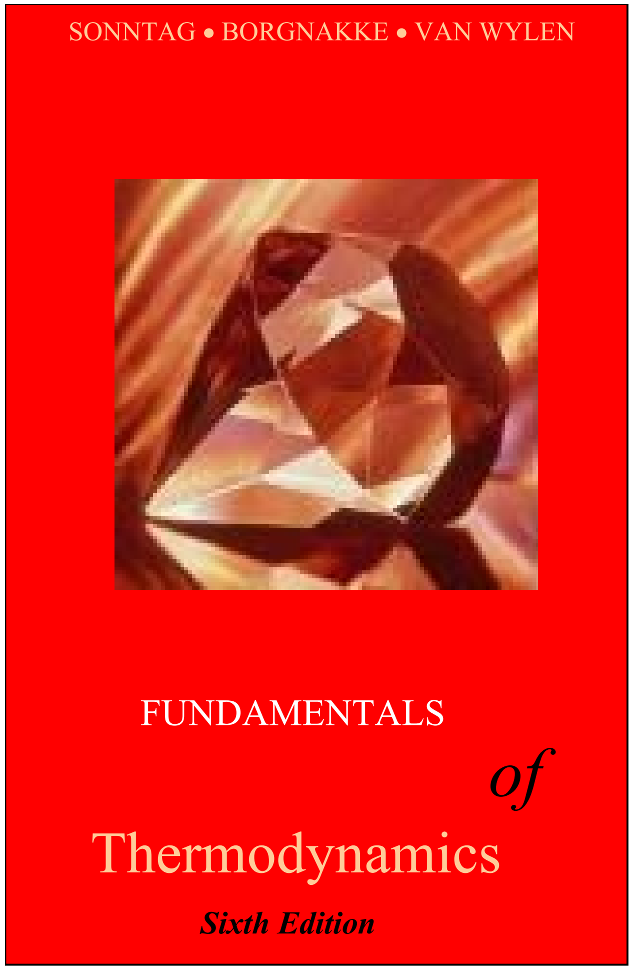
SOLUTION MANUAL
SI UNIT PROBLEMS
CHAPTER 2



CONTENT

Correspondence table
CHAPTER 2 6th edition Sonntag/Borgnakke/Wylen

The correspondence between the problem set in this sixth edition versus the problem set in the 5'th edition text. Problems that are new are marked new and those that are only slightly altered are marked as modified (mod).

Study guide problems 2.1-2.22 and 2.23-2.26 are all new problems.

New	**5th Ed.**	**New**	**5th Ed.**	**New**	**5th Ed.**
27	1	47	new	67	24
28	new	48	16	68	new
29	2	49	17	69	new
30	new	50	new	70	23
31	3	51	new	71	new
32	new	52	19	72	30
33	5	53	new	73	32
34	6	54	34	74	33
35	7	55	29	75	new
36	9	56	new	76	37
37	10	57	28 mod	77	27
38	12	58	new	78	new
39	new	59	20	79	38
40	new	60	26	80	new
41	new	61	new	81	31
42	11	62	21	82	new
43	13	63	new	83	22
44	new	64	new	84	35
45	18	65	15	85	36
46	14	66	new	86	new

English Unit Problems

New	**5th Ed.**	**SI**	**New**	**5th Ed.**	**SI**
87	new	-	97	43E	43
88	new	11	98	new	50
89	new	12	99	new	53
90	new	19	100	45E	70
91	new	20	101	46E	45
92	new	24	102	new	82
93	39E	33	103	48E	55
94	40E	-	104	new	80
95	new	47	105	47E	77
96	42E	42			

Design and Open ended problems 106-116 are from 5th edition problems 2.50-2.60

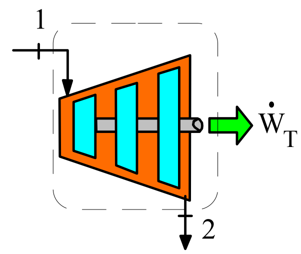
Concept-Study Guide Problems

2.1

Make a control volume around the turbine in the steam power plant in Fig. 1.1 and list the flows of mass and energy that are there.

Solution:

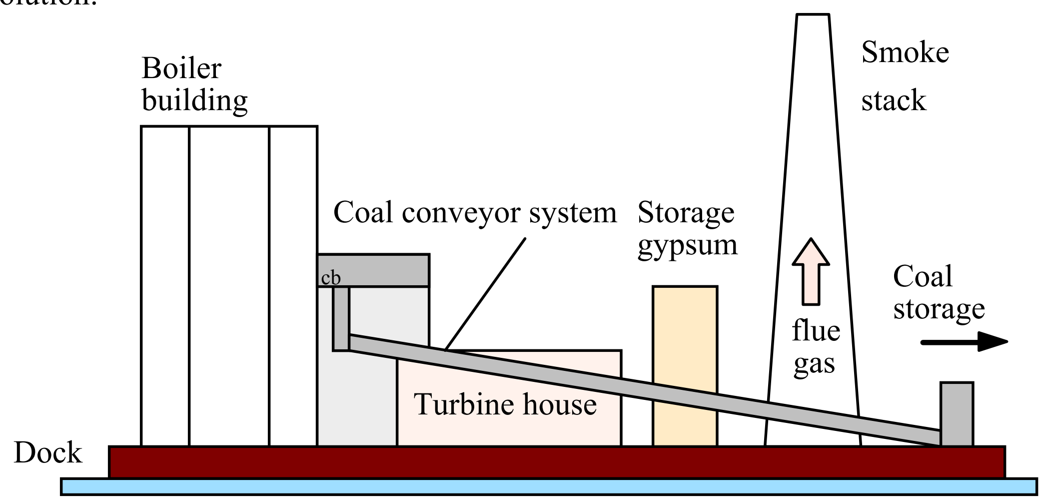
We see hot high pressure steam flowing in at state 1 from the steam drum through a flow control (not shown). The steam leaves at a lower pressure to the condenser (heat exchanger) at state 2. A rotating shaft gives a rate of energy (power) to the electric generator set.

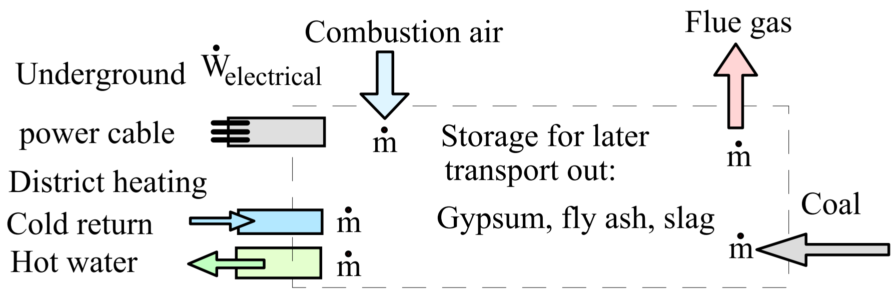


2.2

Make a control volume around the whole power plant in Figure 1.2 and with the help of Fig. 1.1 list what flows of mass and energy are in or out and any storage of energy. Make sure you know what is inside and what is outside your chosen C.V.

Solution:

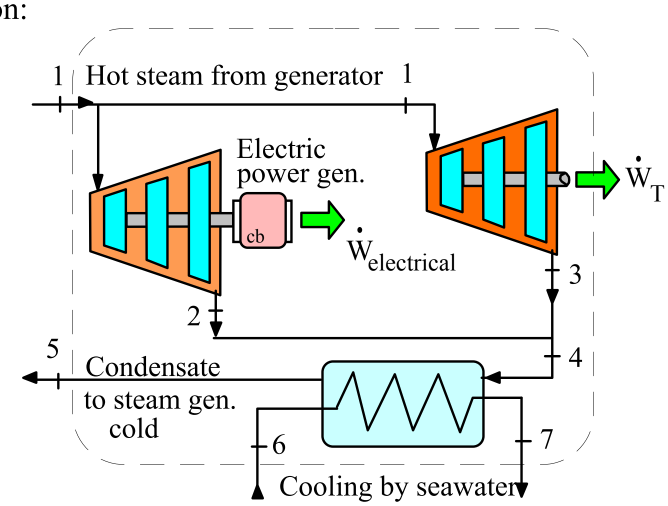




2.3

Make a control volume that includes the steam flow around in the main turbine loop in the nuclear propulsion system in Fig.1.3. Identify mass flows (hot or cold) and energy transfers that enter or leave the C.V.

Solution:



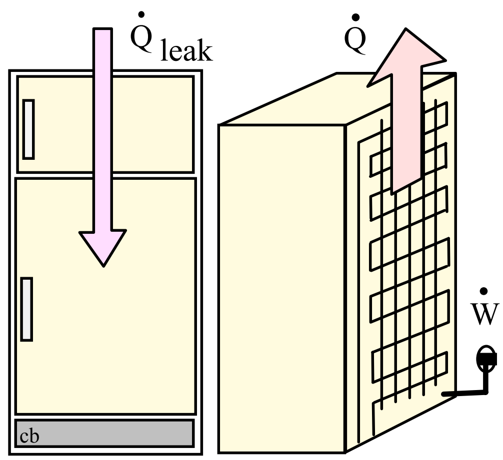
The electrical power also leaves the C.V. to be used for lights, instruments and to charge the batteries.

2.4

Take a control volume around your kitchen refrigerator and indicate where the components shown in Figure 1.6 are located and show all flows of energy transfer.

Solution:

The valve and the cold line, the evaporator, is inside close to the inside wall and usually a small blower distributes cold air from the freezer box to the refrigerator room.



The black grille in the back or at the bottom is the condenser that gives heat to the room air.

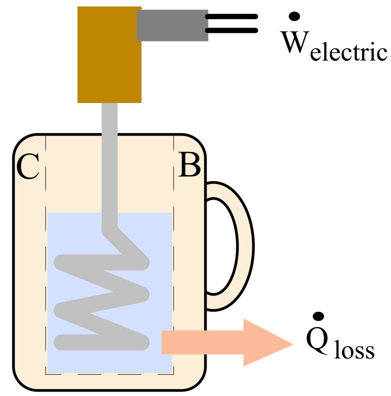
The compressor sits at the bottom.

2.5

An electric dip heater is put into a cup of water and heats it from 20^{o}C to 80^{o}C. Show the energy flow(s) and storage and explain what changes.

Solution:

Electric power is converted in the heater element (an electric resistor) so it becomes hot and gives energy by heat transfer to the water. The water heats up and thus stores energy and as it is warmer than the cup material it heats the cup which also stores some energy. The cup being warmer than the air gives a smaller amount of energy (a rate) to the air as a heat loss.



2.6

Separate the list P, F, V, v, ρ, T, a, m, L, t and **V** into intensive, extensive and non-properties.

Solution:

Intensive properties are independent upon mass: P, v, ρ, T
Extensive properties scales with mass: V, m
Non-properties: F, a, L, t, **V**

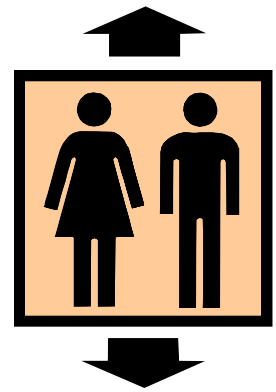
Comment: You could claim that acceleration a and velocity **V** are physical properties for the dynamic motion of the mass, but not thermal properties.

2.7

An escalator brings four people of total 300 kg, 25 m up in a building. Explain what happens with respect to energy transfer and stored energy.

Solution:

The four people (300 kg) have their potential energy raised, which is how the energy is stored. The energy is supplied as electrical power to the motor that pulls the escalator with a cable.



2.8

Water in nature exist in different phases like solid, liquid and vapor (gas). Indicate the relative magnitude of density and specific volume for the three phases.

Solution:

Values are indicated in Figure 2.7 as density for common substances. More accurate values are found in Tables A.3, A.4 and A.5

Water as solid (ice) has density of around 900 kg/m^3
Water as liquid has density of around 1000 kg/m^3
Water as vapor has density of around 1 kg/m^3 (sensitive to P and T)

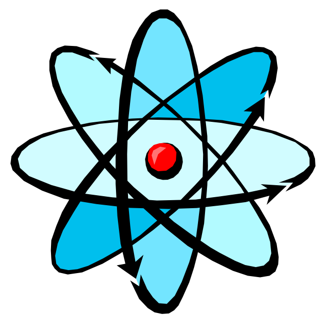
2.9

Is density a unique measure of mass distribution in a volume? Does it vary? If so, on what kind of scale (distance)?

Solution:

Density is an average of mass per unit volume and we sense if it is not evenly distributed by holding a mass that is more heavy in one side than the other. Through the volume of the same substance (say air in a room) density varies only little from one location to another on scales of meter, cm or mm. If the volume you look at has different substances (air and the furniture in the room) then it can change abruptly as you look at a small volume of air next to a volume of hardwood.

Finally if we look at very small scales on the order of the size of atoms the density can vary infinitely, since the mass (electrons, neutrons and positrons) occupy very little volume relative to all the empty space between them.



2.10

Density of fibers, rock wool insulation, foams and cotton is fairly low. Why is that?

Solution:

All these materials consists of some solid substance and mainly air or other gas. The volume of fibers (clothes) and rockwool that is solid substance is low relative to the total volume that includes air. The overall density is

$$\rho = \frac{m}{V} = \frac{m_{solid} + m_{air}}{V_{solid} + V_{air}}$$

where most of the mass is the solid and most of the volume is air. If you talk about the density of the solid only, it is high.

2.11

How much mass is there approximately in 1 L of mercury (Hg)? Atmospheric air?

Solution:

A volume of 1 L equals 0.001 m^3, see Table A.1. From Figure 2.7 the density is in the range of 10 000 kg/m^3 so we get

$$m = \rho V = 10\ 000\ kg/m^3 \times 0.001\ m^3 = \mathbf{10\ kg}$$

A more accurate value from Table A.4 is $\rho = 13\ 580\ kg/m^3$.

For the air we see in Figure 2.7 that density is about 1 kg/m^3 so we get

$$m = \rho V = 1\ kg/m^3 \times 0.001\ m^3 = \mathbf{0.001\ kg}$$

A more accurate value from Table A.5 is $\rho = 1.17\ kg/m^3$ at 100 kPa, 25°C.

2.12

Can you carry 1 m^3 of liquid water?

Solution:

The density of liquid water is about 1000 kg/m^3 from Figure 2.7, see also Table A.3. Therefore the mass in one cubic meter is

$$m = \rho V = 1000 \text{ kg/m}^3 \times 1 \text{ m}^3 = 1000 \text{ kg}$$

and we can not carry that in the standard gravitational field.

2.13

A manometer shows a pressure difference of 1 m of liquid mercury. Find ΔP in kPa.

Solution:

Hg : L = 1 m; ρ = 13 580 kg/m^3 from Table A.4 (or read Fig 2.7)

The pressure difference ΔP balances the column of height L so from Eq.2.2

$$\Delta P = \rho g L = 13\,580 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 1.0 \text{ m} \times 10^{-3} \text{ kPa/Pa}$$

$$= \mathbf{133.2\ kPa}$$

2.14

You dive 5 m down in the ocean. What is the absolute pressure there?

Solution:

The pressure difference for a column is from Eq.2.2 and the density of water is from Table A.4.

$\Delta P = \rho g H$

$= 997\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 \times 5\ \text{m}$

$= 48\,903\ \text{Pa} = 48.903\ \text{kPa}$

$P_{ocean} = P_0 + \Delta P$

$= 101.325 + 48.903$

$=$ **150 kPa**

2.15

What pressure difference does a 10 m column of atmospheric air show?

Solution:

The pressure difference for a column is from Eq.2.2

$$\Delta P = \rho g H$$

So we need density of air from Fig.2.7, $\rho = 1.2 \text{ kg/m}^3$

$$\Delta P = 1.2 \text{ kg/m}^3 \times 9.81 \text{ ms}^{-2} \times 10 \text{ m} = 117.7 \text{ Pa} = \mathbf{0.12 \text{ kPa}}$$

2.16

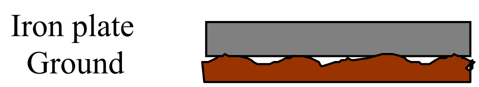
The pressure at the bottom of a swimming pool is evenly distributed. Suppose we look at a cast iron plate of 7272 kg lying on the ground with an area of 100 m^2. What is the average pressure below that? Is it just as evenly distributed?

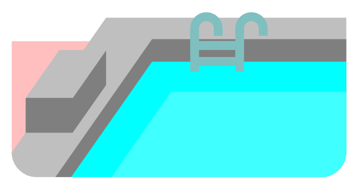
Solution:
The pressure is force per unit area from page 25:

$$P = F/A = mg/A = 7272 \text{ kg} \times (9.81 \text{ m/s}^2) / 100 \text{ m}^2 = \mathbf{713.4\ Pa}$$

The iron plate being cast can be reasonable plane and flat, but it is stiff and rigid. However, the ground is usually uneven so the contact between the plate and the ground is made over an area much smaller than the 100 m^2. Thus the local pressure at the contact locations is much larger than the quoted value above.

The pressure at the bottom of the swimming pool is very even due to the ability of the fluid (water) to have full contact with the bottom by deforming itself. This is the main difference between a fluid behavior and a solid behavior.





2.17

A laboratory room keeps a vacuum of 0.1 kPa. What net force does that put on the door of size 2 m by 1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside} A - P_{inside} A = \Delta P\ A = 0.1\ \text{kPa} \times 2\ \text{m} \times 1\ \text{m} = \mathbf{200\ N}$$

Remember that kPa is kN/m^2.

2.18

A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside} A – P_{outside} A = \Delta P\ A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P\ A = mg$$

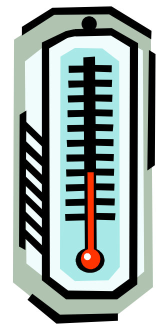
$$\Delta P = mg/A = (1000 \text{ kg} \times 9.807 \text{ m/s}^2)/100 \text{ m}^2 = \mathbf{98 \text{ Pa} = 0.098 \text{ kPa}}$$

Remember that kPa is kN/m^2.

2.19

What is a temperature of $-5^{o}C$ in degrees Kelvin?

Solution:



The offset from Celsius to Kelvin is 273.15 K, so we get

$$T_K = T_C + 273.15 = -5 + 273.15$$
$$= \mathbf{268.15\ K}$$

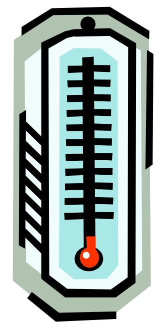
2.20

What is the smallest temperature in degrees Celsuis you can have? Kelvin?

Solution:

The lowest temperature is absolute zero which is at zero degrees Kelvin at which point the temperature in Celsius is negative

$$T_K = 0 \text{ K} = -273.15 \,^{o}\text{C}$$



2.21

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases 10^{o}C how much deeper does a 1 m layer of water become?

Solution:

The density change for a change in temperature of 10^{o}C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of

$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $m = \rho V = \rho AH$, then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \mathbf{0.005\ m}$$

barely measurable.

2.22

Convert the formula for water density in problem 21 to be for T in degrees Kelvin.

Solution:

$$\rho = 1008 - T_C/2 \quad [kg/m^3]$$

We need to express degrees Celsius in degrees Kelvin

$$T_C = T_K - 273.15$$

and substitute into formula

$$\rho = 1008 - T_C/2 = 1008 - (T_K - 273.15)/2 = 1144.6 - T_K/2$$

Properties and units

2.23

A steel cylinder of mass 2 kg contains 4 L of liquid water at 25^{o}C at 200 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of the water

Solution:

Density of steel in Table A.3: $\rho = 7820\ kg/m^3$

Volume of steel: $V = m/\rho = \dfrac{2\ kg}{7820\ kg/m^3} = 0.000\ 256\ m^3$

Density of water in Table A.4: $\rho = 997\ kg/m^3$

Mass of water: $m = \rho V = 997\ kg/m^3 \times 0.004\ m^3 = 3.988\ kg$

Total mass: $m = m_{steel} + m_{water} = 2 + 3.988 =$ **5.988 kg**

Total volume: $V = V_{steel} + V_{water} = 0.000\ 256 + 0.004$

$= \mathbf{0.004\ 256\ \ m^3 = 4.26\ L}$

2.24

An apple "weighs" 80 g and has a volume of 100 cm^3 in a refrigerator at 8^{o}C. What is the apple density? List three intensive and two extensive properties of the apple.

Solution:

$$\rho = \frac{m}{V} = \frac{0.08}{0.0001}\ \frac{kg}{m^3} = 800\ \frac{kg}{m^3}$$

Intensive

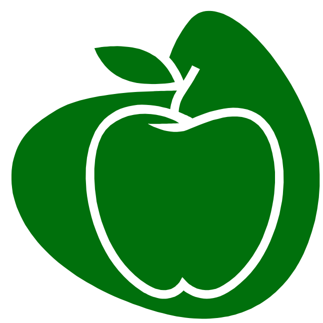
$$\rho = 800\ \frac{kg}{m^3}; \qquad v = \frac{1}{\rho} = 0.001\ 25\ \frac{m^3}{kg}$$

$$T = 8°C; \qquad P = 101\ kPa$$

Extensive

$$m = 80\ g = 0.08\ kg$$

$$V = 100\ cm^3 = 0.1\ L = 0.0001\ m^3$$

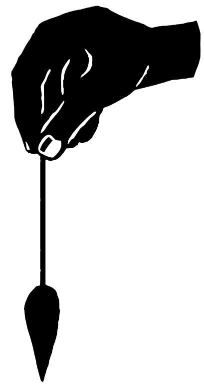


2.25

One kilopond (1 kp) is the weight of 1 kg in the standard gravitational field. How many Newtons (N) is that?

$F = ma = mg$

$1\ \text{kp} = 1\ \text{kg} \times 9.807\ \text{m/s}^2 = \mathbf{9.807\ N}$



2.26

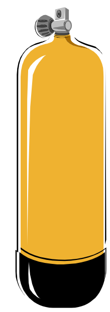
A pressurized steel bottle is charged with 5 kg of oxygen gas and 7 kg of nitrogen gas. How many kmoles are in the bottle?

Table A2 : $M_{O2} = 31.999$; $M_{N2} = 28.013$

$$n_{O2} = m_{O2} / M_{O2} = \frac{5}{31.999} = 0.15625 \text{ kmol}$$

$$n_{O2} = m_{N2} / M_{N2} = \frac{7}{28.013} = 0.24988 \text{ kmol}$$

$$n_{tot} = n_{O2} + n_{N2} = 0.15625 + 0.24988 = \mathbf{0.406\ kmol}$$



Force and Energy

2.27

The "standard" acceleration (at sea level and 45° latitude) due to gravity is 9.80665 m/s^2. What is the force needed to hold a mass of 2 kg at rest in this gravitational field ? How much mass can a force of 1 N support ?

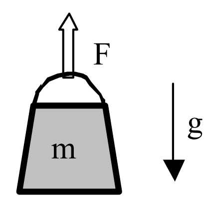
Solution:

$ma = 0 = \sum F = F - mg$

$F = mg = 2 \times 9.80665 =$ **19.613 N**

$F = mg$ =>

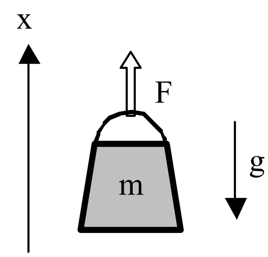
$m = F/g = 1 / 9.80665 =$ **0.102 kg**



2.28

A force of 125 N is applied to a mass of 12 kg in addition to the standard gravitation. If the direction of the force is vertical up find the acceleration of the mass.

Solution:



$$F_{up} = ma = F - mg$$

$$a = \frac{F - mg}{m} = \frac{F}{m} - g = \frac{125}{12} - 9.807$$

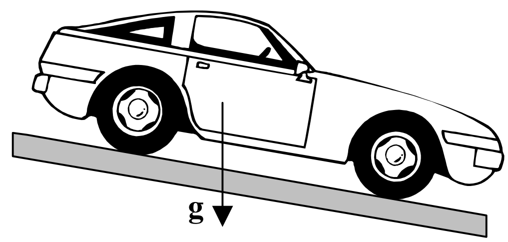
$= \mathbf{0.61\ ms^{-2}}$

2.29

A model car rolls down an incline with a slope so the gravitational "pull" in the direction of motion is one third of the standard gravitational force (see Problem 2.1). If the car has a mass of 0.45 kg find the acceleration.

Solution:

$$ma = \sum F = mg / 3$$
$$a = mg / 3m = g/3$$
$$= 9.80665 / 3 = \mathbf{3.27\ m/s^2}$$



This acceleration does not depend on the mass of the model car.

2.30

When you move up from the surface of the earth the gravitation is reduced as g = $9.807 - 3.32 \times 10^{-6}\, z$, with *z* as the elevation in meters. How many percent is the weight of an airplane reduced when it cruises at 11 000 m?

Solution:

$g_o = 9.807\ ms^{-2}$

$g_H = 9.807 - 3.32 \times 10^{-6} \times 11\,000 = 9.7705\ ms^{-2}$

$W_o = m\, g_o \;;\; W_H = m\, g_H$

$$W_H/W_o = g_H/g_o = \frac{9.7705}{9.807} = 0.9963$$

Reduction = 1 – 0.9963 = 0.0037 or **0.37%**

i.e. we can neglect that for most application

2.31

A car drives at 60 km/h and is brought to a full stop with constant deceleration in 5 seconds. If the total car and driver mass is 1075 kg find the necessary force.

Solution:

Acceleration is the time rate of change of velocity.

$$a = \frac{d\mathbf{V}}{dt} = \frac{60 \times 1000}{3600 \times 5} = 3.333 \text{ m/s}^2$$

$ma = \sum F$;

$F_{net} = ma = 1075 \text{ kg} \times 3.333 \text{ m/s}^2 =$ **3583 N**

2.32

A car of mass 1775 kg travels with a velocity of 100 km/h. Find the kinetic energy. How high should it be lifted in the standard gravitational field to have a potential energy that equals the kinetic energy?

Solution:

Standard kinetic energy of the mass is

$$KIN = ½\ m\ \mathbf{V}^2 = ½ \times\ 1775\ kg \times \left(\frac{100 \times 1000}{3600}\right)^2\ m^2/s^2$$

$$= ½ \times 1775 \times 27.778\ \ Nm = 684\ 800\ J$$

$$= \mathbf{684.8\ kJ}$$

Standard potential energy is

$$POT = mgh$$

$$h = ½\ m\ \mathbf{V}^2 / mg\ = \frac{684\ 800}{1775 \times 9.807} = \mathbf{39.3\ m}$$

2.33

A 1200-kg car moving at 20 km/h is accelerated at a constant rate of 4 m/s^2 up to a speed of 75 km/h. What are the force and total time required?

Solution:

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta\mathbf{V}}{\Delta t} \quad => \qquad \Delta t = \frac{\Delta\mathbf{V}}{a} = \frac{(75-20)\ 1000}{3600 \times 5} = \mathbf{3.82\ sec}$$

$$F = ma = 1200 \text{ kg} \times 4 \text{ m/s}^2 = \mathbf{4800\ N}$$

2.34

A steel plate of 950 kg accelerates from rest with 3 m/s^2 for a period of 10s. What force is needed and what is the final velocity?

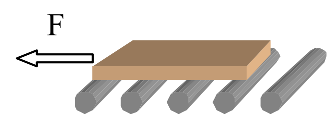
Solution:

Constant acceleration can be integrated to get velocity.

$$a = \frac{d\mathbf{V}}{dt} \quad => \quad \int d\mathbf{V} = \int a\ dt \quad => \quad \Delta\mathbf{V} = a\ \Delta t$$

$$\Delta\mathbf{V} = a\ \Delta t = 3\ \text{m/s}^2 \times 10\ \text{s} = 30\ \text{m/s}$$

$$=> \quad \mathbf{V} = \mathbf{30\ m/s}$$



$$F = ma = 950\ \text{kg} \times 3\ \text{m/s}^2 = \mathbf{2850\ N}$$

2.35

A 15 kg steel container has 1.75 kilomoles of liquid propane inside. A force of 2 kN now accelerates this system. What is the acceleration?

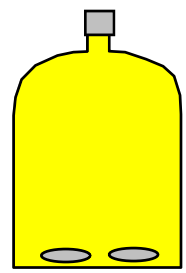
Solution:

The molecular weight for propane is M = 44.094 from Table A.2. The force must accelerate both the container mass and the propane mass.

$$m = m_{steel} + m_{propane} = 15 + (1.75 \times 44.094) = 92.165 \text{ kg}$$

$$ma = \sum F \quad \Rightarrow \quad a = \sum F / m$$

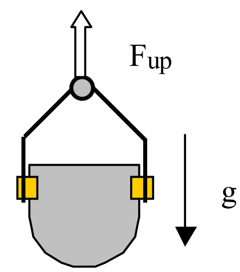
$$a = \frac{2000 \text{ N}}{92.165 \text{ kg}} = \mathbf{21.7\ m/s^2}$$



2.36

A bucket of concrete of total mass 200 kg is raised by a crane with an acceleration of 2 m/s^2 relative to the ground at a location where the local gravitational acceleration is 9.5 m/s^2. Find the required force.

Solution:



$$F = ma = F_{up} - mg$$

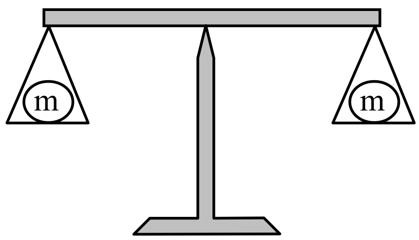
$$F_{up} = ma + mg = 200 (2 + 9.5) = \mathbf{2300\ N}$$

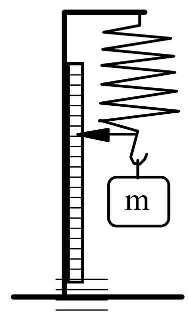
2.37

On the moon the gravitational acceleration is approximately one-sixth that on the surface of the earth. A 5-kg mass is "weighed" with a beam balance on the surface on the moon. What is the expected reading? If this mass is weighed with a spring scale that reads correctly for standard gravity on earth (see Problem 2.1), what is the reading?

Solution:

Moon gravitation is: $g = g_{earth}/6$





Beam Balance Reading is **5 kg**
This is mass comparison

Spring Balance Reading is in kg units
Force comparison length $\propto$ F $\propto$ g

Reading will be $\mathbf{\frac{5}{6}}$ **kg**

Specific Volume

2.38

A 5 m^3 container is filled with 900 kg of granite (density 2400 kg/m^3) and the rest of the volume is air with density 1.15 kg/m^3. Find the mass of air and the overall (average) specific volume.

Solution:

$$m_{air} = \rho \, V = \rho_{air} \left(V_{tot} - \frac{m_{granite}}{\rho} \right)$$

$$= 1.15 \left[5 - \frac{900}{2400} \right] = 1.15 \times 4.625 = \mathbf{5.32\ kg}$$

$$v = \frac{V}{m} = \frac{5}{900 + 5.32} = \mathbf{0.005\ 52\ m^3/kg}$$

Comment: Because the air and the granite are not mixed or evenly distributed in the container the overall specific volume or density does not have much meaning.

2.39

A tank has two rooms separated by a membrane. Room A has 1 kg air and volume 0.5 m^3, room B has 0.75 m^3 air with density 0.8 kg/m^3. The membrane is broken and the air comes to a uniform state. Find the final density of the air.

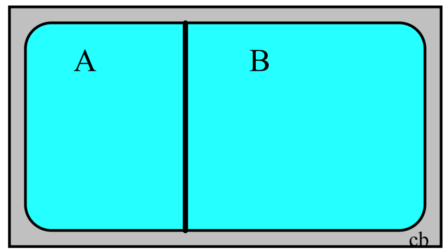
Solution:

Density is mass per unit volume

$$m = m_A + m_B = m_A + \rho_B V_B = 1 + 0.8 \times 0.75 = 1.6 \text{ kg}$$

$$V = V_A + V_B = 0.5 + 0.75 = 1.25 \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{1.6}{1.25} = \mathbf{1.28\ kg/m^3}$$



2.40

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4. Find the average specific volume and density of the masses when you exclude air mass and volume.

Solution:

Specific volume and density are ratios of total mass and total volume.

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\,\rho_{liq} = 0.2 \times 997 = 199.4 \text{ kg}$$

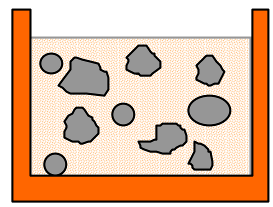
$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} = 400 + 200 + 199.4 = 799.4 \text{ kg}$$

$$V_{stone} = mv = m/\rho = 400/\,2750 = 0.1455 \text{ m}^3$$

$$V_{sand} = mv = m/\rho = 200/\,1500 = 0.1333 \text{ m}^3$$

$$V_{TOT} = V_{stone} + V_{sand} + V_{liq}$$

$$= 0.1455 + 0.1333 + 0.2 = 0.4788 \text{ m}^3$$



$$v = V_{TOT} / m_{TOT} = 0.4788/799.4 = \mathbf{0.000599\ m^3/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 799.4/0.4788 = \mathbf{1669.6\ kg/m^3}$$

2.41

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4 and use air density of 1.1 kg/m^3. Find the average specific volume and density of the 1 m^3 volume.

Solution:

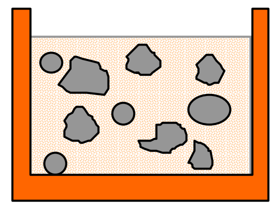
Specific volume and density are ratios of total mass and total volume.

$$V_{stone} = mv = m/\rho = 400/\ 2750 = 0.1455\ m^3$$

$$V_{sand} = mv = m/\rho = 200/\ 1500 = 0.1333\ m^3$$

$$V_{air} = V_{TOT} - V_{stone} - V_{sand} - V_{liq}$$

$$= 1 - 0.1455 - 0.1333 - 0.2 = 0.5212\ m^3$$



$$m_{air} = V_{air}/v_{air} = V_{air}\ \rho_{air} = 0.5212 \times 1.1 = 0.573\ kg$$

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\ \rho_{liq} = 0.2 \times 997 = 199.4\ kg$$

$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} + m_{air}$$

$$= 400 + 200 + 199.4 + 0.573 \approx 800\ kg$$

$$v = V_{TOT} / m_{TOT} = 1/800 = \mathbf{0.00125\ m^3/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 800/1 = \mathbf{800\ kg/m^3}$$

2.42

One kilogram of diatomic oxygen (O_2 molecular weight 32) is contained in a 500-L tank. Find the specific volume on both a mass and mole basis (v and $\bar{v}$).

Solution:

From the definition of the specific volume

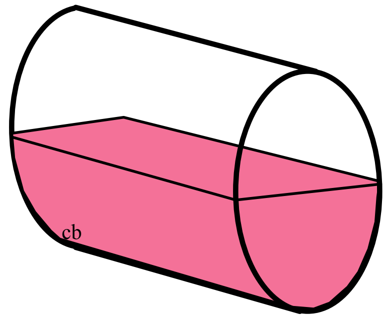
$$v = \frac{V}{m} = \frac{0.5}{1} = \mathbf{0.5\ m^3/kg}$$

$$\bar{v} = \frac{V}{n} = \frac{V}{m/M} = M\ v = 32 \times 0.5 = \mathbf{16\ m^3/kmol}$$

2.43

A 15-kg steel gas tank holds 300 L of liquid gasoline, having a density of 800 kg/m^3. If the system is decelerated with 6 m/s^2 what is the needed force?

Solution:



$$m = m_{tank} + m_{gasoline}$$

$$= 15 \text{ kg} + 0.3 \text{ m}^3 \times 800 \text{ kg/m}^3$$

$$= 255 \text{ kg}$$

$$F = ma = 255 \text{ kg} \times 6 \text{ m/s}^2$$

$$= \mathbf{1530\ N}$$

Pressure

2.44

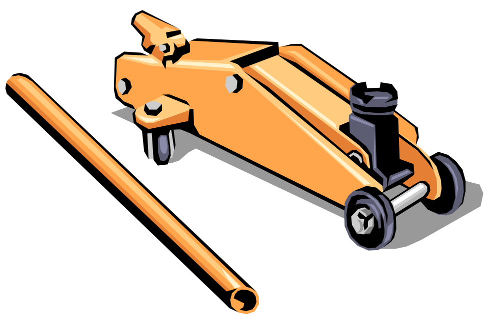
A hydraulic lift has a maximum fluid pressure of 500 kPa. What should the piston-cylinder diameter be so it can lift a mass of 850 kg?

Solution:

With the piston at rest the static force balance is

$$F\uparrow = P\,A = F\downarrow = mg$$
$$A = \pi\, r^2 = \pi\, D^2/4$$
$$PA = P\,\pi\, D^2/4 = mg \quad \Rightarrow \quad D^2 = \frac{4mg}{P\,\pi}$$

$$D = 2\sqrt{\frac{mg}{P\pi}} = 2\sqrt{\frac{850 \times 9.807}{500\,\pi \times 1000}} = \mathbf{0.146\ m}$$



2.45

A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Fig. P2.45. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?

Solution:

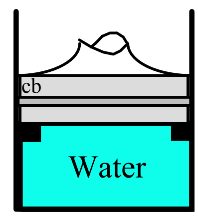
The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $F\uparrow = F\downarrow = PA = m_p g + P_0 A$

Now solve for P (divide by 1000 to convert to kPa for 2nd term)

$$P = P_0 + \frac{m_p g}{A} = 100 \text{ kPa} + \frac{100 \times 9.80665}{0.01 \times 1000}$$

$$= 100 \text{ kPa} + 98.07 \text{ kPa} = \mathbf{198\ kPa}$$



2.46

A vertical hydraulic cylinder has a 125-mm diameter piston with hydraulic fluid inside the cylinder and an ambient pressure of 1 bar. Assuming standard gravity, find the piston mass that will create a pressure inside of 1500 kPa.

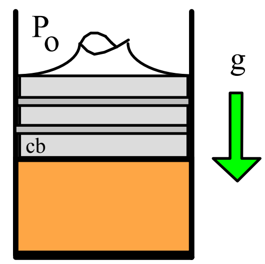
Solution:

Force balance:

$$F\uparrow = PA = F\downarrow = P_0A + m_pg;$$

$$P_0 = 1 \text{ bar} = 100 \text{ kPa}$$

$$A = (\pi/4)\, D^2 = (\pi/4) \times 0.125^2 = 0.01227 \text{ m}^2$$



$$m_p = (P - P_0)\,\frac{A}{g} = (\ 1500 - 100\ ) \times 1000 \times \frac{0.01227}{9.80665} = \mathbf{1752\ kg}$$

2.47

A valve in a cylinder has a cross sectional area of 11 cm^2 with a pressure of 735 kPa inside the cylinder and 99 kPa outside. How large a force is needed to open the valve?

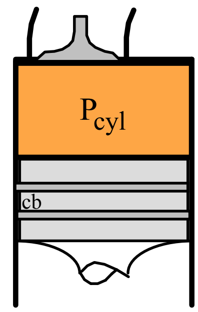
$$F_{net} = P_{in}A - P_{out}A$$

$$= (735 - 99)\ \text{kPa} \times 11\ \text{cm}^2$$

$$= 6996\ \text{kPa cm}^2$$

$$= 6996 \times \frac{\text{kN}}{\text{m}^2} \times 10^{-4}\ \text{m}^2$$

$$= \mathbf{700\ N}$$



2.48

A cannon-ball of 5 kg acts as a piston in a cylinder of 0.15 m diameter. As the gun-powder is burned a pressure of 7 MPa is created in the gas behind the ball. What is the acceleration of the ball if the cylinder (cannon) is pointing horizontally?

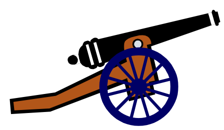
Solution:

The cannon ball has 101 kPa on the side facing the atmosphere.

$$ma = F = P_1 \times A - P_0 \times A = (P_1 - P_0) \times A$$

$$= (7000 - 101) \text{ kPa} \times \pi (0.15^2 /4) \text{ m}^2 = 121.9 \text{ kN}$$

$$a = \frac{F}{m} = \frac{121.9 \text{ kN}}{5 \text{ kg}} = \mathbf{24\ 380\ m/s^2}$$



2.49

Repeat the previous problem for a cylinder (cannon) pointing 40 degrees up relative to the horizontal direction.

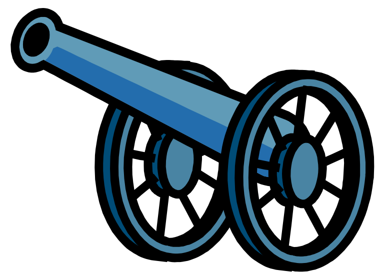
Solution:

$$ma = F = (P_1 - P_0) A - mg \sin 40^0$$

$$ma = (7000 - 101) \text{ kPa} \times \pi \times (0.15^2 / 4) \text{ m}^2 - 5 \times 9.807 \times 0.6428 \text{ N}$$

$$= 121.9 \text{ kN} - 31.52 \text{ N} = 121.87 \text{ kN}$$

$$a = \frac{F}{m} = \frac{121.87 \text{ kN}}{5 \text{ kg}} = \mathbf{24\ 374\ m/s^2}$$



2.50

A large exhaust fan in a laboratory room keeps the pressure inside at 10 cm water relative vacuum to the hallway. What is the net force on the door measuring 1.9 m by 1.1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside}\, A - P_{inside}\, A = \Delta P \times A$$
$$= 10 \text{ cm } H_2O \times 1.9 \text{ m} \times 1.1 \text{ m}$$
$$= 0.10 \times 9.80638 \text{ kPa} \times 2.09 \text{ m}^2$$
$$= \mathbf{2049\ N}$$

Table A.1: 1 m H_2O is 9.80638 kPa and kPa is kN/m^2.

2.51

What is the pressure at the bottom of a 5 m tall column of fluid with atmospheric pressure 101 kPa on the top surface if the fluid is

a) water at 20°C b) glycerine 25°C or c) light oil

Solution:

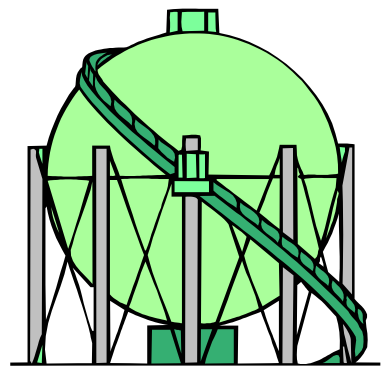
Table A.4: $\rho_{H2O} = 997$ kg/m^3; $\rho_{Glyc} = 1260$ kg/m^3; $\rho_{Oil} = 910$ kg/m^3

$\Delta P = \rho g h$ $P = P_{top} + \Delta P$

a) $\Delta P = \rho g h = 997\times 9.807\times 5 = 48887.9$ Pa
$P = 101 + 48.99 =$ **149.9 kPa**

b) $\Delta P = \rho g h = 1260\times 9.807\times 5 = 61784$ Pa
$P = 101 + 61.8 =$ **162.8 kPa**

c) $\Delta P = \rho g h = 910\times 9.807\times 5 = 44622$ Pa
$P = 101 + 44.6 =$ **145.6 kPa**



2.52

The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?

Solution:

Force acting on the mass by the gravitational field

$$F\downarrow = ma = mg = 740 \times 9.80665 = 7256.9 \text{ N}$$

Force balance: $F\uparrow = (P - P_0)\ A = F\downarrow \quad => \quad P = P_0 + F\downarrow / A$

$$A = \pi\ D^2\ (1 / 4) = 0.031416 \text{ m}^2$$

$$P = 101 + 7256.9 / (0.031416 \times 1000) = \mathbf{332\ kPa}$$

2.53

A 2.5 m tall steel cylinder has a cross sectional area of 1.5 m^2. At the bottom with a height of 0.5 m is liquid water on top of which is a 1 m high layer of gasoline. The gasoline surface is exposed to atmospheric air at 101 kPa. What is the highest pressure in the water?

Solution:

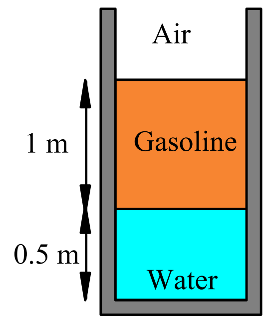
The pressure in the fluid goes up with the depth as

$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}]g$$

The densities from Table A.4 are:



$$\rho_{gasoline} = 750 \text{ kg/m}^3; \quad \rho_{water} = 997 \text{ kg/m}^3$$

$$P = 101 + [750 \times 1 + 997 \times 0.5] \frac{9.807}{1000} = \mathbf{113.2\ kPa}$$

2.54

At the beach, atmospheric pressure is 1025 mbar. You dive 15 m down in the ocean and you later climb a hill up to 250 m elevation. Assume the density of water is about 1000 kg/m^3 and the density of air is 1.18 kg/m^3. What pressure do you feel at each place?

Solution:

$$\Delta P = \rho g h$$

$$P_{ocean} = P_0 + \Delta P = 1025 \times 100 + 1000 \times 9.81 \times 15$$

$$= 2.4965 \times 10^5 \text{ Pa} = \mathbf{250\ kPa}$$

$$P_{hill} = P_0 - \Delta P = 1025 \times 100 - 1.18 \times 9.81 \times 250$$

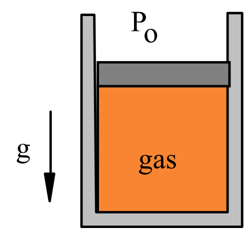
$$= 0.99606 \times 10^5 \text{ Pa} = \mathbf{99.61\ kPa}$$

2.55

A piston, m_P= 5 kg, is fitted in a cylinder, A = 15 cm^2, that contains a gas. The setup is in a centrifuge that creates an acceleration of 25 m/s^2 in the direction of piston motion towards the gas. Assuming standard atmospheric pressure outside the cylinder, find the gas pressure.

Solution:

Force balance: $F\uparrow = F\downarrow = P_0A + m_pg = PA$



$$P = P_0 + \frac{m_pg}{A}$$

$$= 101.325 + \frac{5 \times 25}{1000 \times 0.0015} \frac{\text{kPa kg m/s}^2}{\text{Pa m}^2}$$

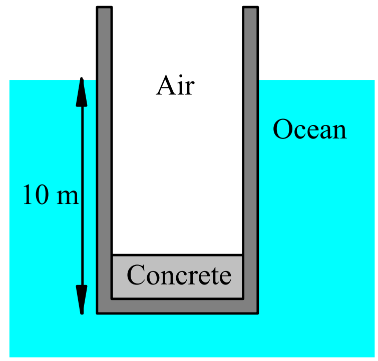
= **184.7 kPa**

2.56

A steel tank of cross sectional area 3 m^2 and 16 m tall weighs 10 000 kg and it is open at the top. We want to float it in the ocean so it sticks 10 m straight down by pouring concrete into the bottom of it. How much concrete should I put in?

Solution:

The force up on the tank is from the water pressure at the bottom times its area. The force down is the gravitation times mass and the atmospheric pressure.



$$F\uparrow = PA = (\rho_{ocean}gh + P_0)A$$

$$F\downarrow = (m_{tank} + m_{concrete})g + P_0A$$

The force balance becomes

$$F\uparrow = F\downarrow = (\rho_{ocean}gh + P_0)A = (m_{tank} + m_{concrete})g + P_0A$$

Solve for the mass of concrete

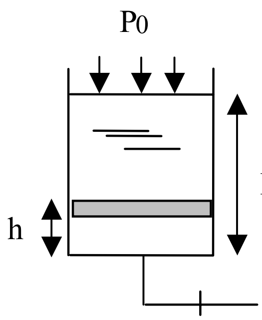
$$m_{concrete} = (\rho_{ocean}hA - m_{tank}) = 997 \times 10 \times 3 - 10\ 000 = \mathbf{19\ 910\ kg}$$

Notice: The first term is the mass of the displaced ocean water. The net force up is the weight (mg) of this mass called bouyancy, P_0 cancel.

2.57

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area *A* and total height *H*. Air is let in under the piston so it pushes up, spilling the water over the edge. Deduce the formula for the air pressure as a function of the piston elevation from the bottom, *h*.

Solution:



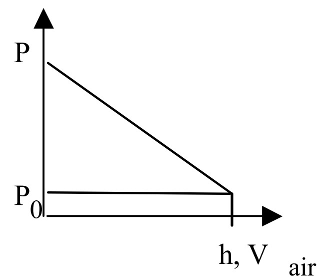
Force balance

Piston: F↑ = F↓

$$PA = P_0A + m_{H_2O}g$$

$$P = P_0 + m_{H_2O}g/A$$

$$\mathbf{P = P_0 + (H - h)\rho g}$$



Manometers and Barometers

2.58

The density of atmospheric air is about 1.15 kg/m^3, which we assume is constant. How large an absolute pressure will a pilot see when flying 1500 m above ground level where the pressure is 101 kPa.

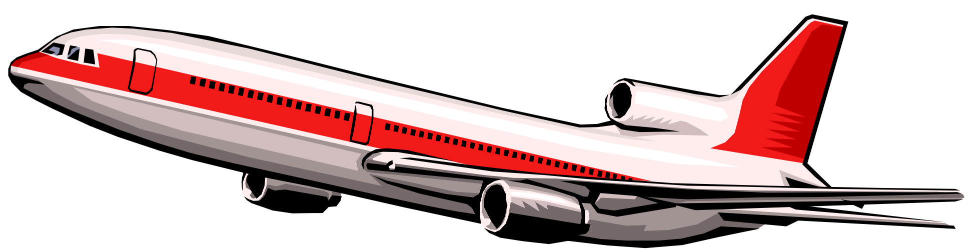
Solution:

Assume g and ρ are constant then the pressure difference to carry a column of height 1500 m is from Fig.2.10

$$\Delta P = \rho g h = 1.15 \text{ kg/m}^3 \times 9.807 \text{ ms}^{-2} \times 1500 \text{ m}$$
$$= 16\,917 \text{ Pa} = 16.9 \text{ kPa}$$

The pressure on top of the column of air is then

$$P = P_0 - \Delta P = 101 - 16.9 = \mathbf{84.1 \text{ kPa}}$$



2.59

A differential pressure gauge mounted on a vessel shows 1.25 MPa and a local barometer gives atmospheric pressure as 0.96 bar. Find the absolute pressure inside the vessel.

Solution:

Convert all pressures to units of kPa.

P_{gauge} = 1.25 MPa = 1250 kPa;

P_0 = 0.96 bar = 96 kPa

$P = P_{gauge} + P_0$ = 1250 + 96 = **1346 kPa**

2.60

Two vertical cylindrical storage tanks are full of liquid water, density 1000 kg/m^3, the top open to the atmoshere. One is 10 m tall, 2 m diameter, the other is 2.5 m tall with diameter 4 m. What is the total force from the bottom of each tank to the water and what is the pressure at the bottom of each tank?

Solution:

$$V_A = H \times \pi D^2 \times (1 / 4) = 10 \times \pi \times 2^2 \times (1 / 4) = 31.416 \text{ m}^3$$

$$V_B = H \times \pi D^2 \times (1 / 4) = 2.5 \times \pi \times 4^2 \times (1 / 4) = 31.416 \text{ m}^3$$

Tanks have the same volume, so same mass of water gives gravitational force

$$F = mg = \rho V g = 1000 \times 31.416 \times 9.80665 = 308\ 086 \text{ N}$$

this is the force the legs have to supply (assuming P_o below the bottom). Tanks have total force up from bottom as

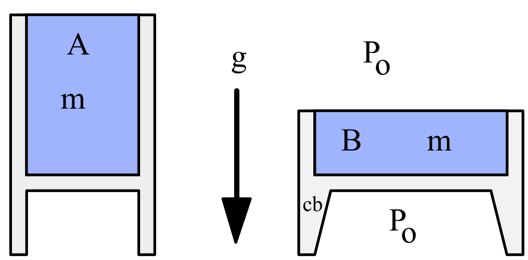
$$F_{tot\ A} = F + P_oA = 308\ 086 + 101325 \times 3.1416 = 626\ 408 \text{ N}$$

$$F_{tot\ B} = F + P_oA = 308\ 086 + 101325 \times 12.5664 = 1\ 581\ 374 \text{ N}$$

$$P_{bot} = P_o + \rho H g$$

$$P_{bot\ A} = 101 + (1000 \times 10 \times 9.80665 / 1000) = \mathbf{199\ kPa}$$

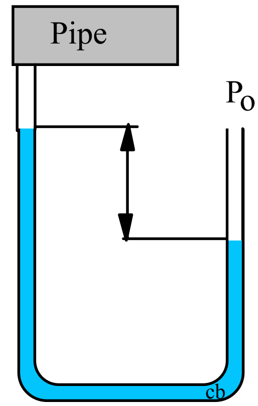
$$P_{bot\ B} = 101 + (1000 \times 2.5 \times 9.80665 / 1000) = \mathbf{125.5\ kPa}$$



2.61

Blue manometer fluid of density 925 kg/m^3 shows a column height difference of 6 cm vacuum with one end attached to a pipe and the other open to P_0 = 101 kPa. What is the absolute pressure in the pipe?

Solution:



Since the manometer shows a vacuum we have

$P_{PIPE} = P_0 - \Delta P$

$\Delta P = \rho gh = 925 \times 9.807 \times 0.06$

$= 544.3 \text{ Pa} = 0.544 \text{ kPa}$

$P_{PIPE} = 101 - 0.544 =$ **100.46 kPa**

2.62

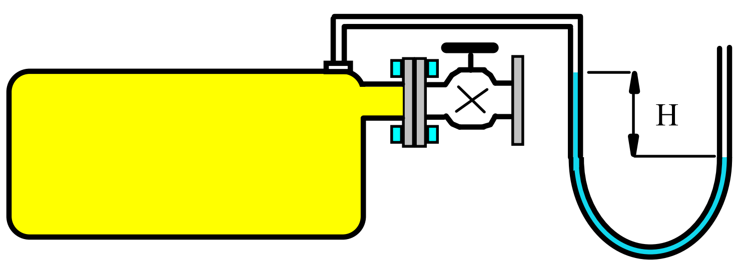
The absolute pressure in a tank is 85 kPa and the local ambient absolute pressure is 97 kPa. If a U-tube with mercury, density 13550 kg/m^3, is attached to the tank to measure the vacuum, what column height difference would it show?

Solution:

$$\Delta P = P_0 - P_{tank} = \rho g\ H$$

$$H = (P_0 - P_{tank}) / \rho g = [(97 - 85) \times 1000] / (13550 \times 9.80665)$$

$$= \mathbf{0.090\ m = 90\ mm}$$



2.63

The pressure gauge on an air tank shows 75 kPa when the diver is 10 m down in the ocean. At what depth will the gauge pressure be zero? What does that mean?

Ocean H_20 pressure at 10 m depth is

$$P_{H20} = P_o + \rho Lg = 101.3 + \frac{997 \times 10 \times 9.80665}{1000} = 199 \text{ kPa}$$

Air Pressure (absolute) in tank

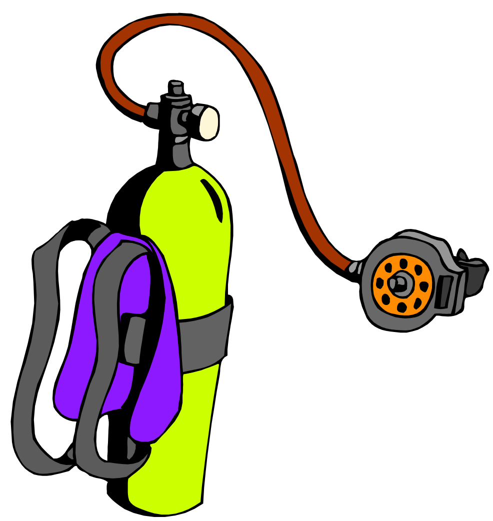
$$P_{tank} = 199 + 75 = 274 \text{ kPa}$$

Tank Pressure (gauge) reads zero at H_20 local pressure

$$274 = 101.3 + \frac{997 \times 9.80665}{1000} L$$

$$L = \mathbf{17.66\ m}$$

At this depth you will have to suck the air in, it can no longer push itself through a valve.



2.64

A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m^3. What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume the atmosphere over the ocean is at 101 kPa, then ΔP is from the 240 m column water.

$$\Delta P = \rho L g = (1030 \text{ kg/m}^3 \times 240 \text{ m} \times 9.807 \text{ m/s}^2) / 1000 = \mathbf{2424\ kPa}$$

2.65

A barometer to measure absolute pressure shows a mercury column height of 725 mm. The temperature is such that the density of the mercury is 13 550 kg/m^3. Find the ambient pressure.

Solution:

Hg : L = 725 mm = 0.725 m; ρ = 13 550 kg/m^3

The external pressure P balances the column of height L so from Fig.2.10

$P = \rho L g = 13\,550 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 0.725 \text{ m} \times 10^{-3} \text{ kPa/Pa}$

$= \mathbf{96.34\ kPa}$

2.66

An absolute pressure gauge attached to a steel cylinder shows 135 kPa. We want to attach a manometer using liquid water a day that P_{atm} = 101 kPa. How high a fluid level difference must we plan for?

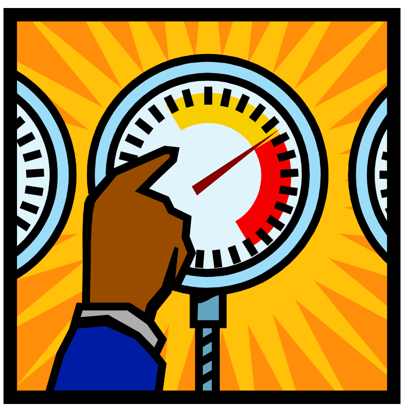
Solution:

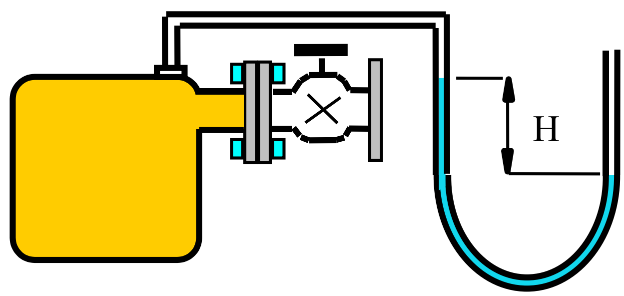
Since the manometer shows a pressure difference we have

$$\Delta P = P_{CYL} - P_{atm} = \rho L g$$

$$L = \Delta P / \rho g = \frac{(135 - 101)\ \text{kPa}}{997\ \text{kg m}^{-3} \times 10 \times 9.807\ \text{m/s}^2} \frac{1000\ \text{Pa}}{\text{kPa}}$$

= **3.467 m**





2.67

The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m^3. What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/ m^3, as manometer fluid?

Solution:

$$\Delta P = \rho_1 g h_1 = 900 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.2 \text{ m} = 1765.26 \text{ Pa} = \mathbf{1.77\ kPa}$$

$$h_{Hg} = \Delta P / (\rho_{hg}\ g) = (\rho_1\ g h_1) / (\rho_{hg}\ g) = \frac{900}{13600} \times 0.2 = \mathbf{0.0132\ m = 13.2\ mm}$$

2.68

An exploration submarine should be able to go 4000 m down in the ocean. If the ocean density is 1020 kg/m^3 what is the maximum pressure on the submarine hull?

Solution:

$$\Delta P = \rho L g = (1020\ kg/m^3 \times 4000\ m \times 9.807\ m/s^2) / 1000$$
$$= 40\ 012\ kPa \approx \mathbf{40\ MPa}$$

2.69

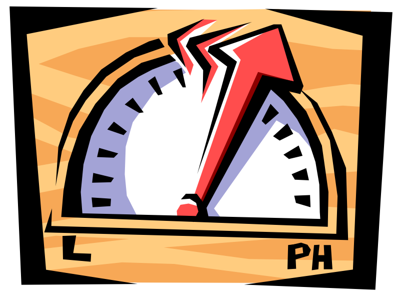
Assume we use a pressure gauge to measure the air pressure at street level and at the roof of a tall building. If the pressure difference can be determined with an accuracy of 1 mbar (0.001 bar) what uncertainty in the height estimate does that corresponds to?

Solution:

$\rho_{air} = 1.169 \text{ kg/m}^3$ from Table A.5

$\Delta P = 0.001 \text{ bar} = 100 \text{ Pa}$

$$L = \frac{\Delta P}{\rho g} = \frac{100}{1.169 \times 9.807} = \mathbf{8.72\ m}$$

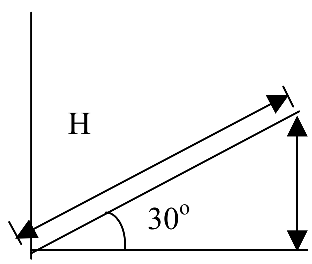


2.70

A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P2.70, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

Same height in the two sides in the direction of g.



$\Delta P = F/A = mg/A = V\rho g/A = h\rho g$

$= 0.25 \times 1000 \times 9.807 = 2452.5$ Pa

$=$ **2.45 kPa**

$h = H \times \sin 30°$

$\Rightarrow H = h/\sin 30° = 2h =$ **50 cm**

2.71

A barometer measures 760 mmHg at street level and 735 mmHg on top of a building. How tall is the building if we assume air density of 1.15 kg/m^3?

Solution:

$$\Delta P = \rho g H$$

$$H = \Delta P/\rho g = \frac{760 - 735}{1.15 \times 9.807} \frac{\text{mmHg}}{\text{kg/m}^2\text{s}^2} \frac{133.32 \text{ Pa}}{\text{mmHg}} = \mathbf{295 \ m}$$

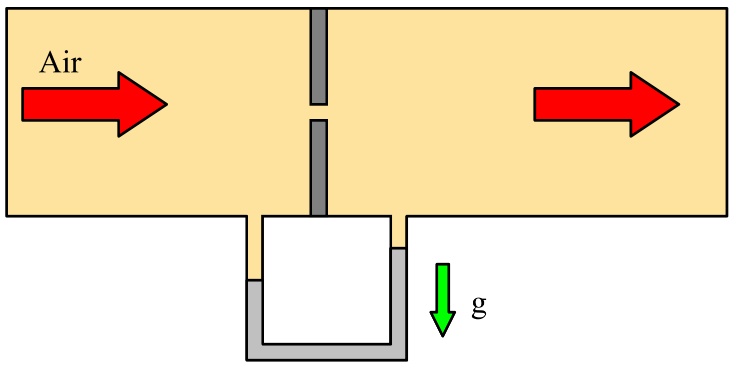
2.72

A piece of experimental apparatus is located where $g = 9.5 \text{ m/s}^2$ and the temperature is 5°C. An air flow inside the apparatus is determined by measuring the pressure drop across an orifice with a mercury manometer (see Problem 2.77 for density) showing a height difference of 200 mm. What is the pressure drop in kPa?

Solution:

$\Delta P = \rho g h$; $\rho_{Hg} = 13600 \text{ kg/m}^3$

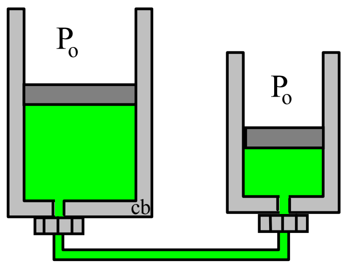
$\Delta P = 13\,600 \text{ kg/m}^3 \times 9.5 \text{ m/s}^2 \times 0.2 \text{ m} = 25840 \text{ Pa} = \mathbf{25.84 \text{ kPa}}$



2.73

Two piston/cylinder arrangements, A and B, have their gas chambers connected by a pipe. Cross-sectional areas are $A_A = 75\ cm^2$ and $A_B = 25\ cm^2$ with the piston mass in A being $m_A = 25$ kg. Outside pressure is 100 kPa and standard gravitation. Find the mass m_B so that none of the pistons have to rest on the bottom.

Solution:



Force balance for both pistons: $F\uparrow = F\downarrow$

A: $m_{PA}g + P_0A_A = PA_A$

B: $m_{PB}g + P_0A_B = PA_B$

Same P in A and B gives no flow between them.

$$\frac{m_{PA}g}{A_A} + P_0 = \frac{m_{PB}g}{A_B} + P_0$$

=> $m_{PB} = m_{PA}\ A_A/\ A_B = 25 \times 25/75 =$ **8.33 kg**

2.74

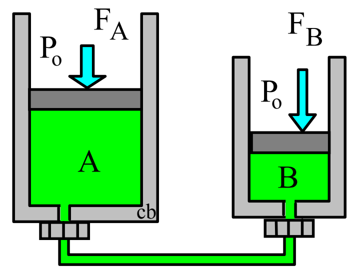
Two hydraulic piston/cylinders are of same size and setup as in Problem 2.73, but with negligible piston masses. A single point force of 250 N presses down on piston A. Find the needed extra force on piston B so that none of the pistons have to move.

Solution:

$A_A = 75\ cm^2$;

$A_B = 25\ cm^2$

No motion in connecting pipe: $P_A = P_B$



Forces on pistons balance

$$P_A = P_0 + F_A / A_A = P_B = P_0 + F_B / A_B$$

$$F_B = F_A \times \frac{A_B}{A_A} = 250 \times \frac{25}{75} = \mathbf{83.33\ N}$$

2.75

A pipe flowing light oil has a manometer attached as shown in Fig. P2.75. What is the absolute pressure in the pipe flow?

Solution:

Table A.3: $\rho_{oil} = 910 \text{ kg/m}^3$; $\rho_{water} = 997 \text{ kg/m}^3$

$$P_{BOT} = P_0 + \rho_{water}\, g\, H_{tot} = P_0 + 997 \times 9.807 \times 0.8$$
$$= P_o + 7822 \text{ Pa}$$

$$P_{PIPE} = P_{BOT} - \rho_{water}\, g\, H_1 - \rho_{oil}\, g\, H_2$$
$$= P_{BOT} - 997 \times 9.807 \times 0.1 - 910 \times 9.807 \times 0.2$$
$$= P_{BOT} - 977.7 \text{ Pa} - 1784.9 \text{ Pa}$$

$$P_{PIPE} = P_o + (7822 - 977.7 - 1784.9) \text{ Pa}$$
$$= P_o + 5059.4 \text{ Pa} = 101.325 + 5.06 = \mathbf{106.4\ kPa}$$

2.76

Two cylinders are filled with liquid water, $\rho = 1000$ kg/m^3, and connected by a line with a closed valve. A has 100 kg and B has 500 kg of water, their cross-sectional areas are $A_A = 0.1$ m^2 and $A_B = 0.25$ m^2 and the height h is 1 m. Find the pressure on each side of the valve. The valve is opened and water flows to an equilibrium. Find the final pressure at the valve location.

Solution:

$$V_A = v_{H_2O} m_A = m_A/\rho = 0.1 = A_A h_A \quad => \quad h_A = 1 \text{ m}$$

$$V_B = v_{H_2O} m_B = m_B/\rho = 0.5 = A_B h_B \quad => \quad h_B = 2 \text{ m}$$

$$P_{VB} = P_0 + \rho g(h_B + H) = 101325 + 1000 \times 9.81 \times 3 = 130\ 755 \text{ Pa}$$

$$P_{VA} = P_0 + \rho g h_A = 101325 + 1000 \times 9.81 \times 1 = 111\ 135 \text{ Pa}$$

Equilibrium: same height over valve in both

$$V_{tot} = V_A + V_B = h_2 A_A + (h_2 - H) A_B \Rightarrow h_2 = \frac{h_A A_A + (h_B + H) A_B}{A_A + A_B} = 2.43 \text{ m}$$

$$P_{V2} = P_0 + \rho g h_2 = 101.325 + (1000 \times 9.81 \times 2.43)/1000 = \mathbf{125.2\ kPa}$$

Temperature

2.77

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\ T\ \text{kg/ m}^3 \qquad T \text{ in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\ L\ g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \text{ kg/m}^3$$

$$\rho_{w} = 13595 - 2.5 \times (-15) = 13632.5 \text{ kg/m}^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7549 \text{ m}$$

$$L_{w} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7480 \text{ m}$$

$\Delta L = L_{su} - L_{w} =$ **0.0069 m = 6.9 mm**

2.78

A mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid Hg due to a change in the density, see problem 2.35. Find the relative change (%) in volume for a change in temperature from 10°C to 20°C.

Solution:

From 10°C to 20°C

At 10°C : ρ_{Hg} = 13595 – 2.5 × 10 = 13570 kg/m^3

At 20°C : ρ_{Hg} = 13595 – 2.5 × 20 = 13545 kg/m^3

The volume from the mass and density is: V = m/ρ

$$\text{Relative Change} = \frac{V_{20} - V_{10}}{V_{10}} = \frac{(m/\rho_{20}) - (m/\rho_{10})}{m/\rho_{10}}$$

$$= \frac{\rho_{10}}{\rho_{20}} - 1 = \frac{13570}{13545} - 1 = \mathbf{0.0018\ (0.18\%)}$$

2.79

Using the freezing and boiling point temperatures for water in both Celsius and Fahrenheit scales, develop a conversion formula between the scales. Find the conversion formula between Kelvin and Rankine temperature scales.

Solution:

$$T_{Freezing} = 0\ ^oC = 32\ F; \qquad T_{Boiling} = 100\ ^oC = 212\ F$$

$$\Delta T = 100\ ^oC = 180\ F \;\Rightarrow\; T_{oC} = (T_F - 32)/1.8 \quad \text{or} \quad T_F = 1.8\ T_{oC} + 32$$

For the absolute K & R scales both are zero at absolute zero.

$$T_R = 1.8 \times T_K$$

2.80

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5 \times 10^{-3}\, z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m expressed in Kelvin and in Celsius?

Solution:

For an elevation of $z = 12\,000$ m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3}\, z = \mathbf{210\ K}$$

To express that in degrees Celsius we get

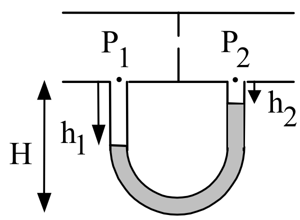
$$T_C = T - 273.15 = \mathbf{-63.15^{o}C}$$

Review Problems

2.81

Repeat problem 2.72 if the flow inside the apparatus is liquid water, $\rho \cong 1000$ kg/m^3, instead of air. Find the pressure difference between the two holes flush with the bottom of the channel. You cannot neglect the two unequal water columns.

Solution:



Balance forces in the manometer:

$$(H - h_2) - (H - h_1) = \Delta h_{Hg} = h_1 - h_2$$

$$P_1A + \rho_{H_2O}h_1gA + \rho_{Hg}(H - h_1)gA$$
$$= P_2A + \rho_{H_2O}h_2gA + \rho_{Hg}(H - h_2)gA$$

$$\Rightarrow P_1 - P_2 = \rho_{H_2O}(h_2 - h_1)g + \rho_{Hg}(h_1 - h_2)g$$

$$P_1 - P_2 = \rho_{Hg}\Delta h_{Hg}g - \rho_{H_2O}\Delta h_{Hg}g = 13600 \times 0.2 \times 9.5 - 1000 \times 0.2 \times 9.5$$

$$= 25840 - 1900 = 23940 \text{ Pa} = \mathbf{23.94\ kPa}$$

2.82

The main waterline into a tall building has a pressure of 600 kPa at 5 m elevation below ground level. How much extra pressure does a pump need to add to ensure a water line pressure of 200 kPa at the top floor 150 m above ground?

Solution:

The pump exit pressure must balance the top pressure plus the column ΔP. The pump inlet pressure provides part of the absolute pressure.

$P_{\text{after pump}} = P_{\text{top}} + \Delta P$

$\Delta P = \rho g h = 997 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times (150 + 5) \text{ m}$

$= 1\,515\,525 \text{ Pa} = 1516 \text{ kPa}$

$P_{\text{after pump}} = 200 + 1516 = 1716 \text{ kPa}$

$\Delta P_{\text{pump}} = 1716 - 600 = \mathbf{1116 \text{ kPa}}$

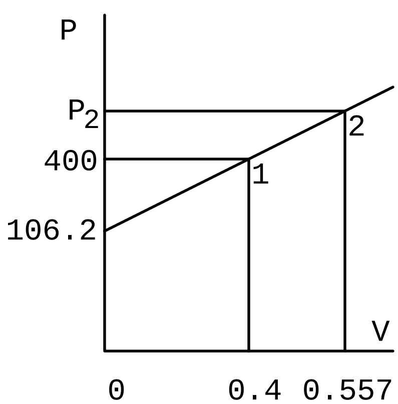
2.83

A 5-kg piston in a cylinder with diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure of 100 kPa. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 400 kPa with volume 0.4 L. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

Solution:

A linear spring has a force linear proportional to displacement. F = k x, so the equilibrium pressure then varies linearly with volume: P = a + bV, with an intersect a and a slope b = dP/dV. Look at the balancing pressure at zero volume (V -> 0) when there is no spring force $F = PA = P_oA + m_pg$ and the initial state. These two points determine the straight line shown in the P-V diagram.

Piston area = $A_p = (\pi/4) \times 0.1^2 = 0.00785\ m^2$



$$a = P_0 + \frac{m_pg}{A_p} = 100\ kPa + \frac{5 \times 9.80665}{0.00785}\ Pa$$

$= 106.2$ kPa intersect for zero volume.

$V_2 = 0.4 + 0.00785 \times 20 = 0.557$ L

$$P_2 = P_1 + \frac{dP}{dV}\Delta V$$

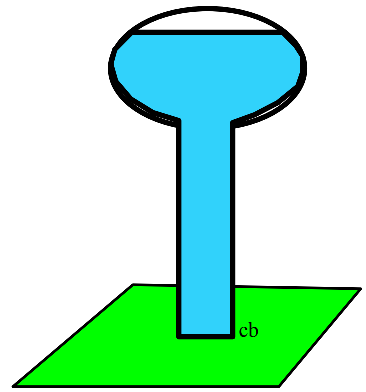
$$= 400 + \frac{(400-106.2)}{0.4 - 0}(0.557 - 0.4)$$

= **515.3 kPa**

2.84

In the city water tower, water is pumped up to a level 25 m above ground in a pressurized tank with air at 125 kPa over the water surface. This is illustrated in Fig. P2.84. Assuming the water density is 1000 kg/m^3 and standard gravity, find the pressure required to pump more water in at ground level.

Solution:



$$\Delta P = \rho \, L \, g$$

$$= 1000 \text{ kg/m}^3 \times 25 \text{ m} \times 9.807 \text{ m/s}^2$$

$$= 245\,175 \text{ Pa} = 245.2 \text{ kPa}$$

$$P_{bottom} = P_{top} + \Delta P$$

$$= 125 + 245.2$$

$$= \mathbf{370 \text{ kPa}}$$

2.85

Two cylinders are connected by a piston as shown in Fig. P2.85. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?

Solution:

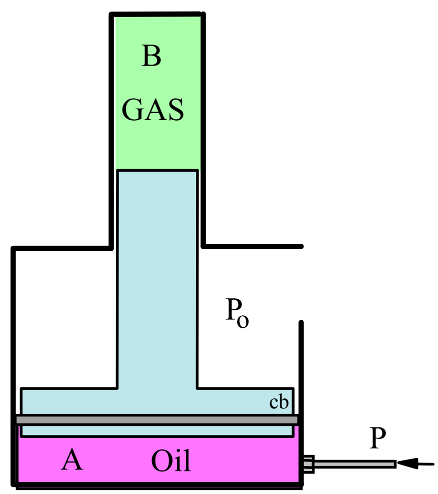
Force balance for the piston: $P_BA_B + m_pg + P_0(A_A - A_B) = P_AA_A$

$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2$; $A_B = (\pi/4)0.025^2 = 0.000\,491 \text{ m}^2$

$P_BA_B = P_AA_A - m_pg - P_0(A_A - A_B) = 500\times 0.00785 - (25 \times 9.807/1000)$

$- 100\,(0.00785 - 0.000\,491) = 2.944 \text{ kN}$

$P_B = 2.944/0.000\,491 = 5996 \text{ kPa} = \mathbf{6.0\ MPa}$



2.86

A dam retains a lake 6 m deep. To construct a gate in the dam we need to know the net horizontal force on a 5 m wide and 6 m tall port section that then replaces a 5 m section of the dam. Find the net horizontal force from the water on one side and air on the other side of the port.

Solution:

$P_{bot} = P_0 + \Delta P$

$\Delta P = \rho gh = 997 \times 9.807 \times 6 = 58\ 665 \text{ Pa} = 58.66 \text{ kPa}$

Neglect ΔP in air

$F_{net} = F_{right} - F_{left} = P_{avg} A - P_0 A$

$P_{avg} = P_0 + 0.5\ \Delta P$ Since a linear pressure variation with depth.

$F_{net} = (P_0 + 0.5\ \Delta P)A - P_0 A = 0.5\ \Delta P\ A = 0.5 \times 58.66 \times 5 \times 6 =$ **880 kN**